Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2021

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS HIGHER LEVEL

Paper 1 – SOLUTIONS

SECTION A

Qn	Solution	Comments
1	$2^{x+1}=\frac{\sqrt{10^x+3}}{5^x}$ $2\cdot 2^x\cdot 5^x=\sqrt{10^x+3}$ $2\cdot 10^x=\sqrt{10^x+3}$ $4\cdot 10^{2x}=10^x+3$ Let $y=10^x$ $4y^2-y-3=0$ $(4y+3)(y-1)=0$ $y=-\frac{3}{4}\quad or \quad 1$ $x=0$	This question was less well-done than expected. Even though most candidates were able to break down to bases 2 and 5, many were not able to use substitution $y=10^x$. A handful were also unable to correctly factorise the resultant quadratic equation. Finally, a few candidates were penalized for not knowing that $lg\,1=0$.
2(a)	$\log_3 x-\log_3(x+4)+2=0$	Generally well done, though there were many who made careless mistake on the last

Qn	Solution	Comments
	$\log_3 \frac{9x}{x+4} = 0$ $\frac{9x}{x+4} = 1$ $9x = x+4$ $x = \frac{1}{2}$	step and erroneously wrote down $x=2$ instead of $x=\frac{1}{2}$.
2(b)	$\log_x 5 + \log_5 x = 2$ $\log_x 5 + \frac{1}{\log_x 5} = 2$ $(\log_x 5)^2 - 2\log_x 5 + 1 = 0$ $(\log_x 5 - 1)^2 = 0$ $\log_x 5 = 1$ $x = 5$	Mostly well done. Most candidates were able to perform change of base for log but a few had difficulty solving the resultant quadratic equation.
3(a)	If $P(X < a) + P(X \le b) = 1$ $P(X \le b) = 1 - P(X < a)$ $P(X \le b) = P(X \ge a)$.	A very simple 2-liner proof which took some candidates many lines. Candidates should not assume upper/lower tail regions to be 0.
3(b)	$\mu = \frac{a+b}{2}$ $\mu = \frac{100}{2}$ $\mu = 50$	Well done.

Qn	Solution	Comments
4(a)	$(1+4x)^{\frac{1}{2}}=\left[(4x)\left(1+\frac{1}{4x}\right)\right]^{\frac{1}{2}}$ $=(4x)^{\frac{1}{2}}\left(1+\frac{1}{4x}\right)^{\frac{1}{2}}$ $=2\sqrt{x}\left(1+\frac{1}{2}\left(\frac{1}{4x}\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{2!}\left(\frac{1}{4x}\right)^2+\ldots\right)$ $=2\sqrt{x}\left(1+\frac{1}{8x}-\frac{1}{128x^2}+\ldots\right)$ $=2\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{1}{64}x^{-\frac{3}{2}}+\ldots$	Disappointingly, many candidates were not able to expand in descending powers. Some who were able to made careless calculation errors. Candidates who did not simplify coefficients or powers of x for final answer were penalized.
4(b)	$\left\|\frac{1}{4x}\right\|<1$ $\Rightarrow \|4x\|>1$ $\Rightarrow \|x\|>\frac{1}{4}$	Mark was awarded as long as $\left\|\frac{1}{4x}\right\|<1$ was seen as most students had difficulty manipulating to the correct final form.
5(a)(i)	$u_{10}=a+9d=10\ \ldots\ldots..(1)$ $S_{10}=\frac{10}{2}\left[2a+9d\right]=10$ $\Rightarrow 2a+9d=2\ldots\ldots..(2)$ Solving (1) and (2) yields: $10-a=2-2a$ $a=-8$ $9d=10-(-8)$ $\Rightarrow d=2$	Generally well done.
5(a)(ii)	$S_{20}=\frac{20}{2}\left[2(-8)+19(2)\right]$ $=10(22)$ $=220$	Generally well done.
5(b)	The general term is given by $u_n=S_n-S_{n-1}=\left(3n^2-2n\right)-\left(3(n-1)^2-2(n-1)\right)$ $=3n^2-2n-3\left(n^2-2n+1\right)+2n-2$ $=6n-5$	Poorly done. Many candidates mistook S_n-S_{n-1} to be the expression for the common difference d. Instead of considering u_n-u_{n-1}, some went to

Qn	Solution	Comments
	Since $u_n = 6n - 5$, then we must have $u_{n-1} = 6(n-1) - 5 = 6n - 11$, so we have $d = (6n-5) - (6n-11) = 6$, which is a constant. Thus the series is an AP.	calculate specific terms u_1, u_2, u_3 instead. Candidates wasted time finding individual expressions for $u_n = S_n - S_{n-1}$ and $u_{n+1} = S_{n+1} - S_n$ instead of using one to obtain the other.
6(a)	$\begin{pmatrix} 1 & -1 & -1 & 1 \\ 3 & 1 & 1 & 3 \\ 2 & -2 & k & 1 \end{pmatrix} \xrightarrow[R3 \to R3 - 2R1]{R2 \to R2 - 3R1} \begin{pmatrix} 1 & -1 & -1 & 1 \\ 0 & 4 & 4 & 0 \\ 0 & 0 & k+2 & -1 \end{pmatrix}$ When $k \neq -2$, there will be a unique solution.	Most candidates were able to perform manual row operations to obtain final RREF matrix. Some candidates were not able to apply the condition for unique solutions to exist.
6(b)	When $k = -2$, there are no solutions.	Well done.
7(a)	Area of sector POQ = $\frac{1}{2}r^2\theta$ Area of triangle POQ = $\frac{1}{2}r^2\sin\theta$ Since ratio of area of sector: area of segment = 4 : 1, then area of sector: area of triangle = 4 : 3. Thus we have $\frac{1}{2}r^2\theta : \frac{1}{2}r^2\sin\theta = 4 : 3$ Thus the ratio is $\theta : \sin\theta = 4 : 3$.	Not as well done as expected. Surprisingly there were a handful candidates who did not know the formulae for the area of segment. Some also made careless mistakes in translating the ratio in question in terms of areas of the two regions.
7(a)	Alternatively: Area of sector POQ = $\frac{1}{2}r^2\theta$ Area of segment POQ = $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$	

<table>
<tr><th>Qn</th><th>Solution</th><th>Comments</th></tr>
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$$\frac{\frac{1}{2}r^2\theta-\frac{1}{2}r^2\sin\theta}{\frac{1}{2}r^2\theta}=\frac{1}{4}$$

$$\Rightarrow 2r^2\theta-2r^2\sin\theta=\frac{1}{2}r^2\theta$$

$$\Rightarrow \frac{3}{2}r^2\theta=2r^2\sin\theta$$

$$\Rightarrow \frac{\theta}{\sin\theta}=2\div\frac{3}{2}=\frac{4}{3}$$

Thus the ratio is $\theta : \sin\theta = 4:3$.

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<tr><td>7(b)</td><td>

$$LHS=\frac{\cos 3\theta}{\sin 2\theta}$$

$$=\frac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{2\sin\theta\cos\theta}$$

$$=\frac{\cos 2\theta}{2\sin\theta}-\sin\theta$$

$$=\frac{1-2\sin^2\theta}{2\sin\theta}-\sin\theta$$

$$=\frac{1}{2}\,csc\,\theta-2\sin\theta$$

$$=RHS\ (\,shown)$$

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Generally well done as long as candidates were able to use Addition Formula on $\cos 3\theta$ and Double Angle Formula on $\sin 2\theta$.

A few candidates went to use De Moivre’s Theorem to expand both numerator and denominator which was not necessary and took more time.

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<tr><td>7(b)</td><td>

Alternative Solution:

$$RHS=\frac{1}{2}csc\,\theta\ -\ 2\sin\theta$$

$$=\frac{1-4\sin^2\theta}{2\sin\theta}$$

$$=\frac{1-4\left(1-\cos^2\theta\right)}{2\sin\theta}$$

$$=\frac{4\cos^2\theta\ -\ 3}{2\sin\theta}$$

$$=\frac{4\cos^3\theta\ -\ 3\cos\theta}{2\sin\theta\cos\theta}$$

$$=\frac{\cos 3\theta}{\sin 2\theta}\ =LHS\ (\,shown)$$

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Only a handful candidates approached the question in this manner.

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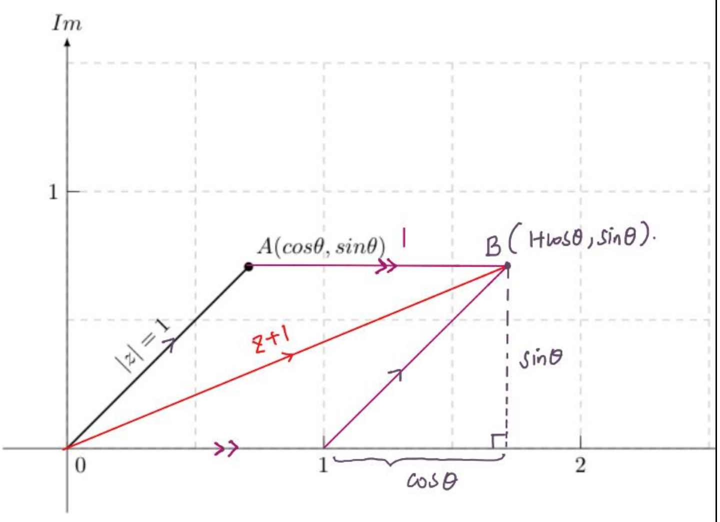
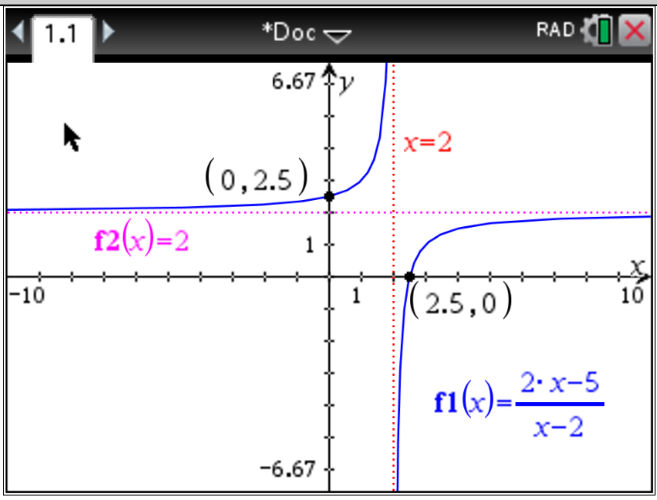
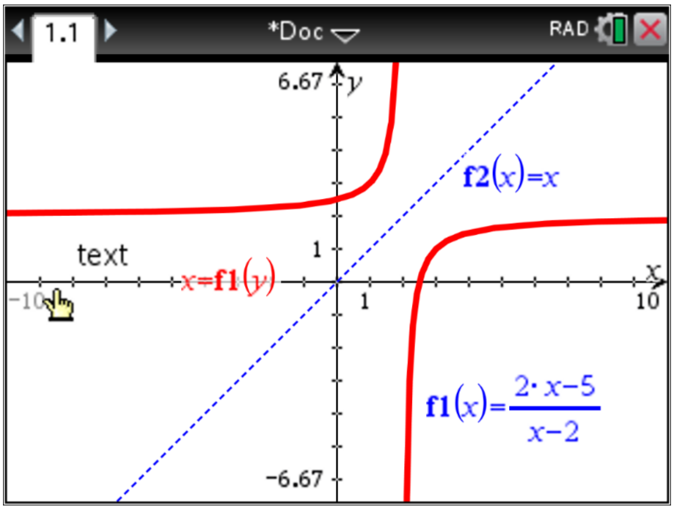
Qn	Solution	Comments
8(a)		Most candidates were able to sketch the point B in the correct region though without the detail provided in the answer.
8(b)	"Hence" by using the fact that triangle *OAB* is an isosceles triangle, and dropping a perpendicular bisector from A to OB: $\frac{1}{2}\left\|z+1\right\| = OB$ $\Rightarrow \frac{1}{2}\left\|z+1\right\| = 1\times\cos\angle AOB = \cos\frac{\theta}{2}$ $\Rightarrow \left\|z+1\right\| = 2\cos\frac{\theta}{2}$ OB is diagonal of parallelogram formed. $arg(z+1) = \frac{1}{2}arg(z)$ $= \frac{\theta}{2}$	Poorly attempted. Many candidates skipped this part entirely. The few who were able to apply geometrical approaches to this question were able to obtain the answers swiftly. Candidates who wrote the correct argument down without any reason or explanation were penalized for their lack of working and brevity.
8(b)	"Hence" by dropping a perpendicular from B to real axis to form a right-angled triangle: $\frac{\sin\theta}{OB} = \sin\frac{\theta}{2}$ $\Rightarrow \frac{\sin\theta}{\left\|z+1\right\|} = \sin\frac{\theta}{2}$ $\Rightarrow \left\|z+1\right\| = \frac{\sin\theta}{\sin\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = 2\cos\frac{\theta}{2}$	Some candidates wrote down the relations but were stuck in applying Half-Angle Formula.

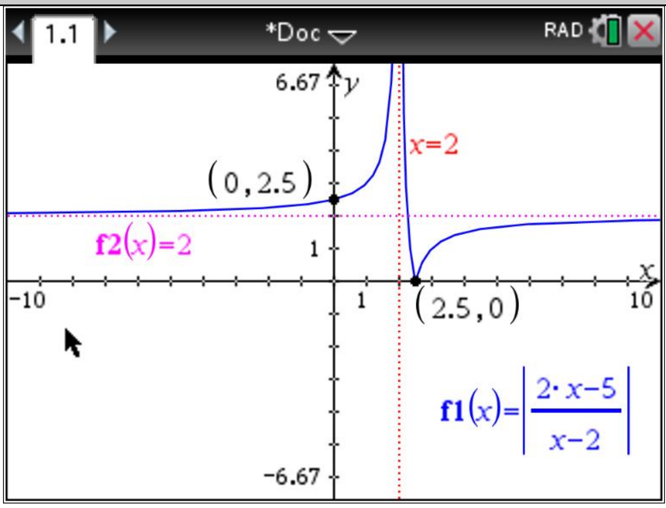
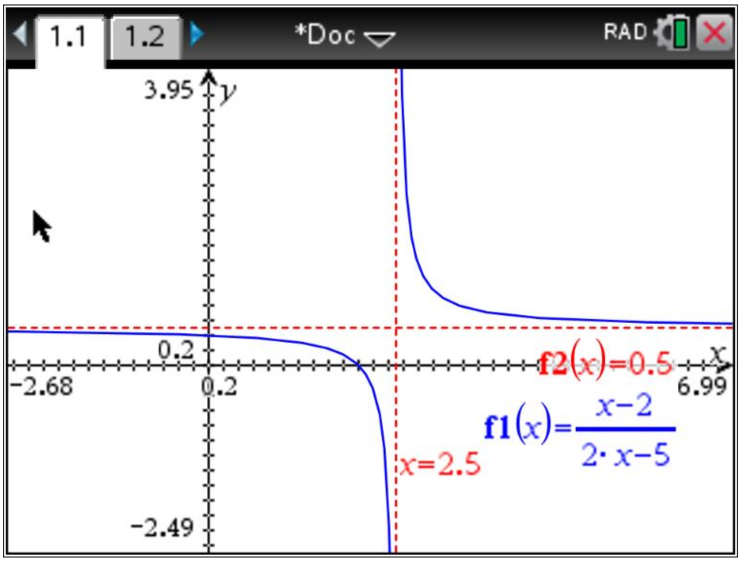
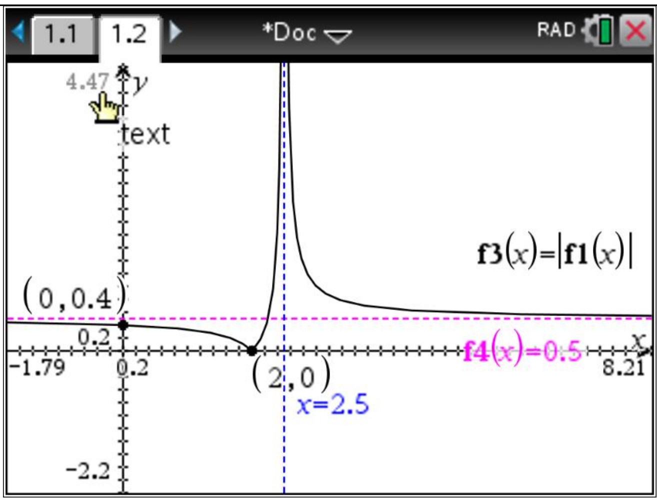
Qn	Solution	Comments
	OB is diagonal of parallelogram formed. $arg(z+1) = \frac{1}{2} arg(z)$ $= \frac{\theta}{2}$	
8(b)	"Hence" by using the fact that triangle *OAB* is an isosceles triangle, $\angle OAB = \pi - \theta$ Sine Rule: $\frac{OB}{\sin\angle OAB} = \frac{OA}{\sin \angle ABO}$ $\Rightarrow \frac{\vert z+1 \vert}{\sin(\pi - \theta)} = \frac{1}{\sin \frac{\theta}{2}}$ $\Rightarrow \vert z+1 \vert = \frac{\sin(\pi-\theta)}{\sin \frac{\theta}{2}}$ $= \frac{\sin\theta}{\sin\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = 2\cos\frac{\theta}{2}$ Base angles of an isosceles triangle: $\angle ABO = \angle AOB$ $arg(z+1) = \angle ABO$ $= \frac{\pi - (\pi - \theta)}{2}$ $= \frac{\theta}{2}$	Few candidates approached the question using Sine Rule. Out of those who did, most were stuck in their attempt at applying Half-Angle formula.
8(b)	"Hence" by using the fact that triangle *OAB* is an isosceles triangle, $\angle OAB = \pi - \theta$ Cosine Rule:	Very few candidates approached the question using this method.

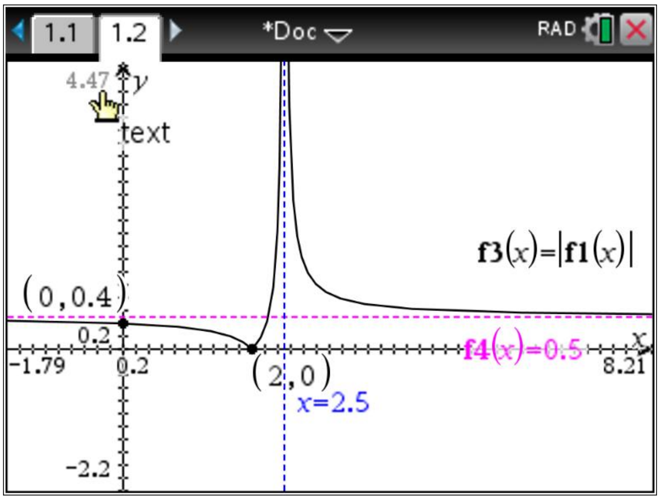
Qn	Solution	Comments
	$(OB)^2=1^2+1^2-2(1)(1)\cos(\pi-\theta)$ $=2-2\cos(\pi-\theta)$ $=2-2\left[1-2\sin^2\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right]$ $=4\sin^2\left(\frac{\pi}{2}-\frac{\theta}{2}\right)$ $\therefore \lvert z+1\rvert=OB=2\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)=2\cos\left(\frac{\theta}{2}\right)$ Base angles of an isosceles triangle: $\angle ABO=\angle AOB$ $arg(z+1)=\angle ABO$ $=\frac{\pi-(\pi-\theta)}{2}$ $=\frac{\theta}{2}$	
8(b)	"Otherwise" method: $z+1=e^{i\theta}+1$ $=e^{i\left(\frac{\theta}{2}\right)}\left(e^{i\left(\frac{\theta}{2}\right)}+e^{-i\left(\frac{\theta}{2}\right)}\right)$ $=2\cos\left(\frac{\theta}{2}\right)e^{i\left(\frac{\theta}{2}\right)}$ Thus we have $\lvert z+1\rvert=2\cos\left(\frac{\theta}{2}\right)$ $arg(z+1)=\frac{\theta}{2}$	A handful of candidates used this method but were not able to read off the modulus and argument even after arrival at the $2\cos\left(\frac{\theta}{2}\right)e^{i\left(\frac{\theta}{2}\right)}$.
8(b)	"Brute force" method:	Most common approach for modulus but most candidates were stuck when they reached $\sqrt{2+2\cos\theta}$, again hindered by inability to manipulate using Half Angle formula. Ditto for the argument calculation.

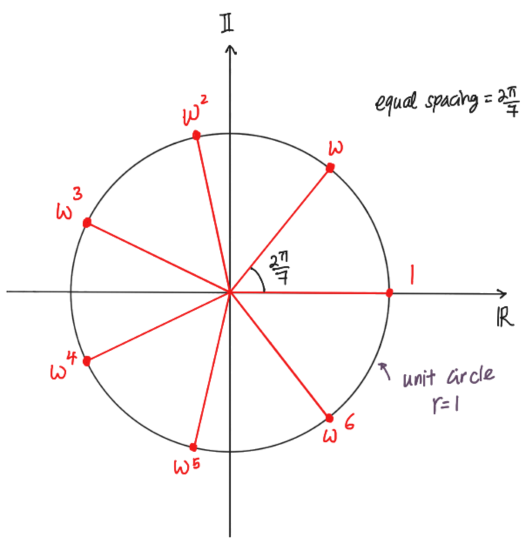
Qn	Solution	Comments
	$\lvert z+1\rvert=\sqrt{(\cos\theta+1)^2+(\sin\theta)^2}$ $=\sqrt{\cos^2\theta+2\cos\theta+1+\sin^2\theta}$ $=\sqrt{2+2\cos\theta}$ $=\sqrt{2+2\left[2\cos^2\left(\frac{\theta}{2}\right)-1\right]}$ $=\sqrt{\left(4\cos^2\left(\frac{\theta}{2}\right)\right)}$ $=2\cos\left(\frac{\theta}{2}\right)$ $arg(z+1)=\tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$ $=\tan^{-1}\left(\frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{1+2\cos^2\left(\frac{\theta}{2}\right)-1}\right)$ $=\tan^{-1}\left(\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}\right)$ $=\frac{\theta}{2}$	
9	Let P_n be the proposition that $x_n<2$ for all positive integers n. $n=1$: $x_1=\frac{5}{3}<2$ Since $x_1<2$, thus P_1 is true. $n=k$: Assume P_k is true for some positive integer k i.e. $x_k<2$ $n=k+1$: To prove P_{k+1} is true i.e. $x_{k+1}<2$	Not well attempted. Some candidates defined the proposition as the recurrence relation. Some candidates used $n=2$ as the first term when the smallest positive integer is $n=1$. Many candidates were penalized for not using induction hypothesis and starting from $x_k<2$, or not showing explicitly that $x_{k+1}<\frac{3}{2}<2$. Do not start by assuming $x_{k+1}<2$. Candidates were also penalized when they did not write the final conclusion properly which showed a lack

Qn	Solution	Comments
	$LHS = x_{k+1}$ $= \sqrt[3]{\left(\frac{11}{8} + x_k\right)}$ $< \sqrt[3]{\left(\frac{11}{8} + 2\right)}$ $= \sqrt[3]{\frac{27}{8}}$ $= \frac{3}{2}$ < 2 Thus, P_{k+1} is true whenever P_k is true. Since P_1 is true and P_{k+1} is true whenever P_k is true, hence by mathematical induction P_n is true for all positive integers n.	of understanding of the Principle of Mathematical Induction. There were some who were penalized for confusing the notation of P_k with x_k. Minor errors to note: -Defining "for all positive integers n" in the proposition -Assume P_k is true for SOME $k \in \mathbb{Z}^+$ (some candidates wrote "all" instead which is a grave error) -Spell out mathematical induction instead of writing MI – this is not a recognized acronym
Section B		
10.	**Total:19**	
(a)	$y = \frac{coeffi \text{ of } x}{coeffi \text{ of } x} = \frac{2}{1}$	Generally well done. Some student went to rewrite into $2 + \frac{k}{x-2}$ but make a mistake in the process, using the formula stated here is easier.
	$y = 2$	Some students mistook horizontal asymptote as $x = 2$

Qn	Solution	Comments
(b)		Generally well done. Quite a number of students drew 2.5 on the left of the vertical asymptote.
(c)	Algebraic Method: $y=\dfrac{2x-5}{x-2}$	
	$xy-2y=2x-5$ $x(y-2)=-5+2y$ $x=\dfrac{2y-5}{y-2}$	Some students prove by showing that $f\circ f^{-1}(x)=x$ which is also acceptable, just very tedious.
	$f^{-1}(x)=\dfrac{2x-5}{x-2}=f(x)$	
	OR: Sketch the inverse function 	Student who stated "reflection about $y=x$ line" is also marked correct. Some students confused between one to one function and self-inverse function.

Qn	Solution	Comments
(d) (i)		Some students forgot to extend the line close to $x=2$. This might be due to the fact that they are afraid to cut the horizontal asymptote. Do note that horizontal asymptote becomes the limit only when x approaches $\pm\infty$, so it is possible to cut at non-infinity values. However, vertical asymptotes are not possible to cut nor touch.
(d) (ii)	Method 1: 	First sketch $\dfrac{1}{f(x)}$
		Reflect the negative graph about the x-axis

Qn	Solution	Comments
	Method 2: 	Directly sketch from graph of $\|f(x)\|$ Original x-intercept becomes the vertical asymptote since you get $\frac{1}{0}$ Original vertical asymptote becomes the x-intercept
(e)	(i) horizontal stretching with scale factor $\frac{1}{2}$	Some students forgot the reciprocate the coefficient of x as the scale factor
	Vertical translation 1 unit downwards	Explain the direction (-1) represents. Though some past year papers just stated (-1) without explanation, it is safest to explain what negative sign indicates in terms of translation.
	(ii) $g(x)=\frac{2(2x)-5}{2x-2}-1$	Some students did $\frac{2(2x-5)}{2x-2}$ which is wrong
	$g(x)=\frac{4x-5-2x+2}{2x-2}$	
	$g(x)=\frac{2x-3}{2x-2}$	
11.	**Total: 19**	
(a)	Substitute $z=\omega=cis\left(\frac{2\pi}{7}\right)$	
	$z^7=\left(cis\left(\frac{2\pi}{7}\right)\right)^7$ $=cis(2\pi)$	Needs to show argument becomes 2π, some omitted

Qn	Solution	Comments
		steps. Try to show as much as possible for show questions
	$=1$	
(b)	$1+\omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6$ $=\dfrac{1-\omega^7}{1-\omega}$ or $\dfrac{\omega^7-1}{\omega-1}$	use sum of GP or long division, GP sum might be faster
	Since $\omega^7=1$ (proven in part (a))	
	$=\dfrac{0}{1-\omega}$	
	$=0$	AF
(c)	 Accept both points and vectors	Modulus =1 for all roots (eg. sketch unit circle or write down modulus for at least one root) Roughly equal spacing between roots since the spacings are all equal Label ω in increasing power anti-clockwise, when multiplied by a complex number, the argument is added to the original number.
(d)	$b=-(\alpha+\alpha^*)$, $c=\alpha\cdot\alpha^*$	Some students forgot to state b as the subject, or forgot the negative sign.
(e)	Method 1: . $\alpha^*=(\omega+\omega^2+\omega^4)^*$	Not well done, students forgot about conjugate properties.

Qn	Solution	Comments
	$=\omega^*+(\omega^2)^*+(\omega^4)^*$	
	$=\omega^6+\omega^5+\omega^3$	
	$=\omega^3+\omega^5+\omega^6$	
	Method 2: $\alpha^*=(\omega+\omega^2+\omega^4)^*$	
	$=\left(\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)+i\left(\sin\left(\frac{2\pi}{7}\right)+\sin\left(\frac{4\pi}{7}\right)+\sin\left(\frac{6\pi}{7}\right)\right)\right)^*$	
	$=\left(\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)-i\left(\sin\left(\frac{2\pi}{7}\right)+\sin\left(\frac{4\pi}{7}\right)+\sin\left(\frac{6\pi}{7}\right)\right)\right)$	
	$=cis\left(-\frac{2\pi}{7}\right)+cis\left(-\frac{4\pi}{7}\right)+cis\left(-\frac{6\pi}{7}\right)$	
	$=\omega^6+\omega^5+\omega^3$	
	$=\omega^3+\omega^5+\omega^6$	
(f)	$b=-(\alpha+\alpha^*)$ $=-(\omega+\omega^2+\omega^4+\omega^3+\omega^5+\omega^6)$	Well done generally, though some students went to substitute individual arguments, and could not derive at the solution. Make use of previous parts, they normally lead you to the answer.
	$=-(0-1)=1$	
	$c=\alpha\cdot\alpha^*$ $=(\omega+\omega^2+\omega^4)\cdot(\omega^3+\omega^5+\omega^6)$	
	$=\omega^4+\omega^6+\omega^7+\omega^5+\omega^7+\omega^8+\omega^7+\omega^9+\omega^{10}$	
	$=\omega^4+\omega^6+1+\omega^5+1+\omega+1+\omega^2+\omega^3$ (since $\omega^7=1$)	Remember any power higher than n can be converted into powers from 1 to n-1 (cyclic property)

Qn	Solution	Comments
	$= 3 + (\omega + \omega^2 + \omega^4 + \omega^3 + \omega^5 + \omega^6)$ $= 3 + (-1)$	
	$= 2$	
12.	**Total: 17**	
(a)(i)	Let $y = e^x$ $(y-5)(y-1) = 0$	Some students did not utilize the quadratic structure and made the question difficult by using quadratic formula method.
	$y = 5, y = 1$	
	$e^x = 5$, $e^x = 1$,	
	$x = \ln 1 = 0$, $x = \ln 5$	
	Mid-point of quadratic form $y = \frac{5+1}{2} = 3$,	
	Minimum point coordinates= $(\ln 3, -4)$	Some students forgot the find x-coordinate from e^x value.
(ii)	$a = \ln 3$	
(iii)	$y = e^{2x} - 6e^x + 5$	
	Method 1: $y = e^{2x} - 6e^x + 9 - 9 + 5$ $y = (e^x - 3)^2 - 4$	Forming complete square is easier to find inverse of a quadratic structured function
	$y + 4 = (e^x - 3)^2$ $e^x - 3 = \pm\sqrt{y+4}$ $e^x = 3 \pm \sqrt{y+4}$	

Qn	Solution	Comments
	$e^x = 3 \pm \sqrt{y+4}$	
	$x = \ln(3 \pm \sqrt{y+4})$ $x = \ln(3 - \sqrt{y+4})$ [since $x \le \ln 3$]	Some students rejected the negative version
	$g^{-1}(x) = \ln(3 - \sqrt{x+4}), -4 \le x < 5$	
(iv)	The graph of $g(x)$ and $g^{-1}(x)$ are symmetrical about the line $y = x$ Or they are reflections about the line $y = x$	Well done
(b)	Suppose/Assume the equation has at least <u>one rational root</u>	Some students wrote the wrong assumption, note that b and c are odd numbers are the conditions, has no rational root is the statement. One assume the negation of the statement, keeping conditions the same for proof by contradiction.
	Method 1: Let the root be $x = \frac{p}{q}$ where p and q has no common factor and $q \ne 0$, $p, q \in \mathbb{Z}$	A number of students did not state the condition for x to be rational number, should emphasize "no common factor"
	$\left(\frac{p}{q}\right)^2 + b\left(\frac{p}{q}\right) + c = 0$	
	$p^2 + bpq + cq^2 = 0$	
	Or $x = \frac{-b \pm \sqrt{b^2 - 4c}}{2} = \frac{p}{q}$	

Qn	Solution	Comments
	$-bq \pm q\sqrt{b^2-4c} = 2p$ $\pm q\sqrt{b^2-4c} = 2p+bq$ $q^2(b^2-4c) = 4p^2+4bpq+b^2q^2$ $p^2+bpq+cq^2=0$	
	If p and q are both odd, LHS is odd since b,c are odd and cannot be equal to 0 (which is even) If p is odd and q is even without loss of generality, LHS p^2 is odd, bpq is even and c is odd, so the sum must be odd as well, which again cannot be equal to 0.	Some students discussed even and odd numbers while the equation is still in fraction. Note that we only discuss odd/even for integers, so the denominator should be eliminated first from the equation.
	There is contradiction with the assumption. Therefore by contradiction, the equation has no rational root.	Some omitted the final statement. Some did not state clearly where the contradiction is.
	Method 2: $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$ $= \alpha \pm \sqrt{\beta}$, where α, β are rational numbers. 2 Real roots are either be: both rational or both irrational.	
	Assume that both roots are rational. (Proof by contradiction) $\alpha \pm \sqrt{\beta}$ is rational. $\sqrt{\beta}$ must be rational β must be a perfect square, $\beta = n^2$, where n is integer. ($n=0$ trivial case.)	

Qn	Solution	Comments
	$x^2+bx+c=0$ where b,c are odd numbers. $b=$ sum of roots $=\alpha+\sqrt{\beta}+\alpha-\sqrt{\beta}=2\alpha$ (given odd number) $c=$ product of roots=$\left(\alpha+\sqrt{\beta}\right)\left(\alpha-\sqrt{\beta}\right)=\alpha^2-\beta$ (given odd number) Given $\alpha^2-\beta$ is odd, $4\left(\alpha^2-\beta\right)$ is even: $4\alpha^2-4\beta$ $=(2\alpha)^2-4\beta$ $=(odd)^2-even$ $\quad$ $(Given\ 2\alpha$ is odd$)$ $=odd-even$ $\quad$ $(odd^2$ is odd$)$ $=odd$ $\quad$ $\left(Contradiction\right.$ with $4\left(\alpha^2-\beta\right)$ is even$\left.\right)$ $\therefore$ Assumption is false. Roots cannot be rational numbers.	