

Name: _____ () Class: _____



華僑中學

HWA CHONG INSTITUTION

Paper D

HIGHER MATHEMATICS

Level : Secondary Three

Duration : 1 hour

Do not open this booklet until you are told to do so.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Answer **4 out of 7** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 40.

You are expected to use an electronic calculator to evaluate explicit numerical expressions. **The use of a graphing calculator is not permitted.**

You are reminded of the need for clear presentation in your answers. Omission of essential working will result in loss of marks.

This question paper consists 4 printed pages, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

Answer 4 out of 7 questions

- 1** **(a) (i)** By considering $(x^2 + 2y^2)^2$, factorise $x^4 + 4y^4$ completely. [3]
- (ii)** Hence, determine all distinct prime factors of $5^8 + 2^6$. [3]
- (b) (i)** State the range of values of $\cos x$ for $-360^\circ \leq x \leq 360^\circ$. [1]
- (ii)** Hence or otherwise, solve the equation $1 - \tan^6 x = \sec^6 x$ for $-360^\circ \leq x \leq 360^\circ$. [3]
- 2** **(a)** Simplify $\sqrt{24}(\sqrt{8+4\sqrt{3}} + \sqrt{8-4\sqrt{3}})$. [3]
- (b)** The parabola $y = -x^2 + 1$ is transformed into $3y = -\frac{1}{4}(x+1)^2 + 1$ under three successive transformations.
- (i)** Describe the transformations in order. [3]
- (ii)** Given that the area enclosed by the parabola $y = -x^2 + 1$ and the x -axis is $\frac{4}{3}$ square units. By considering **(i)**, calculate the area enclosed by $3y = -\frac{1}{4}(x+1)^2 + 1$ and the x -axis. Explain your working clearly. [4]
- 3** **(a)** A point P moves along the parabola $y^2 - 4x - 2y + 5 = 0$ so that its x -coordinate is always equal to its distance from a fixed point Q . Find the coordinates of point Q . [3]
- (b) (i)** Find the range of values of x satisfying $\frac{2x}{(x+1)(x-2)} \geq -1$. [4]
- (ii)** Hence or otherwise, solve the inequality $\frac{2e^{-x}}{(e^{-x}+1)(1-2e^{-x})} \geq -1$, leaving you answer in the exact form. [3]

- 4 (a) By using a suitable substitution, solve the equation

$$\sqrt{\frac{2x+4}{x-2}} - 3\sqrt{\frac{x-2}{2x+4}} = -2. \quad [5]$$

- (b) Prove the identity $\frac{\sin x + \cos x}{\tan x} + \frac{\sin x + \cos x}{\cot x} = \frac{1}{\sin x} + \frac{1}{\cos x}.$ [5]

- 5 A quartic polynomial $P(x) = x^4 + ax^3 + bx^2 + cx + d$ leaves a remainder of $3x - 4$ when divided by $x^2 - x - 2$ and a remainder of $x - 4$ when divided by $x^2 + x - 2.$

- (i) Prove that $P(x) - x^2 - 2x + 6$ has both factors $x^2 - x - 2$ and $x^2 + x - 2.$ [6]
(ii) Hence or otherwise, find $P(x).$ Explain your working clearly. [4]

- 6 (a) Given that $45^a = 75^b = 15^c,$ find the value of $\frac{c}{a} + \frac{c}{b}.$ [4]

- (b) Two functions f and g are defined by: $f : x \mapsto 2x^3 - 1$ and $g : x \mapsto \sqrt[3]{\frac{x+1}{2}}.$

- (i) Show that functions f and g are inverse functions. [2]

- (ii) Hence, solve the equation $\underbrace{fgfg \dots fgfg}_{2017 \text{ terms of "fg"}}(x) = g^{-1}(x).$ [4]

- 7 (i) In the Cartesian plane, a circle passes through the origin $O,$ point $X(-1,1)$ and point $Y(7,7).$ Find the equation of the circle. [5]
(ii) Calculate the greatest distance between $(2, 6)$ and a point on the circumference of the circle. Leave your answer in exact form. [2]
(iii) Two tangent lines are drawn from point $Z(-2, -8)$ to the circle. Find the size of the acute angle formed by the two tangents drawn. [3]

End of Paper