

Name: _____ () Class: _____



華 僑 中 學

HWA CHONG INSTITUTION

Paper F

INTEGRATED MATHEMATICS

Level : Secondary Three

Duration : 2 hours 30 minutes

Do not open this booklet until you are told to do so.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Answer **all** questions on foolscap paper except

Question 13 on graph paper provided and

Question 15 on the space provided on question paper.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place. For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

You are expected to use an electronic calculator to evaluate explicit numerical expressions. **The use of a graphing calculator is not permitted.**

You are reminded of the need for clear presentation in your answers. Omission of essential working will result in a loss of marks.

This question paper consists 9 printed pages, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Mensuration

Arc Length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

- 1 (a) Simplify $\frac{(2x)^3 - 27}{2x - 3} - 5$ [3]
- (b) Evaluate $9|\sqrt{5} - 2| - 4|2 - \sqrt{5}|$ without the use of calculator.
Leave your answer in exact form. [2]
- 2 (a) Solve the equation $\sqrt{3 - 2x} - x = 0$ [3]
- (b) Solve $2x^2 - 7x = 2 - \frac{1}{x}$, giving your answers in exact form. [5]
- 3 (a) Find the range of values of x for which $(x - 1)(5 - x) > 3$. [3]
- (b) Find the values of c for which the equation $4x^2 - (c + 2)x + c = 1$
has equal roots. [4]
- 4 The expression $ax^3 + x^2 + bx$ has remainder 3 when it is divided by $x + 1$. When the expression is divided by $x - 2$, the remainder is 6. Find the remainder when the expression is divided by $x - 3$. [5]
- 5 (a) Given that $4\lg(x\sqrt{y}) = 1.5 + \lg x - \lg y$, where x and y are both positive,
express, in simplest form, y in terms of x . [4]
- (b) Solve the simultaneous equations

$$\log_2 7 - \log_2(11q - 2p) = 1,$$

$$3^p = 9(27)^q.$$
 [4]

[Turn Over]

- 6 (a) Find the range of the function $f : x \mapsto 2x^2 - 6x + 7$ for the domain $0 < x < 5$. [4]

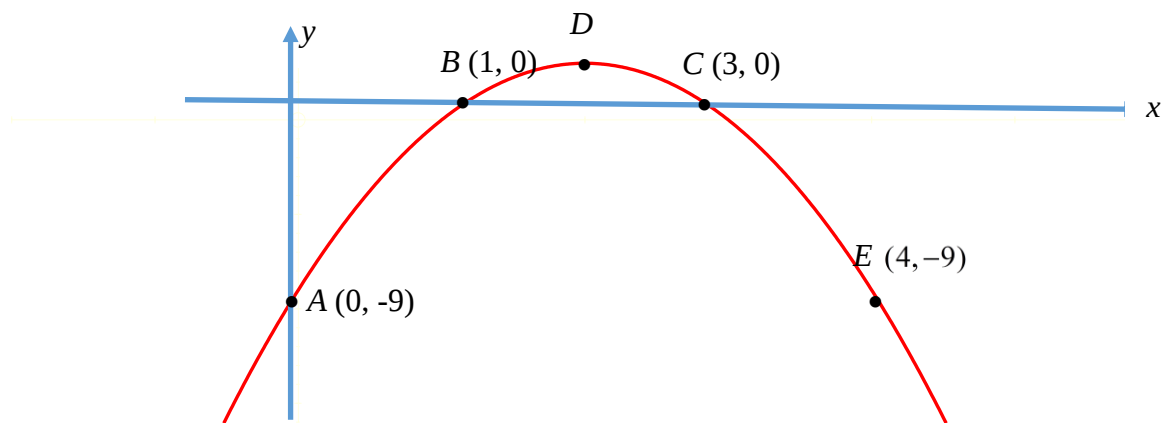
- (b) The function g is defined by $g : x \mapsto \frac{x+1}{x-2}, x \neq 2$.

Find g^{-1} , stating its domain clearly. [3]

- 7 The sketch below shows part of a quadratic graph, $y = ax^2 + bx + c$, where a , b and c are constants.

Points A , B and C have coordinates $(0, -9)$, $(1, 0)$ and $(3, 0)$ respectively.

The curve passes through the maximum point D and point E , $(4, -9)$.



- (i) Evaluate a , b , and c . [3]
- (ii) Find the coordinates of D . [2]
- 8 (i) Sketch the graph of $y = |5 - 2x| - 2$, for $0 \leq x \leq 5$, labelling the intercepts clearly. [3]
- (ii) Using your sketch in part (i) or otherwise, find the range of values of x for which
- (a) y is negative, [2]
- (b) $|y| \leq 1$. [2]

- 9 The ACCM Steel Pte Ltd is a producer of stainless steel and aluminum containers.

Table A shows the production figures for a particular day.

Material	Capacity Type	Quantity
Stainless Steel	10 – gallon	500
Stainless Steel	5 – gallon	350
Stainless Steel	1 – gallon	400
Aluminium	10 – gallon	700
Aluminium	5 – gallon	500
Aluminium	1 – gallon	850

Table B shows the amount of material required to produce each type of container.

Capacity Type	Amount of material in kg
10 – gallon	6.8
5 – gallon	3.6
1 – gallon	1.4

Table C shows the cost for each type of material.

Material	Cost
Stainless Steel	\$2.20
Aluminium	\$2.50

- (i) Write down a 2 by 3 matrix representing the data in Table A. [1]
- (ii) Use the data in Table B and matrix multiplication to determine the amount of each type of material used on that day. [2]
- (iii) Use the data in Table C and matrix multiplication to determine the total cost of the day's production, giving your answer to the nearest dollars. [2]

[Turn Over

10 The solution to this question by accurate drawing will not be accepted.

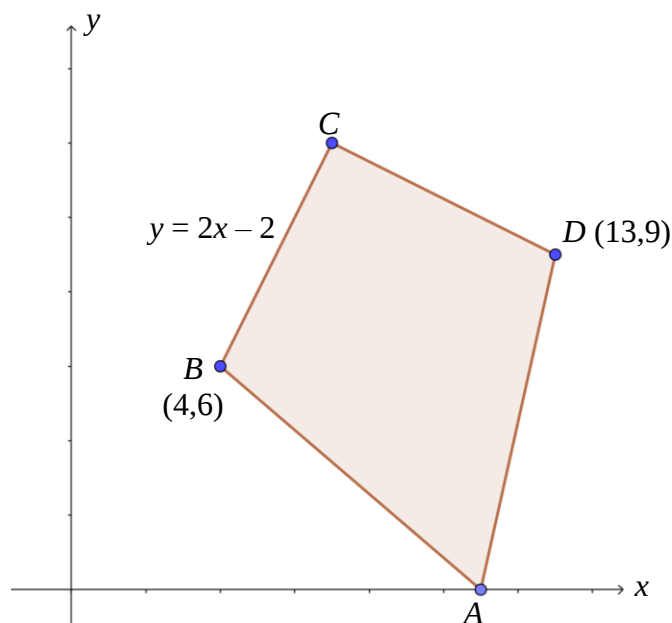
The diagram show a kite $ABCD$ with $AB = AD$ and $CB = CD$.

Point A lies on the x -axis, point B is $(4,6)$, point D is $(13,9)$ and

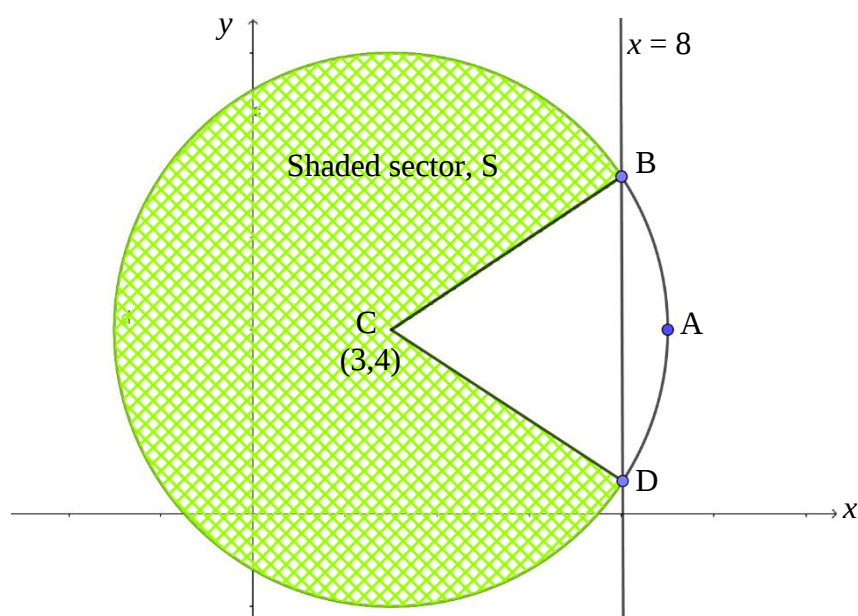
the equation of BC is $y = 2x - 2$.

Find

- (i) the gradient of BD , [2]
- (ii) Show that the equation of AC is
 $y = -3x + 33$, [3]
- (iii) the coordinates of A and of C , [4]
- (iv) the area of the kite $ABCD$ [2]



11



The figure shows a circle with centre $C (3,4)$. The line $x = 8$ intersects the circle at points B and D . Point $A, (9,4)$ is another point on the circle.

- (i) Find the equation of the circle. [2]
- (ii) Show that $\angle BCD = 1.171$ radians. [2]
- (iii) Hence or otherwise, find the area of the shaded sector. [2]

- 12 (a) (i) Sketch the graph of $y = 3\sin x + 1$ for the domain $0 \leq x \leq 2\pi$,
showing clearly the y -intercept of graph. [3]
- (ii) State the range of y . [1]
- (b) Find all the angles of z , between 0° and 360° inclusive which satisfy the equation
 $\tan(z - 18^\circ) = -2$. [3]

13 **Answer the whole of this question on the graph paper provided.**

The table shows experimental values of two variables x and y .

x	1	2	3	4	5
y	2.40	3.15	3.76	4.25	4.67

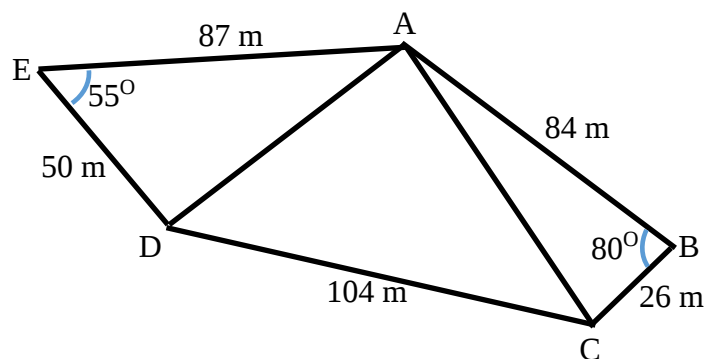
It is known that $y = \ln(ax^2 + b)$ and $x > 0$.

- (i) Express this equation in a form suitable for drawing a straight line. [1]
- (ii) Plot this graph using the given data. [3]
- (iii) Use your graph to estimate
- (a) the value of a and b , [3]
- (b) the value of y when $x = 2.5$. [2]

[Turn Over]

- 14 The figure below shows a picture of the Kerið crater lake, a 3,000 year old volcanic crater lake in South Iceland. The actual lake itself is relatively shallow. The water in the crater lake does not drain out but just rise and fall according to changes in the water level. As a result, the crater resembles a window on the groundwater. Jack would like to explore the possibility of having a water paddling zone for tourists during non-winter months. Hence, there is a need to find out the area of this Kerið crater lake.

To approximate the area of the lake, he walks around the perimeter of the lake, taking the measurements shown in the illustration. Using the cosine rule on the three triangles shown, find the area of the lake, round off to nearest whole number. [6]



Detach this page and stapled it together with your solutions on foolscap papers.

15 The diagram shows the function $y = f(x)$ over the domain $-4 \leq x \leq 2$.

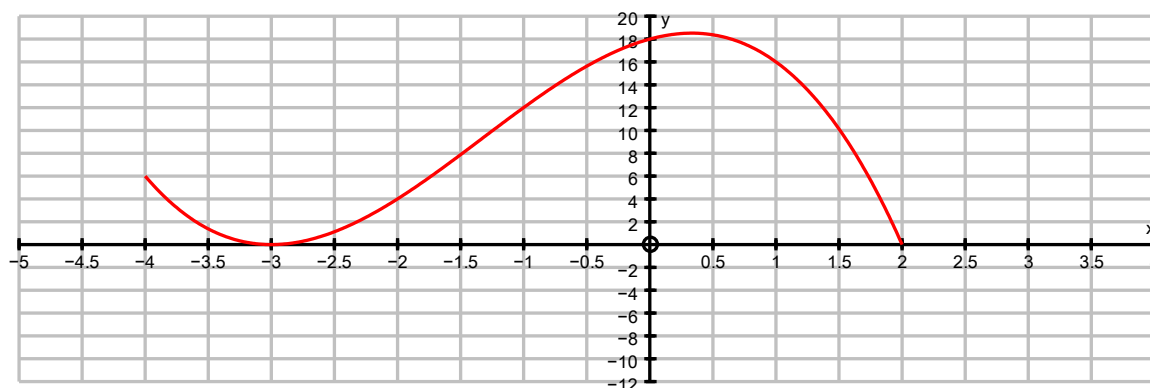
The function is undefined for all other values of x .

On space provided below, sketch the functions given by

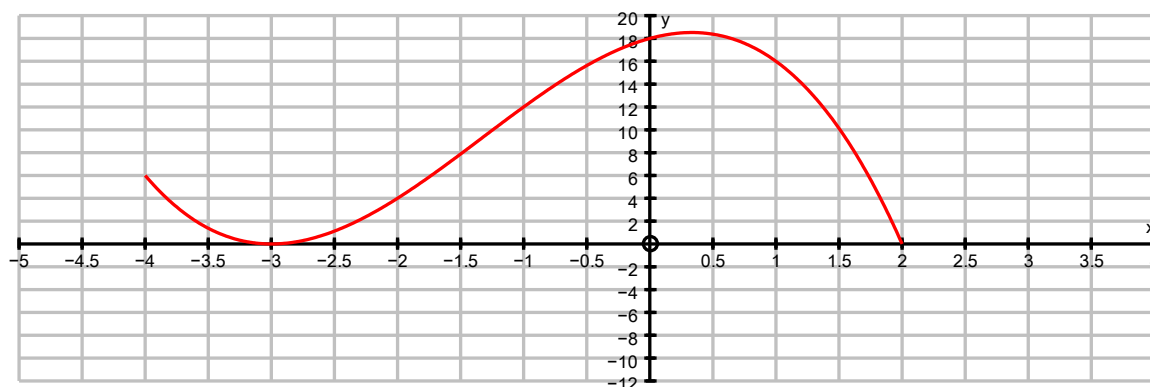
(i) $y = -\frac{1}{2}f(x)$, [2]

(ii) $y = f(2x) - 6$. [2]

(i) $y = -\frac{1}{2}f(x)$



(ii) $y = f(2x) - 6$



End of Paper