Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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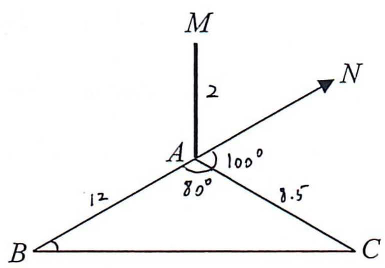
MA304.4.2 – 3D Problems

At the end of this unit, you should be able to
• **solve three-dimensional problems using trigonometry**

With the aid of trigonometry, we can also solve some problems in geometry involving three dimensional figures.

E1. A mast stands vertically at point A such that A, B and C are on level ground with the bearing of C is 100° from A and B is due south of A. Given that $AB = 12\,\text{m}$, $AC = 8.5\,\text{m}$ and $AM = 2\,\text{m}$, find



(a) the bearing of C from B,

$\angle BAC = 180^\circ - 100^\circ$
$= 80^\circ$
$BC^2 = 12^2 + 8.5^2 - 204\cos(80^\circ)\ \text{cm}^2$
$BC = \sqrt{216.25 - 204\cos(80^\circ)}\ \text{cm}$
$\dfrac{\sin\angle ABC}{8.5} = \dfrac{\sin(80^\circ)}{\sqrt{216.25 - 204\cos(80^\circ)}}\ \text{cm}$

Bearing of C from B = 038.5° (3s.f.)

(b) the bearing of B from C,

Bearing of B from C = $180^\circ + 038.5^\circ$
$= 218.5^\circ$ (1d.p.)

(c) the shortest distance from A to the line BC,

$\sin\angle ABC = \dfrac{Ax}{12\,\text{cm}}$
$= 7.47\ \text{cm}$ (3s.f.)

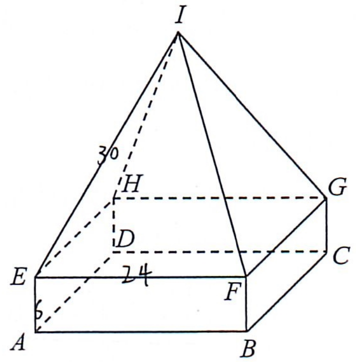
(d) the angle of elevation of M from D, which is the point of shortest distance from A from the line BC.

$\tan(\theta) = \frac{2}{AD}$

$\theta = 15.0°$ (1d.p.)

P1. A regular square pyramid sits on top of a cuboid such that I lies in the intersection of the diagonals of the square $EFGH$ which is of side 24 cm. If the height of the cuboid is 6 cm and the length of each edge of the pyramid is 30 cm, find

(a) the angle of elevation of I from E, and



$\frac{1}{2}EX = 12$ cm

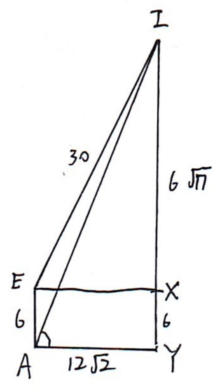
$EX = \sqrt{12^2+12^2}$ cm

$= \sqrt{288}$ cm (5s.f.)

Angle of elevation $= \cos^{-1}\left(\frac{\sqrt{288}}{30}\right)°$

$= 55.6°$ (1d.p.)

(b) the angle of elevation of I from A.



Let Y be the center of ABCD.

$AY = EX$

$= 12\sqrt{2}$ cm

$IX = \sqrt{EI^2 - EX^2}$

$= 6\sqrt{17}$ cm

$\measuredangle IAY = \tan^{-1}\left(\frac{6\sqrt{17}+6}{12\sqrt{2}}\right)°$

$= 61.1°$ (1d.p.)

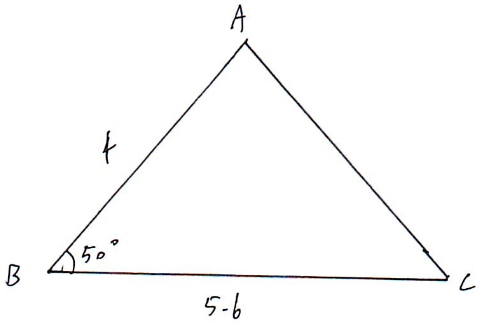
Angle of elevation is $61.1°$.

P2. Three masts of height 2 m, 5 m and 7.5 m stands vertically at points A, B and C respectively which are on level ground. It is given that $\measuredangle ABC = 50^\circ$, $AB = 4$ m and $BC = 5.6$ m. Denoting the top of the masts by A', B' and C' in the same order, calculate

(a) the length of AC,

$AC = \sqrt{4^2 + 5.6^2 - 2 \times 4 \times 5.6 \times \cos(50^\circ)}$

$= 4.31$ m (3 s.f.)



(b) the angle of elevation of C' from B', and

Since $B'B \parallel C'C$ and $B'D \parallel BC$,

$B'D = BC$

$= 5.6$ m

$C'D = C'C - B'B$

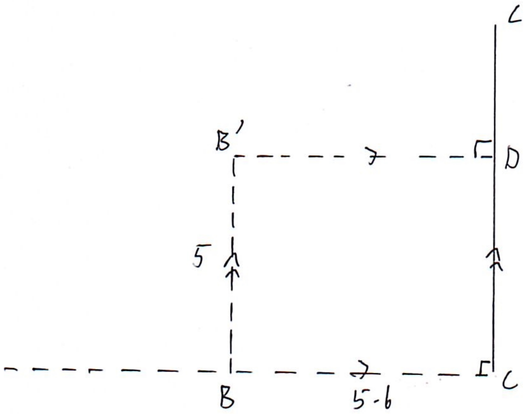
$= 7.5 - 5$ m

$= 7.5$ m

$\measuredangle C'B'D = \tan^{-1}\left(\frac{2.5}{5.6}\right)$

$= 24.1^\circ$ (1 d.p.)

Angle elevation is 24.1°



(c) the angle of depression of A' from B'.

Since $EA \parallel B'B$ and $EB' \parallel AB$,

$EB'' = AB$

$= 4$ m

$EA' = B'B - A'A$

$= 5 - 2$ m

$= 3$ m

Since $\tan(\angle EB'A') = \frac{EA'}{EB'}$

$\measuredangle EB'A' = \tan^{-1}\left(\frac{3}{4}\right)$

$= 36.9^\circ$ (1 d.p.)

Angle of depression is 36.9°.