RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Additional Practice Questions for Chapter 7C:

Maclaurin Series

1 **(a)** Find the fifth term in the expansion of $\left(x^3-\dfrac{1}{2x}\right)^6$ in descending powers of x. **[2]**

(b) Find the term independent of x in the expansion of $\left(9x-\dfrac{1}{3\sqrt{x}}\right)^{18}$. **[3]**

(c) Find the coefficient of x^{-18} in the expansion of $\left(3x+\dfrac{1}{x^2}\right)^{12}$. **[3]**

(d) Expand $(2+3x+4x^2)^{10}$, up to the term in x^3. **[3]**

(e) The coefficient of x^2 in the expansion of $(2x+k)^6$ is equal to the coefficient of x^5 in the expansion of $(2+kx)^8$. Find k. **[3]**

(f) The expansion of $\left(1+kx\right)^n$ up to and including the term in x^3 are $1+36x+66k^2x^2+hx^3$, where n is a positive integer. Find the values of n, k and h. **[4]**

2 Express $\dfrac{2+5x+15x^2}{\left(2-x\right)\left(1+2x^2\right)}$ in partial fractions. **[2]**

Hence or otherwise, obtain the expansion of $\dfrac{2+5x+15x^2}{\left(2-x\right)\left(1+2x^2\right)}$ in ascending powers of x, giving the first five terms in the expansion. **[3]**
Find the exact set of values of x for which the expansion is valid. **[2]**

3 **MI Promo 9758/2020/PU2/P1/Q6**

(a) Using standard series from the List of Formulae, expand $\mathrm{e}^{3x}\ln\left(1+ax\right)$ as far as the term in x^3, where a is a non-zero constant. Given that there is no term in x^2, determine the coefficient of x^3. [5]

(b) Find the expansion of $\dfrac{1+3x}{\sqrt{9-x^2}}$ in ascending powers of x, up to and including the term in x^3. State the set of values of x for which the expansion is valid. [4]

4 **9740/2015/01/Q6**

(i) Write down the first three non-zero terms in the Maclaurin series for $\ln(1+2x)$, where $-\frac{1}{2} < x \le \frac{1}{2}$, simplifying the coefficients. [2]

(ii) It is given that the three terms found in part **(i)** are equal to the first three terms in the series expansion of $ax(1+bx)^c$ for small x. Find the exact values of the constants a, b and c and use these values to find the coefficient of x^4 in the expansion of $ax(1+bx)^c$, giving your answer as a simplified rational number. [6]

5 Given that $y = \dfrac{1}{\sqrt{1+2x}+\sqrt{1+x}}$ where $x > -0.5$. Show that, provided x is non-zero, then $y = \dfrac{1}{x}\left(\sqrt{1+2x}-\sqrt{1+x}\right)$. **[1]**

Hence

(i) express y as a series of ascending powers of x up to and including the term in x^2 **[4]**

(ii) show that $\dfrac{10}{\sqrt{102}+\sqrt{101}} \approx \dfrac{79407}{160000}$. **[2]**

6 Expand $\dfrac{\sqrt{1-x}}{1+2x}$ in ascending powers of x up to and including the term in x^2. **[4]**

(i) In an attempt to estimate $\sqrt{2}$, a student substituted $x = -1$ in the above expansion. Explain why this does not give a good estimate. **[1]**

(ii) By putting $x = \dfrac{1}{9}$ in the expansion, show that $\sqrt{2} \approx \dfrac{p}{1296}$, where p is a positive integer to be determined. **[2]**

7 For $n > 0$, the expansion of $(1+mx)^{-n}$, in ascending powers of x, is $1+8x+48x^2+.....$.

(i) Find the constants m and n. **[5]**

(ii) Show that the coefficient of x^{400} is in the form $a(4)^k$ where a and k are constants to be determined. **[2]**

8 **RIPromo9740/2015/Q8**

(a) **(i)** Expand $\mathrm{f}(x) = \dfrac{2}{2-x} - \dfrac{1}{(1+x)^2}$ as a series in ascending powers of x up to and including the term in x^2. **[3]**

(ii) State the equation of the tangent to the curve $y = \mathrm{f}(x)$ at the origin. **[1]**

(b) Using the standard series given in the List of Formulae (MF26) or otherwise, show that the first three non-zero terms in the Maclaurin series for $\mathrm{e}^{\sqrt{1+x}}$ can be expressed as $\mathrm{e}\left(1+px+qx^3+...\right)$ where p and q are constants to be determined. **[6]**

9 RVHS Promo 9758/2020/Q3

It is given that $\tan^{-1}\left(y\right)=1-\mathrm{e}^{-x}$.

(i) Show that $\mathrm{e}^{x}\dfrac{\mathrm{d}y}{\mathrm{d}x}=1+y^{2}$. [1]

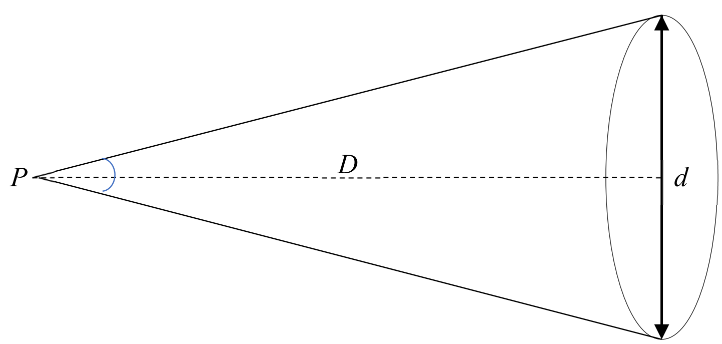
(ii) By further differentiation of the expression in part (i), find the Maclaurin series for y, up to and including the term in x^{2}. [4]

(iii) Hence, or otherwise, find the Maclaurin series for $\dfrac{1+y^{2}}{\mathrm{e}^{x}}$, up to and including the term in x. [1]

10 EJC Prelim 9758/2020/01/Q11

(a) The angular diameter of an object is the angle the object makes (subtends) as seen by an observer.

As shown in the diagram below, ϕ denotes the angular diameter (measured in radians) of a circle whose plane is perpendicular to the line between the point of view (point P) and the centre of said circle. D denotes the distance from point P to the centre of the circle and d denotes the diameter of the circle.

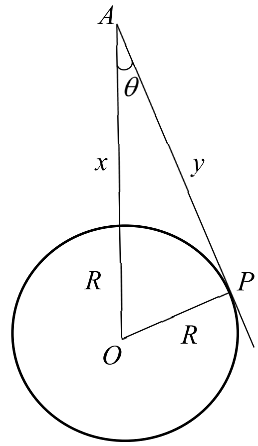


(i) Show that, if ϕ is sufficiently small, $\phi D=d$. [2]

The equation in **(i)** is often used in astronomy to estimate the diameters of stars from the angular diameter, assuming their shapes to be approximately circular.

(ii) If the angular diameter of a star is measured to be 0.00873 rad and the distance of the star from the earth is 9.46×10^{12} km, estimate the diameter of the star. [1]

(b) An astronaut A is at a large distance x km from the surface of the earth. The radius of the earth is assumed to be a constant R km. The furthest point on the earth's surface that the astronaut can see is a point P such that $AP = y$ km and the angle $OAP = \theta$, where O is the centre of the earth (see diagram).



(i) Show that $y = x\left(1+\dfrac{2R}{x}\right)^{\frac{1}{2}}$. [3]

(ii) It is given that R is small compared to x. Show that, if $\alpha = \dfrac{R}{x}$,

$\tan\theta \approx \alpha - \alpha^2 + 1.5\alpha^3$. [4]

(iii) It is also given that $\theta = 0.0345$ rad and the astronaut A is 180,000 km from the surface of the earth, find α and hence estimate the radius of the earth. Leave your answer to the nearest km. [3]

11 If $y = \ln(\cos 2x)$, prove that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = -4$.

Hence, by repeated differentiation of this result, or otherwise, obtain the Maclaurin expansion of y in ascending powers of x up to and including the term in x^4.

Using $x = \dfrac{\pi}{6}$, show that $\ln 2 \approx \dfrac{\pi^2}{36}\left(2 + \dfrac{\pi^2}{27}\right)$. **[10]**

12 Given that $y = \dfrac{2}{\sqrt{1+\sin 2x}}$, wherever it is defined, show that

$$(1+\sin 2x)\frac{\mathrm{d}y}{\mathrm{d}x} + y\cos 2x = 0.$$

Hence, by further differentiation of this result, obtain the Maclaurin expansion of y in ascending powers of x, up to the term in x^3.

By putting $x = \dfrac{\pi}{12}$, find an approximate value of $\sqrt{\dfrac{2}{3}}$, correct to 2 decimal places. **[10]**

13 RI Prelim 9758/2020/02/Q4

Given that $y = \mathrm{e}^{\tan^{-1}\left(\frac{x}{2}\right)}$, show that $\left(4+x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y.$ [2]

(i) By repeated differentiation of the above result, find the Maclaurin series for $\mathrm{e}^{\tan^{-1}\left(\frac{x}{2}\right)}$ up to and including the term in x^3. [5]

(ii) Hence find the Maclaurin series for $\dfrac{\mathrm{e}^{\tan^{-1}\left(\frac{x}{2}\right)}}{(1+x)^2}$ up to and including the term in x^2. [3]

14 ACJCPrelims9740/2006/01/Q16OR

Let $y = \ln(1 + \tan x)$

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^y + 2\mathrm{e}^{-y} - 2$. **[3]**

(ii) By differentiating this result repeatedly, show that

$$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \left(\mathrm{e}^y - 2\mathrm{e}^{-y}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \left(\mathrm{e}^y + 2\mathrm{e}^{-y}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2.$$ **[4]**

(iii) Find the series expansion of y in ascending powers of x up to and including the term in x^3. **[3]**

(iv) Show that, when x is small enough for powers above the third to be neglected,

$$\ln\left(\frac{1+\tan x}{1+x}\right) \approx \frac{1}{3}x^3.$$ **[2]**

15 NJCPrelims9740/2006/01/Q6

Given that a curve is defined by

$$(\ln y + 1)\frac{\mathrm{d}y}{\mathrm{d}x} + \tan^{-1}(2x) = 1$$

and passes through (0,1).

Find the Maclaurin Series for y, up to and including the term in x^3. **[5]**

Deduce the equation of tangent to the curve at $x = 0$. **[1]**

16 ACJCPrelims9233/2005/01/Q14

If $y = \mathrm{e}^{\cos^{-1} x}$, where $\cos^{-1} x$ denotes the principal value, show that

$$(1-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} - y = 0.$$

By further differentiation, obtain the expansion of y in ascending powers of x up to and including the term in x^3 in terms of $\mathrm{e}^{\frac{\pi}{2}}$. Hence show that for small values of x,

$$\mathrm{e}^{\cos^{-1} x} \approx \mathrm{e}^{\frac{\pi}{2}-x} + kx^3,$$

where the value of k is to be found. **[9]**

17 **TJCPrelims9740/2006/01/Q1 (modified)**

The equation $4\cos 2x + 10\sin 3x + 3 = 0$ has a root α close to zero. By using suitable approximations, find an approximate value for α. **[4]**

18 **NYJCPrelims9740/2006/01/Q1**

Show that, if x is so small that terms in x^3 and higher powers may be neglected, then

$$\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right) \approx \frac{1}{2}-x^2.$$ **[3]**

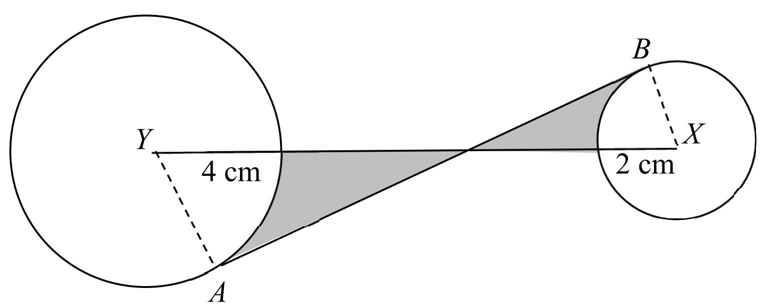
19 **NJCPrelims9740/2004/01/Q4**

The diagram below shows two circles of radius 2 cm and 4 cm, with centre X and Y respectively. The line AB is tangent to both circles and angle AYX is $\left(\frac{\pi}{3}-x\right)$ radians.

Show that the area of the shaded region is $10\left(\tan\left(\frac{\pi}{3}-x\right)-\left(\frac{\pi}{3}-x\right)\right)$.

If x is sufficiently small for x^3 and higher powers of x to be neglected, find the approximate area of the shaded region in terms of x. **[6]**

[You may use the result: $\left(1+\sqrt{3}\,x\right)^{-1} \approx 1-\sqrt{3}\,x+3x^2$].



20 **HCIPrelims9740/2021/01/Q9**

Given that $y=(\tan x+\sec x)^2$, show that $\cos x\dfrac{\mathrm{d}y}{\mathrm{d}x}=2y$. [2]

(i) By repeat differentiation, find the Maclaurin series for y up to and including the term in x^3. [4]

(ii) Using the result obtained in (i) estimate the value of $\tan 1^\circ+\sec 1^\circ$ to 4 decimal places. [2]

(iii) By expressing y in terms of sine and cosine, use the standard series in MF26 to find the series expansion for y up to and including the term in x^3. [3]

(iv) Comment on the results obtained from part **(i)** and part **(iii)**. [1]