NAME	CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2021

SUBJECT	:	Additional Mathematics Paper 1 (4049/01)	DATE	:	17 August 2021
LEVEL	:	Secondary 4 Express	DURATION	:	2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.
Write in dark blue or back pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use	
Score	**/90**

This question paper consists of 23 printed pages, including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

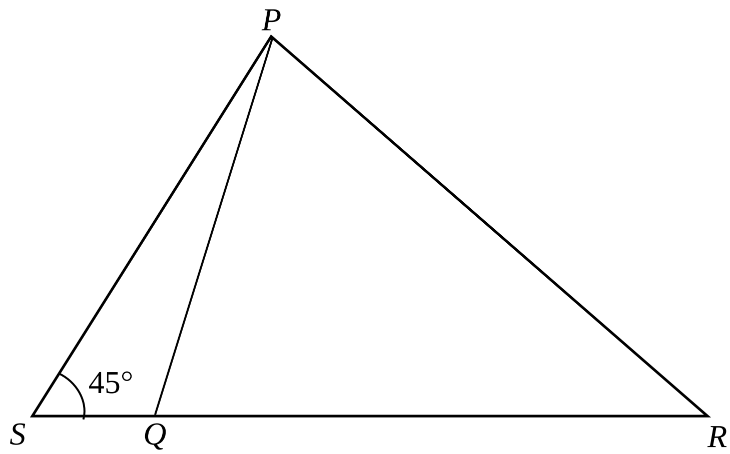
$$\text{Area of } \Delta = \frac{1}{2}bc\sin A$$

1. Given that $x^2 + 2x - 3$ is a factor of $f(x)$, where $f(x) = x^4 + 4x^3 + 2x^2 + ax - b$.

   (a) Find the values of a and b. [4]

   (b) Hence, find the other quadratic factor of $f(x)$. [2]

2.



The diagram shows a triangle *PQR* with an area of 18 cm^2. The base *QR* has a length of $(4\sqrt{2}+8)$ cm. The line *RQ* is extended to the point *S* where the angle *PSR* = 45°.

(a) Find the perpendicular distance from *P* to *QR*, leaving your answer in the form $a+b\sqrt{2}$, where *a* and *b* are rational numbers. [3]

(b) Find the exact length of *PS*, leaving your answer in the form $m+n\sqrt{2}$, where m and n are integers. [2]

3. Given the curve $y=\frac{1}{2}e^{2x}-\frac{1}{3}e^{-3x}+\frac{1}{3}$ and the normal to the curve at point $P\left(\ln 2,\ \frac{55}{24}\right)$ crosses the y – axis at Q.

(a) Show that $\frac{dy}{dx}=\frac{e^{5x}+1}{e^{3x}}$. [2]

(b) Find the equation of the normal at $P\left(\ln 2,\ \frac{55}{24}\right)$, leaving your answer in exact form. [3]

(c) Hence, find the coordinates of Q. [1]

4. (a) Find the coordinates of the stationary points on the graph of $y = 2x^3 - 9x^2 + 13$. [4]

(b) Hence, determine the nature of the stationary point(s). [3]

5. (a) Differentiate $2x\cos 2x$ with respect to x. [2]

(b) Hence, evaluate $\int_0^{\frac{\pi}{2}} \left(1-4x\sin 2x\right)\, dx$. [5]

6. It is given that $y=(x-2)(2x-5)^3$.

(a) Obtain an expression for $\frac{dy}{dx}$ in the form $(ax+b)(2x-5)^2$, where a and b are integers. [2]

(b) Determine the values of x for which y is a decreasing function. [2]

The variables x and y are such that, when $x = 3$, y is increasing at a rate of 0.35 units per second.

(c) Find the rate of change of x when $x = 3$. [2]

It is given further that the variable z is such that $z = y^2$.

(d) Show that, when $x = 3$, z is increasing at twice the rate of y. [2]

7. (a) Given that $\sqrt{125^x} = \dfrac{5^{1-x}}{25}$, find the value of $\sqrt{125^x}$. [4]

(b) Show that $\log_2 x - \log_{16} x = \dfrac{3\lg x}{4\lg 2}$. [4]

8. (a) Write down the general term, T_{r+1}, for the binomial expansion of $\left(2x+\frac{1}{2x^3}\right)^{12}$. [1]

(b) Find the term independent of x in the expansion of $\left(\frac{1}{x^4}-1\right)\left(2x+\frac{1}{2x^3}\right)^{12}$. [5]

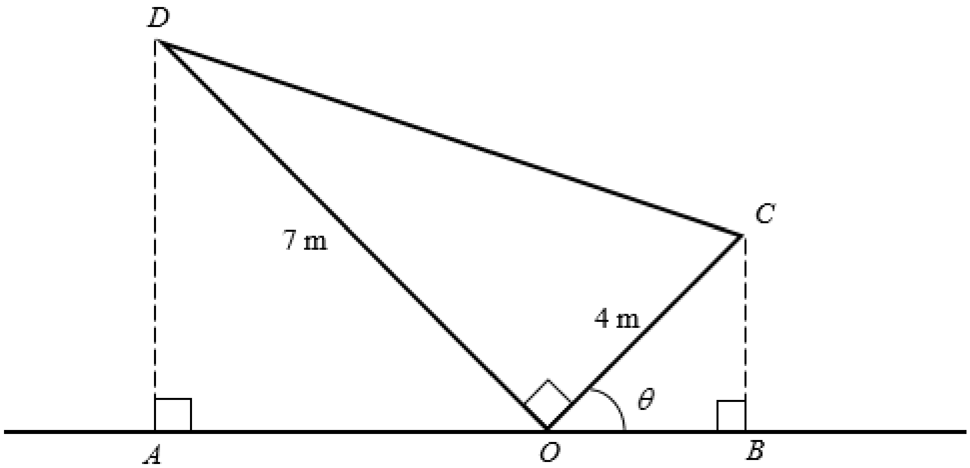
(c) Explain whether there are any terms containing x^n, where n is odd in the expansion of $\left(2x+\frac{1}{2x^3}\right)^{12}$. [2]

9. (a) Prove that $\dfrac{\sec^2 x}{1+\sec^2 x} = \dfrac{2}{\cos 2x + 3}$. [3]

(b) Hence, solve the equation $\dfrac{\sec^2 2x}{1+\sec^2 2x}=\dfrac{4}{7}$ for $0 \le x \le 3$. [4]

(c) State the number of solutions for $-2\pi \le x \le 2\pi$. [1]

10.



In the diagram, $OC = 4$ m, $OD = 7$ m and angle CBO = angle DOC = angle $DAO = 90°$.
It is also given that angle $COB = \theta$ such that $0° \leq \theta \leq 90°$.

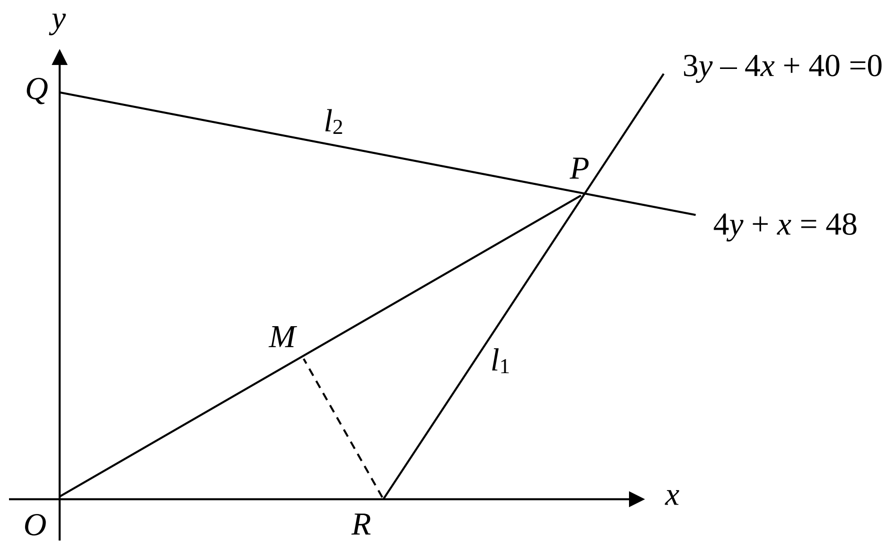
(a) Express AB in the form $p \sin \theta + q \cos \theta$ where p and q are integers. [2]

(b) Hence, express AB in the form $R \sin (\theta + \alpha)$ where R is positive and α is acute. [3]

(c) State the maximum length of AB and its corresponding value of θ. [3]

(d) Find the value of θ when $AB = \sqrt{55}$ m. [3]

11.



The diagram shows lines l_1 and l_2 intersecting at P. Line l_1 has equation $3y - 4x + 40 = 0$ and l_2 has equation $4y + x = 48$. The point M is the midpoint of OP. The line l_2 intersects the y-axis at Q and the line l_1 intersects the x-axis at R.

(a) Show that OP is perpendicular to MR. [5]

(b) Find the area of *ORPQ*. [3]

12. The table shows experimental values of two variable, x and y, which are connected by the equation $y = ae^{bx-1}$, where a and b are constants.

x	1	2	3	4	5
y	1.89	2.3	2.82	3.44	4.20

(a) On the grid given on page 23, using a scale of 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 0.2 units on the vertical axis, plot a straight line graph of $\ln y$ against x. [3]

(b) Use your graph to estimate the value of a and of b. [3]

(c) By drawing a suitable line on your graph, solve the equation $2.46 = ae^{bx-1}$. [2]

End of Paper

NAME	CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2021

SUBJECT	:	**Additional Mathematics** **Paper 2 (4049/02)**	**DATE**	:	**23 August 2021**
LEVEL	:	**Secondary 4 Express**	**DURATION**	:	**2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.
Write in dark blue or back pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use	
Score	**/90**

This question paper consists of 21 printed pages, including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{Area of } \Delta = \frac{1}{2}bc\sin A$$

[Turn over

1. (a) Express $y = 2x^2 - 4x + 7$ in the form $a(x - b)^2 + c$, where a, b and c are constants. [3]

(b) Hence, explain why the curve lies completely above the x-axis. [2]

2. A virus is spreading in a country. The number of infected cases, I, after t days, can be modelled by $I = \dfrac{a}{b + 2^{20-t}}$, where a and b are constants. The number of infected cases was 3600 after 16 days. Four days later, the number of infected cases increased to 9000.

   (a) Show that $a = 90000$ and $b = 9$. [3]

[Turn over

(b) Given that the number of infected cases first exceeds 100 after n days, where n is an integer, find the value of n. [3]

(c) Describe what happens to the number of infected cases in the long run. [1]

3. (a) Without the use of calculator, show that $\sin 75 = \dfrac{\sqrt{2}+\sqrt{6}}{4}$. [3]

(b) In a city in the United Kingdom, the number of daylight hours is given by $h(t) = -4\cos(bt) + 12$, where t is an integer ranging from 1 to 12 inclusive that represents the month of January to December. It takes 12 months before the number of daylight hours reaches its lowest point again.

(i) Explain, with workings clearly shown, why $b = \dfrac{\pi}{6}$. [2]

(ii) Sketch the graph of $h(t)$, where $0 \leq t \leq 18$. [3]

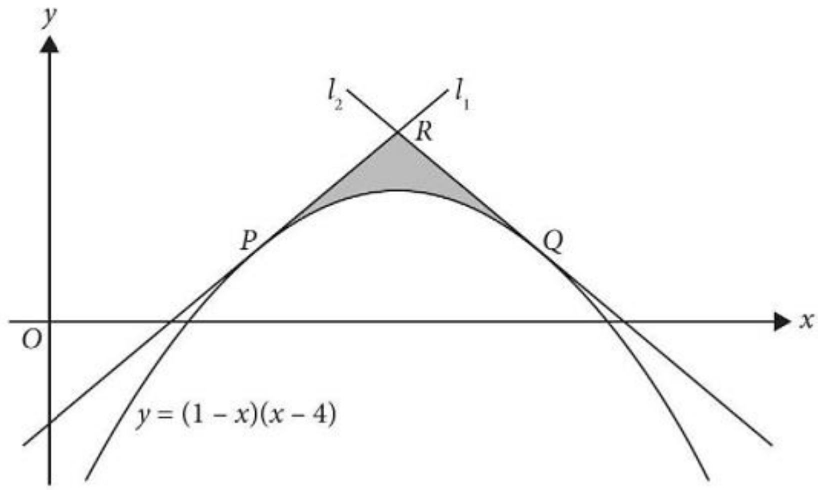
4. (a) Show that $\frac{d}{dx}\left(\frac{\ln x}{x^4}\right) = \frac{1}{x^5} - \frac{4\ln x}{x^5}$. [3]

(b) Hence, $\int \frac{\ln x}{x^5}\, dx$. [3]

(c) Given that the curve $y = \mathrm{f}(x)$ passes through the point $\left(1, \frac{3}{4}\right)$ and is such that

$\mathrm{f}'(x) = \frac{\ln x}{x^5}$, find $\mathrm{f}(x)$. [3]

5.



The diagram shows part of the curve $y=(1-x)(x-4)$ and two lines l_1 and l_2. The lines l_1 and l_2 are tangents to the curve at points P and Q and have gradients 2 and -2 respectively.

(a) Show that coordinates P and Q are $\left(\frac{3}{2},\frac{5}{4}\right)$ and $\left(\frac{7}{2},\frac{5}{4}\right)$ respectively. [3]

The lines l_1 and l_2 intersect at point R.

(b) Find the coordinate of point R. [2]

(c) Hence, find the shaded area PQR. [4]

6. (a) Find the set of values of x for which $5x^2 + 12x > 3x + 2$. [3]

(b) The line $y = mx + c$ does not intersect the curve $x^2 + y^2 = 8$.

(i) Prove that $8m^2 + 8 < c^2$. [4]

(ii) Hence, determine whether the line $y = 2x + 5$ intersects the curve $x^2 + y^2 = 8$. [2]

7. The equation of a circle is $x^2 + y^2 + 2x - 8y - 8 = 0$.

(a) Find the coordinates of the centre, C and the radius of the circle. [3]

(b) Find the equation of the tangent to the circle at the point $P(-2, 8)$. [3]

(c) If the circle is reflected in the line $y = 1$, find the equation of the reflected circle. [2]

(d) Find the area of the biggest square that can be drawn inside the circle. [2]

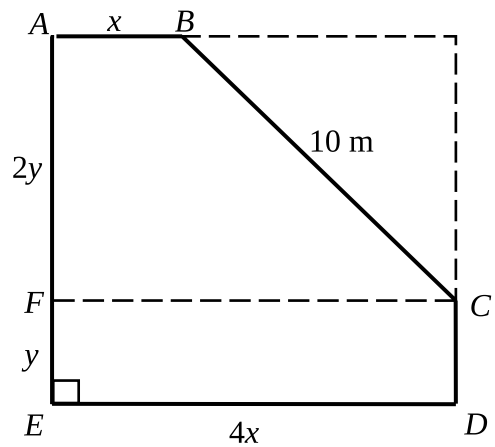
8. (a) Express $\dfrac{8x^3+3x+1}{x(2x+1)^2}$ as a sum of a polynomial and a proper fraction. [1]

(b) Hence, express $\dfrac{8x^3+3x+1}{x(2x+1)^2}$ as a sum of a polynomial and a partial fraction. [5]

Space for Q8(b)

(c) Hence, $\int_{1}^{3} \frac{8x^3+3x+1}{x(2x+1)^2}\,dx$ [4]

9.



ABCDEF is the cross-sectional plan of a temporary spectator stand, where *AB* is parallel to *FC* and *ED* and *AE* is parallel to *CD*.
Given that $AB = x$, $ED = 4x$, $FE = y$, $AF = 2y$ and $BC = 10$ m.

(a) Express y in terms of x. [2]

(b) Show that the area, S, of the cross-sectional plan of the spectator stand is given by
$S = \frac{9}{2}x\sqrt{100-9x^2}$. [3]

(c) Given that x varies, find the stationary value of S. [5]

(d) Hence, determine whether it is a maximum or minimum value. [1]

10. A particle moves in a straight line such that its velocity, v m/s, is given by $v = 9\sin\left(\frac{t}{2}\right)$,

where t is the time in seconds after passing a fixed point O.

(a) Find the value of t for which the particle is at its first instantaneous rest after leaving O. [3]

(b) Find the value of t for which the particle is at its first maximum velocity. [2]

(c) Calculate the distance travelled by the particle in the first 10 seconds. [4]

(d) Find the time that the particle passes through the point O again. [3]

End of Paper

NAME	CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2022

SUBJECT	**Additional Mathematics (4049/01)**	**DATE**	**11 AUGUST 2022**
LEVEL	**Secondary 4 Express**	**DURATION :**	**2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Score	**/90**

This question paper consists of 18 printed pages, including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

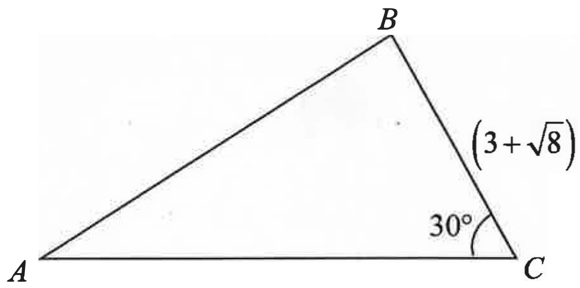
$$\textit{Area of } \Delta = \frac{1}{2}bc\sin A$$

1. Find the range of values of m for the which the graph of the straight line $y = mx - 1$ is always below that of $y = 2x^2 + 7$ for all the real values of x. [4]

2. (i) Simplify $\log_4 32 + \log_5 125 - \log_{\sqrt{3}} 27$. [3]

(ii) Solve $5 + 2e^{y} = 3e^{-y}$. [4]

3. In the diagram, area of triangle ABC is $\left(14+9\sqrt{2}\right)$ cm^2, $BC = \left(3+\sqrt{8}\right)$ cm and angle $ACB = 30°$. Find AC in the form of $p(q-r\sqrt{2})$. [5]



4. (i) Find $\frac{d}{dx}\left(x^3 \ln x\right)$. [2]

(ii) Hence, find $\int x^2 \ln x \, dx$. [3]

5. Find the equation of the tangent to the curve $y = e^{1+\sin x}$ at the point where $x = \pi$, leaving your answer in terms of π and e. [5]

6. It is given that $\tan\theta = -2\sqrt{2}$ and $\sin\theta > 0$.

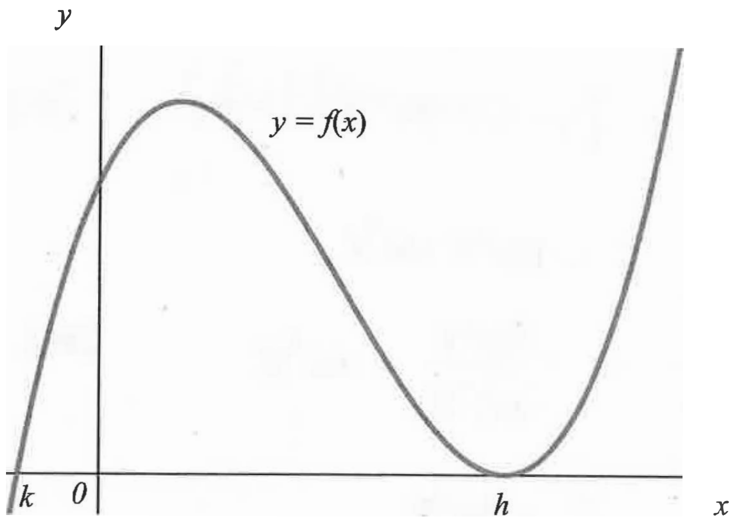
Find the exact value of

(i) $\cos\theta$, [2]

(ii) $\tan 2\theta$, [2]

(iii) $\sin\frac{1}{2}\theta$ [3]

7.



The diagram represents a cubic function $f(x)$ which touches the x-axis at $x = h$ and intersects the x-axis at $x = k$. The coefficient of x^3 term is 2 and the y-intercept is 50.
$f(x)$ is divisible by $(x-5)$. [2]

(i) State the value of h.

(ii) Show that $k = -1$. [2]

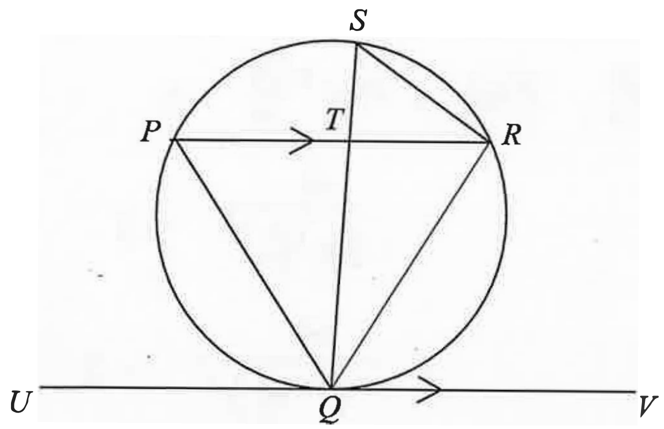
(iii) Find the remainder when $f(x)$ is divided by $(x - 1)$. [2]

8. (i) Prove that $\left(1-\sec^2 x\right)(1-\sin^2 x) = -\dfrac{1}{\operatorname{cosec}^2 x}$ [3]

(ii) Hence, solve $\left(1-\sec^2 2A\right)(1-\sin^2 2A) = -\dfrac{1}{4}$, for $0 \le \overset{A}{\cancel{x}} \le \pi$.

Leave your answer in terms of π. [4]

9. The diagram shows triangles PQR and QRS whose vertices lie on the circumference of a circle. The chords QS and PR intersect at T and PR is parallel to UV. UV is a tangent to the circle at Q.



Show that

(i) triangle QRS is similar to triangle QTR, [3]

(ii) $QR^2 = QS \times QT$, [2]

(iii) Triangle PQR is an isosceles triangle. [2]

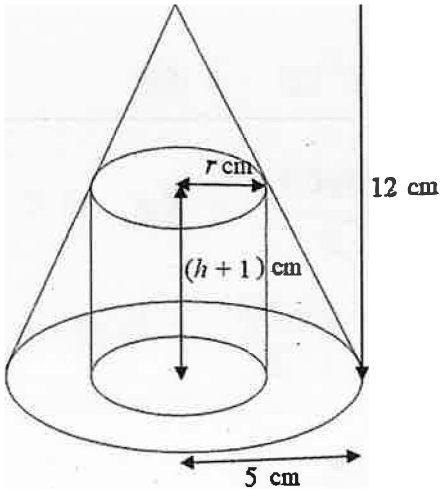
10. The curve $y = f(x)$ is such that $f'(x) = 4\sec^2 2x - 3$.

(i) Find the gradient of the normal to the curve at the point where $x = \dfrac{5\pi}{12}$. [2]

(ii) Explain why the curve $y = f(x)$ has no stationary point. [2]

(iii) Given that the curve passes through the point (0, 5), find an expression for $f(x)$. [3]

11.



The diagram shows a cylinder of radius r cm and height $(h + 1)$ cm, inscribed in a right circular cone with a radius of 5 cm and height 12 cm.

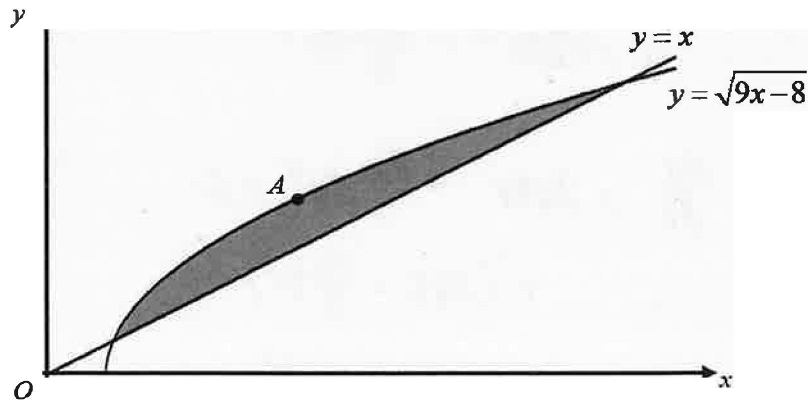
(i) Show that $h = 11 - \frac{12}{5}r$. [2]

(ii) Show that the volume of the cylinder, V cm^3, is given by $V = \frac{1}{5}\pi r^2(60 - 12r)$. [2]

(iii) Given that r can vary, find the value of r which gives a stationary value of V. [3]

(iv) Explain why this value of r gives the largest volume possible. [2]

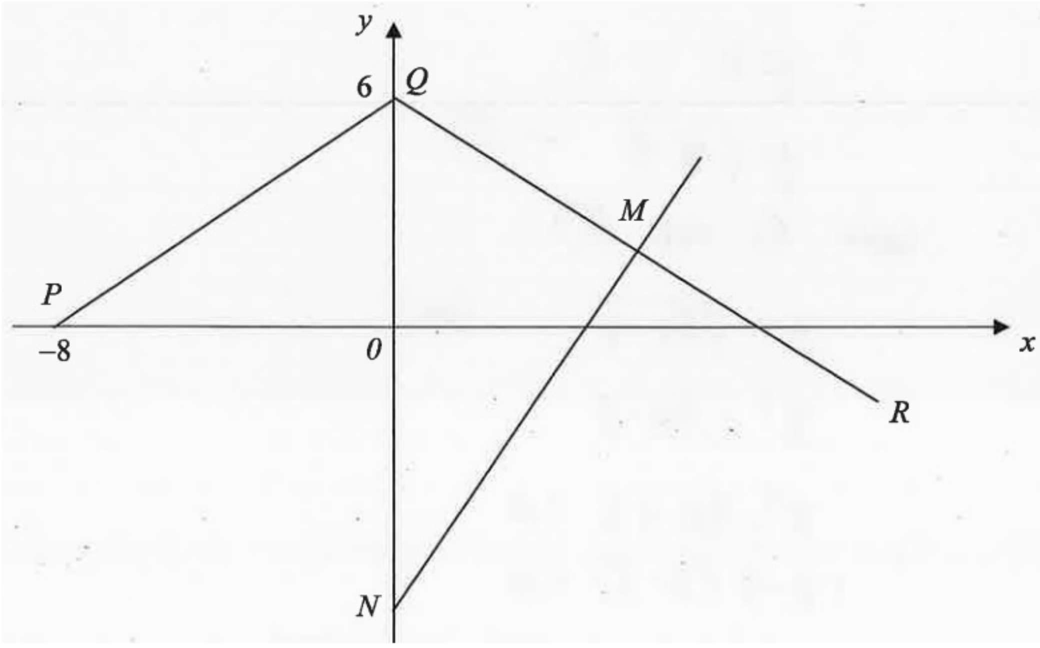
12. The diagram shows part of the curve $y = \sqrt{9x - 8}$ intersecting the line $y = x$.
The point A lies on the curve and the normal at A is perpendicular to the line $y = x$.



(i) Find the x-coordinate of A. [4]

(ii) Find the area of the shaded region. [6]

13. In the diagram, P and Q intersects the x and y axes at -8 and 6 respectively.



(i) Find the distance of PQ. [1]

(ii) QR cuts the x-axis with angle θ such that $\tan\theta = \frac{3}{4}$. Find the equation of QR. [2]

(iii) Given that $\frac{QR}{PQ} = 1.5$, show that the coordinates of R is $(12, -3)$. [4]

(iv) MN is the perpendicular bisector of QR. Find the y-intercept of line MN. [2]

(v) Find the area of ~~parallelogram~~ $PQMN$. [2]

End of Paper

NAME	CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2022

SUBJECT : **Additional Mathematics (4049/02)** **DATE** : **19 AUGUST 2022**

LEVEL : **Secondary 4 Express** **DURATION** : **2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Score	**/90**

This question paper consists of 17 printed pages, including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{Area of } \Delta = \frac{1}{2}bc\sin A$$

1. Salmonella bacteria is commonly found in food that is not hygienically prepared. The number of bacteria, n, can be modelled by $n = Be^{kt}$ where n is the number of bacteria t minutes after the food is prepared, and B and k are constant.

   (i) State the initial number of bacteria. [1]

   (ii) Show that $k = 0.0330$ if the number of bacteria doubles every 21 minutes. [2]

   (iii) It is found that food containing at least 10^6 bacteria will cause illness when consumed. To protect consumers, the Singapore Food Agency requires all food catering companies to indicate on the food packaging the "CONSUME BY" time that is not later than 4 hours from the time the food is cooked.

   Find the maximum value of B, the initial number of bacteria, in order that the food is safe for consumption at $t = 240$ minutes. [3]

2. The equation of a circle is $x^2 + y^2 - 8x + 6y - 75 = 0$.

(i) Find the radius of the circle and the coordinates of its centre. [3]

(ii) AB is a diameter of the circle. Find the coordinates of B given $A(12, 3)$. [2]

(iii) Given $P(-8, 2)$, find the greatest distance from P to the circle. [2]

3. (i) In the expansion of $(2+3x)^n$,
the coefficient of x^3 is four times the coefficient of x^2.
Show that $n = 10$. [4]

(ii) Find the term independent of x in the expansion of $\left(4-\dfrac{1}{128x^3}\right)(2+3x)^{10}$. [3]

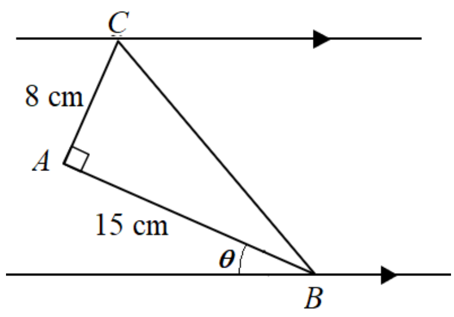
4. Given that $f(x) = 4\cos\left(\frac{x}{2}\right) + 2$,

(i) state the amplitude and period of $f(x)$. [2]

(ii) sketch the graph of $f(x)$ for $0 \le x \le 4\pi$.
Indicate clearly the coordinates of any maximum or minimum points. [3]

(iii) Hence, by using (b), sketch a suitable straight line, state the number of solutions to the equation $4\cos\frac{x}{2} - \frac{3x}{2\pi} = -4$ for $0 \le x \le 4\pi$. [3]

5.



The diagram shows two parallel lines and a right-angled triangle BAC with AB = 15 cm and AC = 8cm. AB makes an acute angle θ with one of the lines as shown.

(i) Show that the distance between the parallel lines, $D = 15\sin\theta + 8\cos\theta$ cm. [2]

(ii) Express D in the form $R\sin(\theta+\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. [2]

(iii) Find the greatest possible value of D and the value of θ at which this occurs. [2]

6. (i) Given that $y = x^2\sqrt{2x+1}$, show that $\dfrac{dy}{dx} = \dfrac{x(5x+2)}{\sqrt{2x+1}}$. [3]

(ii) Hence, evaluate $\displaystyle\int_1^5 \frac{5x^2+2x-3}{\sqrt{2x+1}}dx$. [4]

7. The equation of a curve $y = 3xe^{-2x}$.

(i) Find the value of x for which the curve has a stationary point. [3]

(ii) Express $\frac{d^2y}{dx^2}$ in the form $ae^{-2x}(b + x)$, where a and b are constants. [3]

(iii) Hence, determine the nature of this stationary point. [2]

8. An object in motion with velocity v m/s is subjected to a resistive force of R Newton against its motion given by the equation

$$R = kv^{\beta} \quad \text{, where } k \text{ and } \beta \text{ are constants.}$$

The table shows experimental values of v and R.

v	10	20	30	40	50
R	96	290	530	840	1200

(a) Express $\ln R$ in terms of $\ln v$. [1]

(b) Draw the graph of $\ln R$ plotted against $\ln v$, using a scale of 2 cm to 1 unit on the $\ln R$ axis and a scale of 4 cm to 1 unit on the $\ln v$ axis. [4]

(c) Use your graph to estimate the values of each of the constants k and β. [4]

9. (i) Express $\dfrac{5x^2+4x-3}{x^2(2x-1)}$ in partial fractions. [4]

(ii) Hence, show that $\displaystyle\int_1^5 \frac{5x^2+4x-3}{x^2(2x-1)}dx = \frac{12}{5} + \ln 75$. [5]

BLANK PAGE

10. (i) A curve has equation $y = f(x)$, where $f(x) = \dfrac{5x-2}{x-3}$, $x > 3$.

(a) Find expression for $f'(x)$. [2]

(b) Determine whether $y = f(x)$ is an increasing function or a decreasing function. Explain your answer. [2]

(ii) Liquid gold is poured into a mold at a rate of 9 cm^3/s.
The volume, V cm^3 of liquid gold in the mold when the depth of liquid is x cm,
is given by $V = \frac{1}{30}(2x+5)^3 - \frac{2}{3}$.
Find the
(a) rate of increase in the depth of liquid gold when $x = 5$, [3]

(b) depth of liquid gold when the rate of increase of the depth is 1.2 cm/s. [3]

11. A particle moves in a straight line such that t seconds after leaving a fixed point O, its velocity, v m s^{-1}, is given by $v = (0.5t - 3)^4 - 16$.

(i) Find the values of t when the particle is at instantaneous rest. [3]

(ii) Express the acceleration of the particle in terms of t. [1]

(iii) Using your answer in (ii), explain why the velocity is decreasing, only when $0 < t < 6$. [3]

(iv) Express the displacement of the particle from O in terms of t. [3]

(v) Calculate the distance travelled during the first 5 seconds. [3]

End of Paper

NAME	CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2023

SUBJECT : **Additional Mathematics Paper 1 (4049/01)**

DATE : **17 August 2023**

LEVEL : **Secondary 4G3**

DURATION : **2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Score	/90

This question paper consists of 21 printed pages, including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2} bc \sin A$$

1 If the roots of the equation $x^2 + 2x(1-k) + 3k - 5 = 0$ are real, find the range of values of k. [4]

2 A curve has equation $y = \dfrac{2x+5}{x-1} - 12x$, where $x \neq 1$.

Explain, with working, whether y is an increasing or a decreasing function of x. [4]

3 **(a)** Express $y = 2x^2 - 10x + c$ in the form $a(x-h)^2 + k$ where a, c, h and k are constants. [2]

(b) If the minimum value of y is $-\dfrac{19}{2}$, find the value of c. [2]

4 The speed, v m/s, of an aircraft, t s after passing a fixed point T on the runway, is given, for $t \geq 0$, by $v = 10 + Me^{kt}$, where M and k are constants. The table below shows corresponding values of t and v.

t	1	2	3	4
v	14.4	15.7	17.4	19.4

(a) Draw the graph of ln $(v - 10)$ plotted against t, using a scale of 2 cm for 1 unit on the t-axis and a scale of 5 cm for 1 unit on the ln $(v - 10)$-axis. [2]

(b) Use the graph to estimate

(i) the value of M, [2]

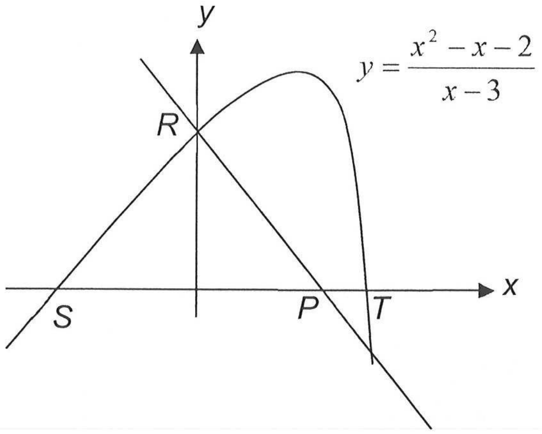
(ii) the time when the speed of the aircraft is 18 *m/s*. [2]

4

5 For a particular curve $\frac{d^2y}{dx^2} = 9\cos 3x - 5\sin x$. The curve passes through the point $P\left(\frac{3\pi}{2}, \frac{15\pi}{2}\right)$ and the gradient of the curve at P is 8. Find the equation of the curve. [6]

6 Express $\dfrac{x^3+1}{(x^2+1)(x-2)}$ in partial fractions. [6]

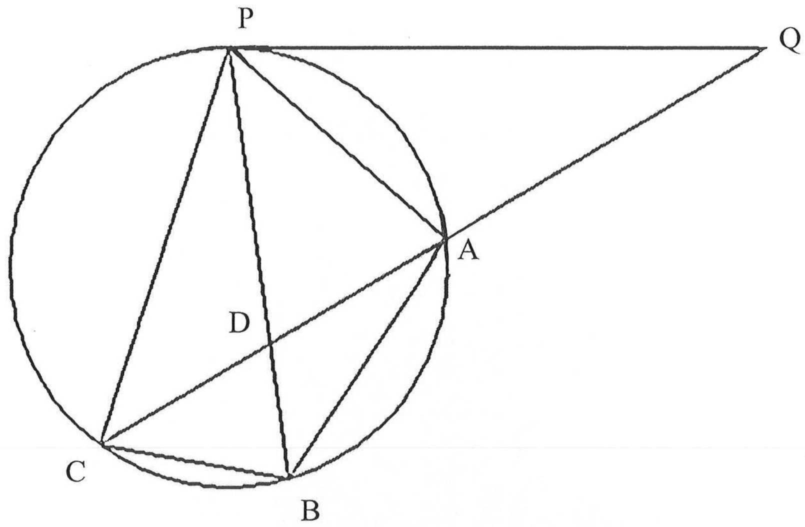
7 The diagram shows part of the graph of the curve $y = \dfrac{x^2 - x - 2}{x - 3}$.



(a) Given that $\dfrac{dy}{dx} = 1 - \dfrac{k}{(x-3)^2}$. Find the value of k. [3]

7 **(b)** R, S and T are intersections of the graph with the axes. PR is the normal to the curve at R. Find the equation of PR. [3]

8 The diagram shows a point P on a circle and PQ is a tangent to the circle at P. Points A, B and C lie on the circle such that PA bisects angle QPB. QAC is a straight line. The lines QC and PB intersect at D.



(a) Prove that $AP = AB$. [3]

(b) Prove that CD bisects angle PCB. [3]

8 **(c)** Prove that triangles *CDP* and *CBA* are similar. [2]

9 **(a)** Show that the curve $y = 2e^{\frac{1}{2}x} + 5x$ has no stationary points. [3]

(b) Find the equation of the normal to the curve $y = \dfrac{2}{3\sqrt{2x-1}}$, $x \neq \dfrac{1}{2}$ at the point where the tangent is parallel to the line $6y + 8x = 1$. [5]

10 A boy cycling passes a point A at the bottom of the slope with a speed of 17 m/s. He then cycled up the slope and passes point B at the top of the slope, 5 seconds later, with a speed of 4.5 m/s. Between A to B, his velocity v m/s, is given by $v = 0.5t^2 - pt + q$, where t is the time in seconds since passing A, and p and q are constants.

(a) Show that $q = 17$ and find the value of p. [2]

(b) Find the deceleration of the cyclist when his speed is 6.12 m/s. [4]

10 (c) Find the distance *AB* in metres. [3]

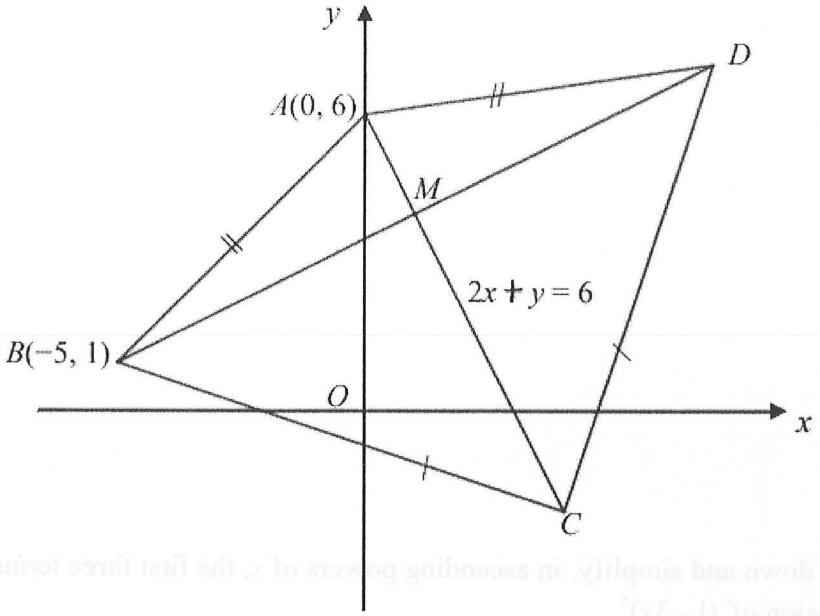
11 **(a)** The thirteenth term in the expansion of $\left(x^3 - \frac{4}{x}\right)^n$, is independent of x.

Find the value of n. [3]

(b) (i) Write down and simplify, in ascending powers of x, the first three terms in the expansion of $(1-3x)^5$. [2]

(ii) Given that there is no term containing x in the expansion of $(x-a)^2(1-3x)^5$, find the value of a, where a is a non-zero constant. [4]

12 The diagram shows a kite $ABCD$ with $AB = AD$ and $CB = CD$. The diagonals intersect at M. It is given that the coordinates of A and B are $(0,6)$ and $(-5,1)$ respectively and the equation of AC is $2x + y = 6$.



Find

(a) the equation of BD. [2]

12 (b) the coordinates of M and of D. [4]

12 Given further that the area of the triangle *ABD* is $\frac{1}{3}$ of the area of the triangle *CBD*,

(c) find the coordinates of *C*, [2]

(d) find the area of the kite *ABCD*. [2]

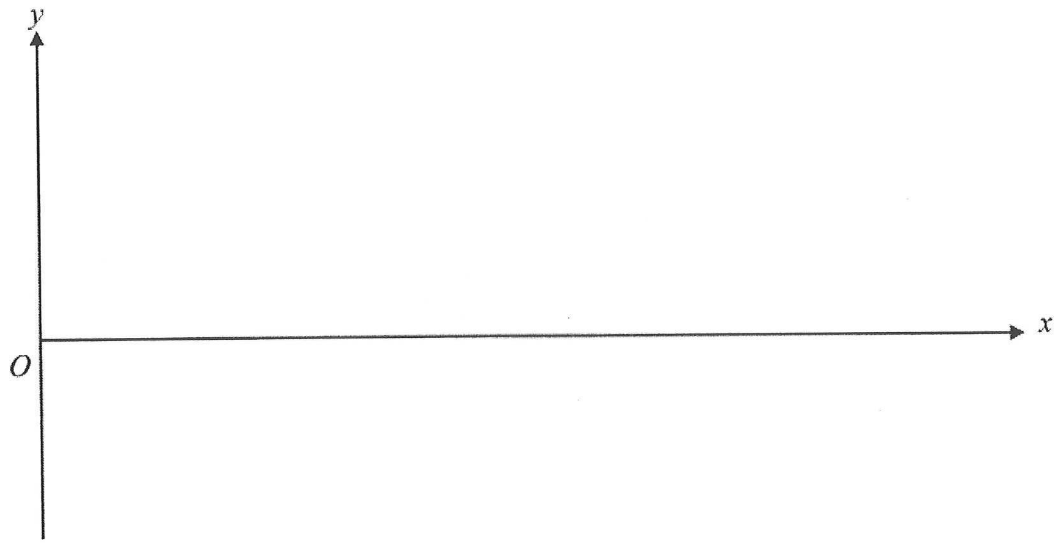
13 (a) State the values between which the principal value of $\sin^{-1} x$ lies. [1]

(a) Find the amplitude and period of

(i) $g(x) = \sin\frac{1}{2}x$ [1]

(ii) $f(x) = 1 + \cos 2x$ [2]

(iii) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \le x \le 2\pi$. [4]



(iv) Hence, state the number of solutions of the equation $\sin\frac{1}{2}x - 1 = \cos 2x$ for $0 \le x \le 2\pi$. [1]

(v) The number of solutions of the equation $m\sin\frac{x}{2} = 1 + \cos 2x$, where m is a constant, for $0 \le x \le 2\pi$ is exactly 3. Find the value of m. [1]

End of Paper

NAME	CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2023

SUBJECT : **Additional Mathematics (4049/02)**

DATE : **22 AUGUST 2023**

LEVEL : **Secondary 4G3**

DURATION : **2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Score	/90

This paper contains 23 printed pages including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A student learns a new topic from a teacher. At t days after learning the topic, the percentage, P %, of the topic that the student remembers can be modelled by the equation $P = 80e^{-kt} + 20$, where k is a constant.

(a) Explain why the student remembers 100% of what he learns from the teacher on the day that it is taught. [1]

(b) A week later, the student remembers 40% of what he has learnt from the teacher. Find the value of k. [2]

1 **(c)** How many days will have elapsed when a student remembers at **most** half of what he has learnt from the teacher?

Give your answer to the nearest whole number. [3]

(d) A parent claims that in the long run, the student remembers nothing about the topic that he has learnt from the teacher. Do you agree with the parent? Explain. [2]

2 Air is pumped into a spherical air balloon which changes the surface area at the rate of $400\ \text{cm}^2/\text{s}$.

Find the rate of increase of the radius when the volume is $288\pi\ \text{cm}^3$, leaving your answer in terms of π. [5]

[*Volume of sphere* $= \dfrac{4}{3}\pi r^3$]

3 It is given that $f(x) = 4x^3 + 6x^2 - 9x + 2k$, where k is a constant.

(a) The expression $f(x)$ has a remainder of -125 when divided by $x+2$.
Find the value of k. [2]

(b) Show that $(x-3)$ is a factor of $f(x)$. [2]

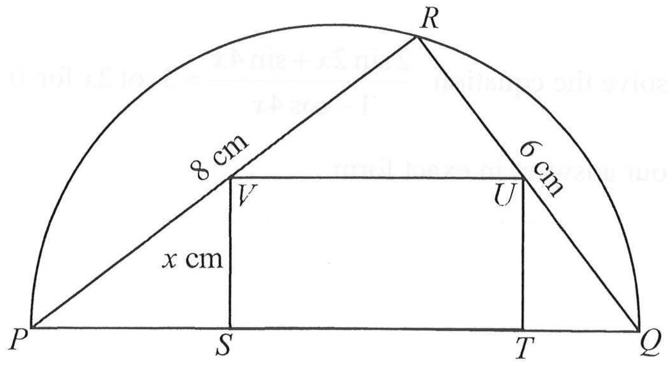
3 **(c)** Explain why $f(x) = 0$ has only one real root. Show all your workings clearly. [3]

4 **(a)** Prove the identity $\dfrac{2\sin x + \sin 2x}{1 - \cos 2x} = \dfrac{\sin x}{1 - \cos x}$. [4]

4 (b) **Hence**, solve the equation $\dfrac{2\sin 2x+\sin 4x}{1-\cos 4x}=3\cot 2x$ for $0<x<\pi$.

Leave your answers in exact form. [7]

5



The diagram shows a semicircle with diameter PQ, with the point R on the circumference of the semicircle. A rectangle $STUV$ is drawn within the triangle PQR where ST lies on PQ. U and V are points on QR and PR respectively. It is given that $PR = 8$ cm and $QR = 6$ cm.

(a) Find the perpendicular distance from R to PQ. [3]

(b) It is given that SV is x cm.
Show that the area of rectangle $STUV$, A m^2, is given by $A = 10x - \frac{25}{12}x^2$. [2]

5 (c) Calculate the value of x for which A has a stationary value. [3]

(d) Determine whether this value of x gives a maximum or minimum value for the area of rectangle *STUV*. [2]

6 **A calculator must not be used in this question.**

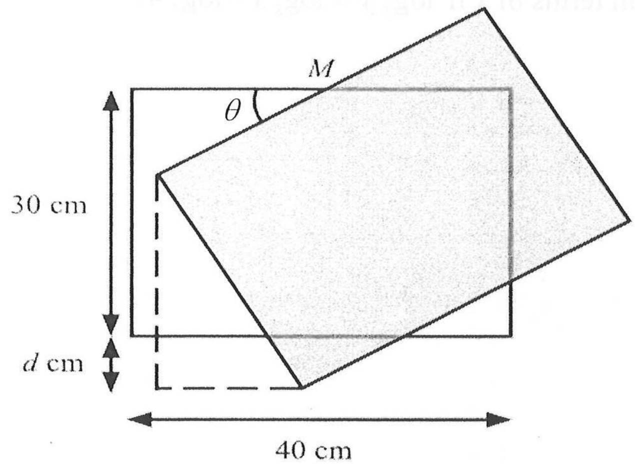
(a) Show that $\cos 105° = \dfrac{\sqrt{2}-\sqrt{6}}{4}$. [4]

6 **(b)** Use the result from **part (a)** to find an expression for $\sec^2 105°$, in the form $a + b\sqrt{3}$, where a and b are integers. [3]

7 **(a)** Given that $3^{x+1} \times 2^{2x+1} = 2^{x+2}$, evaluate 6^x. [4]

7 (b) Express y in terms of x if $\log_2 y = \log_8 x - \log_2 4$. [4]

8



The diagram shows a rectangular picture frame 40 cm by 30 cm hung on a wall.

The picture frame is rotated through an angle θ about the midpoint, M of the top edge.

(a) Show that the vertical displacement, d cm, of the picture frame below its original bottom edge is given by $d = 20\sin\theta + 30\cos\theta - 30$. [2]

(b) Express d in the form of $R\sin(\theta + \alpha) - 30$ where $R > 0$ and $0° \le \alpha \le 90°$. [4]

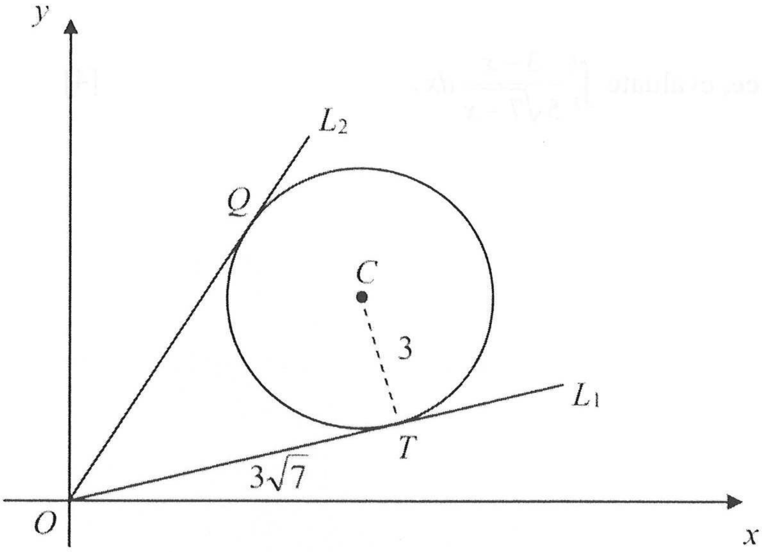
8 **(c)** Find the value of d and the corresponding value of θ that will give the greatest vertical displacement of the picture frame below its original bottom edge. [3]

9 A curve has the equation $y=(x+5)\sqrt{7-x}$, where $x>7$.

(a) Show that $\dfrac{dy}{dx}$ can be expressed in the form $\dfrac{a+bx}{2\sqrt{7-x}}$. [4]

9 (b) **Hence,** evaluate $\int_3^6 \frac{3-x}{5\sqrt{7-x}}\,dx$. [4]

10



The diagram shows a circle with centre C and radius 3 units. The tangent to the circle, L_1, at the point T passes through the origin and $OT = 3\sqrt{7}$ units.

The x – coordinate of C is 6.

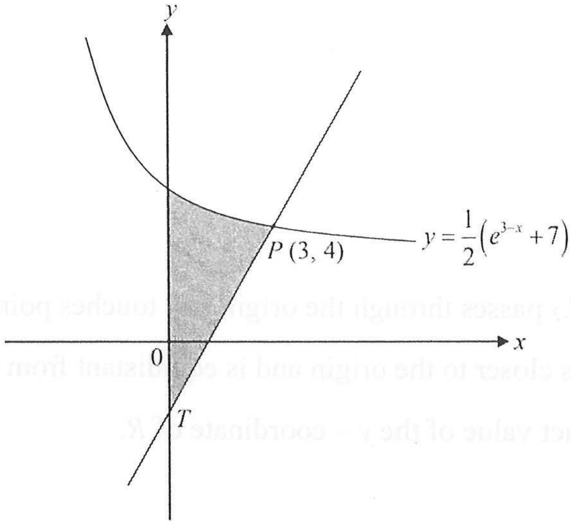
(a) Find the equation of the circle. [4]

10 **(b)** State the coordinates of the point on the circle which is furthest from the y - axis. [1]

A second tangent L_2 passes through the origin and touches point Q. Point R lies on the circle such that it is closer to the origin and is equidistant from the two tangents.

(c) Find the exact value of the y – coordinate of R. [4]

11



The diagram shows part of the curve $y = \frac{1}{2}\left(e^{3-x} + 7\right)$ and the normal to the curve at the point P (3, 4) which meets the y – axis at T.

(a) Find the coordinates of the point T. [4]

11 (b) Find the area of the shaded region bounded by the curve, the normal and the $y-$axis, leaving your answer in the form $(A + Be^3)$ square units, where A and B are constants to be found. [5]

End of Paper 2

NAME	FORM CLASS	ACAD CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2025

SUBJECT	**:**	**Additional Mathematics (4049/01)**	**DATE**	**:**	**18 August 2025**
LEVEL	**:**	**Secondary 4 Express**	**DURATION**	**:**	**2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Form Class, Acad Class and Index No. in the spaces at the top of the page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value of 3.142, unless the equation requires the answer in terms of π

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90 .

<table>
<tr><th colspan="4">For Examiner's Use</th></tr>
<tr><td colspan="2">Type of Penalties</td><td>Question Number</td><td rowspan="5">90</td></tr>
<tr><td rowspan="2">Units</td><td>–1</td><td rowspan="2"></td></tr>
<tr><td>–2</td></tr>
<tr><td rowspan="2">Presentation</td><td>–1</td><td rowspan="2"></td></tr>
<tr><td>–2</td></tr>
</table>

This paper consists of 20 printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The equation of a curve is $y = kx^2 + kx + p$, where p and k are constants.

(a) Show that $p > \frac{k}{4}$ for which the curve lies completely above the x-axis [3]

(b) In the case where $k = 2$ and $p = 4$, find the values of $\boldsymbol{m}$ for which the line $y = mx - 4$ is a tangent to the curve. [4]

2 **(a)** Express $\dfrac{2x^3+7x^2+5x+6}{x(x^2+2)}$ as the sum of a polynomial and a proper fraction. [1]

(b) Hence, express $\dfrac{2x^3+7x^2+5x+6}{x(x^2+2)}$ in partial fractions. [5]

3 Prove the identity $\dfrac{\cos x}{\operatorname{cosec} x - 1} + \dfrac{\cos x}{\operatorname{cosec} x + 1} = 2\tan x$. [5]

4 **(a)** Given that $y = (x-1)\sqrt{4x+1}$, show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{px+q}{\sqrt{4x+1}}$ where p and q are constants. [4]

(b) **Hence** evaluate $\displaystyle\int_2^6 \frac{x}{\sqrt{4x+1}}\,dx$ [5]

5 Solve the equation $3\cos A - \sec A = 5\tan A$ for $0 \leq A \leq 2\pi$. [5]

6 It is given that $y = x^3 + px^2 + qx + 10$ where p and q are integers. The only values of x for which y is a decreasing function of x are those values for which $3 < x < 7$.
Find the value of p and of q. [4]

7 The line $2x+3y=12$ intersects the curve $y^2=4x-8$ at points A and B.
Find the value of p and q for which the length of AB can be expressed as $p\sqrt{q}$. [6]

8 The equation of a curve is $y = 5e^{-\frac{1}{2}x} + e^{\frac{1}{2}x}$.

(a) Find the coordinates of the stationary point, in logarithmic and/or surd from where appropriate. [5]

(b) Determine the nature of the stationary point of the curve. [2]

(c) Explain why the gradient is an increasing function for all real values of x. [2]

9 A particle moves along the curve $y = \dfrac{3(x+6)}{x+4}$, where $x \neq -4$, in such a way that the y-coordinate of the particle is increasing at a constant rate of $\dfrac{4}{27}$ units per second.

Find the x-coordinates of the particle at that instant that the x-coordinates of the particle is decreasing at a rate of 2 units per second. [5]

10 Jade is a parkour enthusiast who enjoys jumping between built structures or obstacles.

One day, Jade decides to jump from one 2-metre-high structure to another of the same height.

The path of Jade's jump can be modeled by the quadratic equation:

$h = -\frac{5}{2}t^2 + 5t + 1$ where h is the height in metres above the ground after t seconds.

(a) Rewrite h in the form $b - k(t-a)^2$. [3]

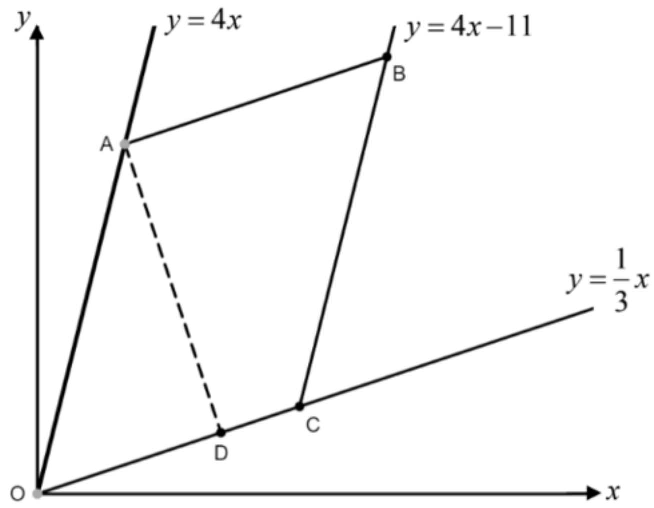
(b) Hence determine

(i) the maximum height that Jade reaches during the jump, [1]

(ii) the time at which this maximum height occurs. [1]

(c) Find the total time Jade spends in the air before landing on the other building. [3]

11



The diagram shows a parallelogram $OABC$ where O is the origin and A is the point (1,4).

The equations OA, OC and CB are $y = 4x$, $y = \frac{1}{3}x$ and $y = 4x - 11$ respectively.

The perpendicular from A to OC meets OC at the point D.

Find

(a) the coordinates of C, of B and of D. [6]

Continuation of working space for question 11 (a)

(b) area of parallelogram *OABC*. [2]

12 A radioactive substance undergoes decay. Its mass, M grams, after t days is defined by $M = ae^{-kt}$, where a and k are constants.

The table below shows some measurements of M and t.

t (days)	1	4	7	10	12
M (grams)	1.40	1.06	0.82	0.63	0.53

(a) Plot $\ln M$ against t and draw a straight line graph to illustrate the information. [2]

(b) Use your graph to estimate the value of a and of k. [4]

(c) Use your graph to estimate the time taken, to the nearest day, when at least 60% of the radioactive substance has decayed. [2]

13 A circle has a diameter AB. The point A has coordinates $(1, -6)$ and the equation of the tangent to the circle at B is $3x + 4y = k$.

(a) Show that the equation of the normal to the circle at the point A is $4x - 3y = 22$. [3]

(b) It is also given that line $x = -1$ touches the circle of the point $(-1, -2)$.
Find the coordinates of the centre and the radius of the circle.
Hence, state the equation of the circle. [5]

(c) Find the value of k. [2]

BLANK PAGE

NAME	FORM CLASS	ACAD CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2025

SUBJECT	**:**	**Additional Mathematics (4049/02)**	**DATE**	**:**	**21 August 2025**
LEVEL	**:**	**Secondary 4 Express**	**DURATION**	**:**	**2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST
Write your Name, Form Class, Acad Class and Index No. in the spaces at the top of the page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value of 3.142, unless the equation requires the answer in terms of π

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90 .

For Examiner's Use			
Type of Penalties		Question Number	
Units	–1		
	–2		
Presentation	–1		
	–2		**90**

This paper consists of 19 printed pages including this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

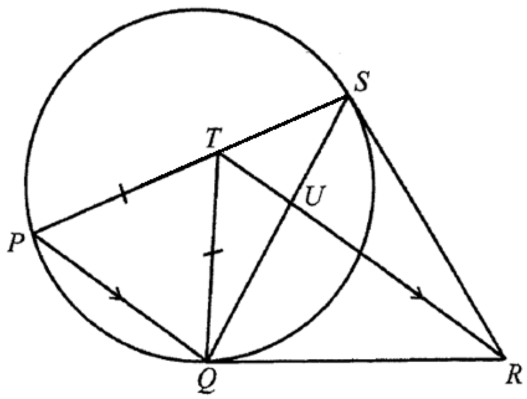
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1



In the diagram, P, Q and S lie on a circle. P,T and S lie on a straight line. The tangents to the circle at Q and S meet at R, and PQ is parallel to TR. SQ and TR intersect at U and $PT = QT$.

(a) Prove that ΔTQU is similar to ΔSRU . [5]

(b) Hence, show that a circle can be drawn passing through Q, R, S and T. [1]

(c) What conclusion can you make of angle QTS and QRS ? [1]

2 **(a)** Solve the equation $\log_2 x^2 + \log_{16} x = -2.5$. [3]

(b) A cylinder of volume $\left(3\sqrt{7}-6\right)\pi$ cm^3 has a height of $\left(2+\sqrt{7}\right)$ cm and a radius of r cm.
Without using a calculator, obtain expression for r^2 in the form $\left(h+k\sqrt{7}\right)$, where h and k are integers.

[4]

3 A man suffering from insomnia took a sleeping pill at 9:30 p.m. The initial drug level in his blood is 6 mg per litre. After T hours, the amount of drug in his blood is N, where $N = 6(0.7)^T$ mg per litre.

(a) Calculate the amount of drug at midnight in per litre of blood. [2]

(b) It is found that when the amount of drug has fallen to below $\frac{3}{4}$ of the original level, the man would fall asleep.
Estimate the time, to the nearest minutes, that the man is expected to fall asleep. [3]

4 **(a)** Given that $(1+kx)^n = 1-12x+63x^2+\dots$, find the value of k and of n. [4]

(b) In the expansion of $\left(1-2x^{3}\right)\left(3x^{2}+\dfrac{1}{2x}\right)^{9}$, find the term independent of x. [5]

5 **(a)** A, B, and C are the angles of a triangle.
Show that $\tan C = -\tan(A+B)$ [2]

(b) **(i)** State the exact value of $\cos(390°)$ [1]

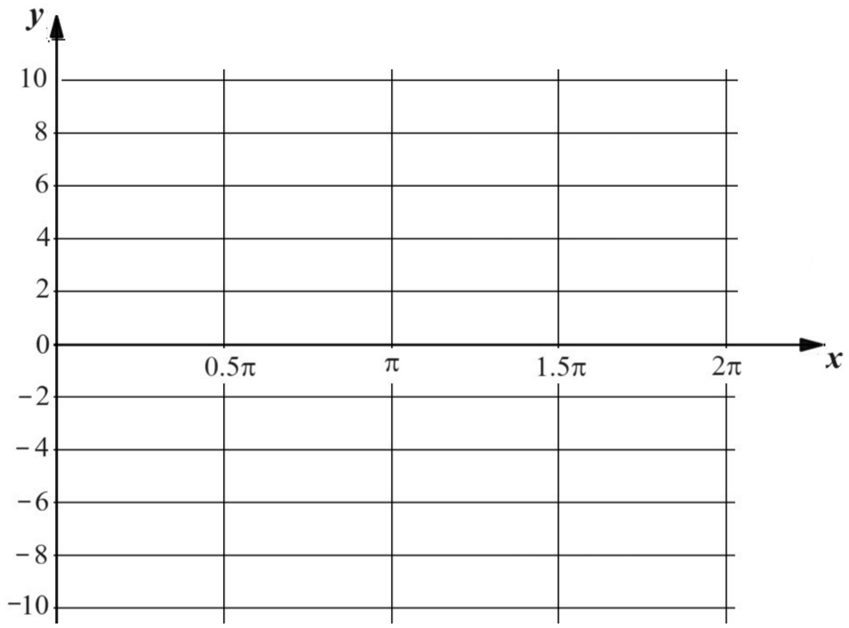
(ii) Hence, by using double angle formula, find the exact value of $\sin(195°)$. [4]

6 **(a)** The graph of $y = a\sin bx + h$ has a minimum point at $\left(\frac{3\pi}{4}, -2\right)$ followed by a maximum point at $\left(\frac{5\pi}{4}, 8\right)$.

Find the values of the constants a, b and h. [3]

(b) Hence,

(i) sketch the graph of $y = a\sin bx + h$ in the grid below. [2]



(ii) By drawing a suitable straight line on the axes above, find the number of solutions for $a\sin bx + h = \frac{12x}{\pi} - 4$. [2]

7 **(a)** Show that $(x+2)$ is a factor of $4x^3-3x^2-16x+12$ and hence factorise $4x^3-3x^2-16x+12$ completely. [4]

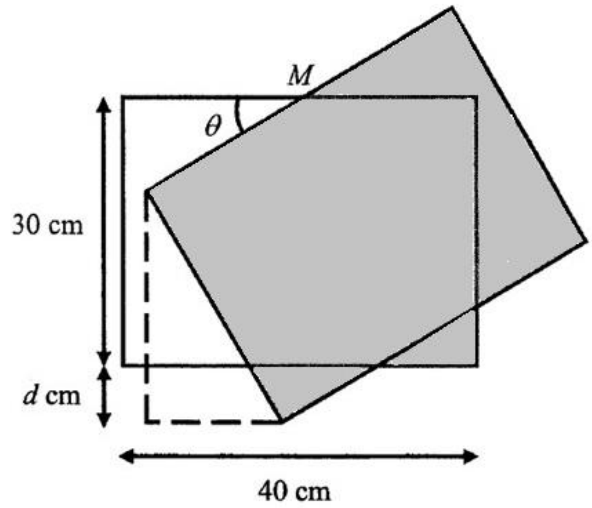
(b) Hence, solve the equation $4(9^{y})+12(3^{-y})=3^{y+1}+16$. [5]

8 A particle, travelling in a straight line, has velocity, v m/s, given by $v = 4\cos 3t + 2$, where t is the time, in seconds, after passing through O.

(a) Find the first two instances when the particle changes direction of motion. [3]

(b) Find the total distance travelled by the particle from $t = 0\,\text{s}$ to $t = \dfrac{\pi}{2}\,\text{s}$. [6]

9



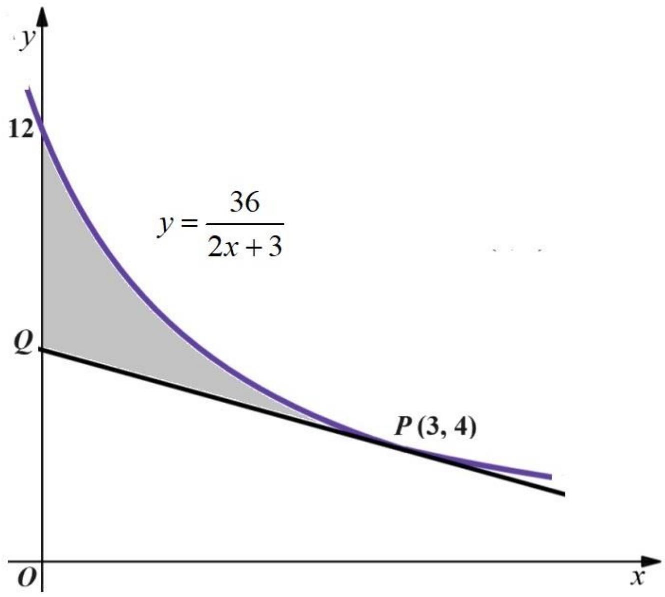
The diagram shows a rectangular picture frame 40 cm by 30 cm hung on the wall. The picture frame is rotated through an angle θ about the midpoint, M of the top edge of the frame.

(a) Show that the vertical displacement, d cm, of the picture frame below its original bottom edge is given by $d = 20\sin\theta + 30\cos\theta - 30$. [3]

(b) Express d in the form of $R\sin(\theta + \alpha) - 30$, where $R > 0$ and $0 < \alpha < 90°$. [3]

(c) Find the maximum value of d and the corresponding value of θ. [2]

10

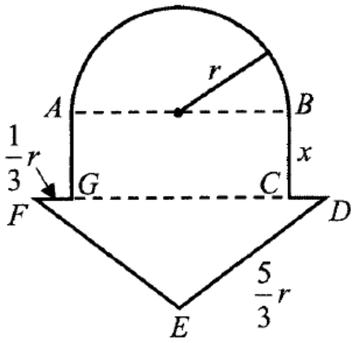


The diagram shows part of the curve $y = \dfrac{36}{2x+3}$ and its y-intercept is 12.

The tangent to the curve at the point $P(3, 4)$ intersects the y-axis at Q.
Find the exact area of the shaded region. [10]

Continuation of working space for Question 10.

11



A baker uses 131 cm of wire to enclose a cake mould of the shape shown in the diagram. The shape consists of a semicircle with diameter AB, a rectangle $ABCG$ and an isosceles triangle FED such that $FE = ED$.

It is given that $AB = 2r$ cm, $BC = x$ cm, $ED = \frac{5}{3}r$ cm and $FG = CD = \frac{1}{3}r$ cm.

(a) Express x in terms of r and π. [3]

(b) Show that the area of the mould, A cm^2, is given by $A = 131r - \frac{\pi}{2}r^2 - \frac{8}{3}r^2$. [4]

(c) Given that r can vary, find the stationary value of A and determine its nature. [5]

End of Paper