Name: MARKERS' REPORT .... Register no: .... Class: ....

NGEE ANN SECONDARY SCHOOL

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS 4049/02

Paper 2

31 August 2026

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Total	**/90**

Checked by student: ………………………………… Date: …………………

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The simultaneous equations $y = x^2 + px - 6$ and $y = 2x + \frac{3}{2}$ have a common solution for which $x = 3$. Find the value of p, and hence solve the simultaneous equations. [5]

SOLUTIONS:

Given the common solution has $x = 3$,

$$y = 2(3) + \frac{3}{2} = \frac{15}{2}$$

Subst. into $y = x^2 + px - 6$:

$$\frac{15}{2} = 3^2 + 3p - 6$$
$$p = \frac{3}{2}$$

Hence

$$x^2 + \frac{3}{2}x - 6 = 2x + \frac{3}{2}$$
$$2x^2 - x - 15 = 0$$
$$(2x+5)(x-3) = 0$$
$$x = 3 \quad \text{or} \quad x = -\frac{5}{2}$$

Using $y = 2x + \frac{3}{2}$,

$$x = 3,\ y = \frac{15}{2}$$
$$x = -\frac{5}{2},\ y = -\frac{7}{2}$$

Conceptual gaps

Some students did not know that given $x = 3$ is a common solution (from question), we can sub in $x = 3$ to the equations to find the value of p.

Many students did not find the values of y after solving for x.

2 **(a)** **(i)** In an architectural design, a support beam is modelled by the line

$$y = 2x + k$$

and a parabolic arch is modelled by

$$y = x^2 - 4x + 7\ .$$

The beam is to touch, but not cut, the arch. Find the value of *k*. [3]

SOLUTIONS:

$$x^2 - 4x + 7 = 2x + k$$
$$x^2 - 6x + (7 - k) = 0$$

For tangency, discriminant = 0.

$$(-6)^2 - 4(1)(7 - k) = 0$$
$$36 - 28 + 4k = 0$$
$$k = -2$$

Algebraic manipulation mistakes
Most students knew to equate discriminant to 0, but a few students made algebraic mistakes related to the signs.

(ii) The beam is then raised vertically by 3 units from this tangent position. Without drawing or referring to a graph, determine the number of points at which the beam intersects the arch. [2]

SOLUTIONS:

If raised vertically by 3 units, $k = 1$.

Discriminant = $(-6)^2 - 4(1)(6) = 12 > 0$

Hence, there will be 2 points of intersection.

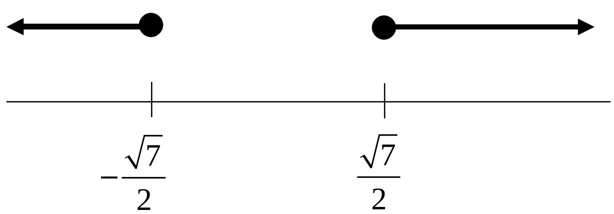
Did not read question properly/ understand the question
Some students made reference to graph(s) to answer this question when it was explicitly mentioned in question not to.
Conceptual gaps
Some students did not know that shifting beam up vertically by 3 units means the equation of the beam is now $y = 2x + 1$.

(b) Represent the solution set of $(2x+1)^2 \geq 4x+8$ on a number line. [3]

SOLUTIONS:

$$(2x+1)^2 \geq 4x+8$$
$$4x^2+4x+1 \geq 4x+8$$
$$4x^2-7 \geq 0$$
$$\left(2x-\sqrt{7}\right)\left(2x+\sqrt{7}\right) \geq 0$$

$$x \leq -\frac{\sqrt{7}}{2} \quad \text{or} \quad x \geq \frac{\sqrt{7}}{2}$$



Conceptual gaps

Some students did not know how to solve this quadratic inequality. In particular, they write $x^2 \geq \pm\frac{\sqrt{7}}{2}$ which is wrong. (What you should do) For quadratic inequalities, we typically make RHS = 0, then factorise LHS to identify the roots (i.e. x-intercepts).

3 A cubic polynomial is given as $\mathrm{P}(x) = x^3 + ax^2 + bx + c$. The graph of $y = \mathrm{P}(x)$ cuts the y-axis at $(0, 2)$. The remainder when $\mathrm{P}(x)$ is divided by $x+1$ is -3, and $x-2$ is a factor of $\mathrm{P}(x)$.

(a) Find the values of *a*, *b* and *c*. [4]

SOLUTIONS:

Since the graph cuts the y-axis at $(0, 2)$,

$\mathrm{P}(0) = 2 \quad \Rightarrow \quad c = 2$

Since $x-2$ is a factor,

$$\mathrm{P}(2) = 0$$
$$8 + 4a + 2b + 2 = 0$$
$$2a + b = -5 \quad \text{--- (1)}$$

The remainder when $\mathrm{P}(x)$ is divided by $x+1$ is -3, so

$$\mathrm{P}(-1) = -3$$
$$-1 + a - b + 2 = -3$$
$$a - b = -4 \quad \text{--- (2)}$$

Solving (1) and (2), $a = -3,\ b = 1,\ c = 2$.

Overall Performance
Most students were able to apply the remainder and factor theorem correctly and finding the values of *a*, *b* and *c*.
Common Mistakes
Mostly careless mistakes including taking remainder as 3 instead of -3, manipulation errors or computational errors.

(b) Hence solve the equation $\mathrm{P}(x)=0$, leaving your answers in exact form. [3]

SOLUTIONS:

$\mathrm{P}(x)=x^3-3x^2+x+2$

Since $x-2$ is a factor, $\mathrm{P}(x)=(x-2)(x^2+dx-1)$.

Comparing coefficients of *x*,

$-2d-1=1$

$d=-1$

$\Rightarrow \quad \mathrm{P}(x)=(x-2)(x^2-x-1)$

Hence,

$(x-2)(x^2-x-1)=0$

$x-2=0$ or $(x^2-x-1)=0$

$x=2$ or $x=\dfrac{-(-1)\pm\sqrt{1^2-4(1)(-1)}}{2}$

$=\dfrac{1+\sqrt{5}}{2}$

$x=2,\ \dfrac{1+\sqrt{5}}{2},\ \dfrac{1-\sqrt{5}}{2}$

Overall Performance

Most students were able to factorise P(*x*) by applying long division method to find the quadratic factor before factorising completely.

Common Mistakes

Mostly error carried forward from (a) leading to wrong factors and solutions. A couple of students left the answers in significant figures instead of exact form.

4 The amount of medicine remaining in a patient's bloodstream t hours after it is taken is M mg. It is modelled by the equation

$$\ln\left(\frac{M}{5}\right) = \ln 16 - \frac{t}{3}\ln 2 .$$

(a) Show that the model can be written in the form

$$M = 80e^{-\frac{t}{3}\ln 2} .$$ [2]

SOLUTIONS:

$$\ln\left(\frac{M}{5}\right) = \ln 16 - \frac{t}{3}\ln 2$$

$$\ln\left(\frac{M}{5}\right) = \ln 16 + \left(-\frac{t}{3}\ln 2\right)\ln e$$

$$\ln\left(\frac{M}{5}\right) = \ln 16 + \ln e^{-\frac{t}{3}\ln 2}$$

$$\ln\left(\frac{M}{5}\right) = \ln\left(16e^{-\frac{t}{3}\ln 2}\right)$$

$$\frac{M}{5} = 16e^{-\frac{t}{3}\ln 2}$$

$$M = 80e^{-\frac{t}{3}\ln 2} \quad \text{(shown)}$$

Overall Performance

Many students were able to show how to convert from logarithmic form to exponential form but some did not apply the concept $e^{\ln x} = x$.

Common Mistakes

Most students were unable to show the conversion clearly, there were several trials and errors or incomplete solutions.

(b) Find the time taken for the medicine to decrease to 20 mg. [3]

SOLUTIONS:

When $M = 20$,

$$80e^{-\frac{t}{3}\ln 2} = 20$$

$$e^{-\frac{t}{3}\ln 2} = \frac{1}{4}$$

$$\ln\left(e^{-\frac{t}{3}\ln 2}\right) = \ln\left(\frac{1}{4}\right)$$

$$-\frac{t}{3}\ln 2 = \ln\left(\frac{1}{4}\right)$$

$$t = \frac{-3\ln\left(\frac{1}{4}\right)}{\ln 2}$$

$$t = 6$$

Time taken is 6 hours.

Overall Performance
Most students were able to determine the value of *t* correctly.
Common Mistakes
There were a small number of computational errors.

(c) A second dose should be taken only when the amount of medicine remaining in the bloodstream has fallen to less than 15 mg.

A patient takes the medicine at 07 00. The patient asks whether the second dose can be taken at 14 20.

Use the model to determine whether the second dose can be taken at 14 20.
Give a reason for your answer. [3]

SOLUTIONS:

From 07 00 to 14 20, $t = \frac{22}{3}$.

$$M = 80e^{-\frac{22}{9}\ln 2}$$

$$= 14.7 \text{ mg} < 15$$

Since the amount remaining has fallen below 15 mg,

the <u>second dose **can** be taken at 14 20</u>.

Overall Performance
Most students were able to show that the residue in bloodstream allowed for 2nd dose, almost an equal number of students substituted $M = 15$ to find the corresponding t instead.
Common Mistakes
There were a small number of computational errors.

5 The graph of $y = a\cos(bx) + c$ has adjacent minimum points at $(0,-1)$ and $(4\pi,-1)$. It also has a maximum point at $(2\pi, 5)$.

(a) Find the values of *a*, *b* and *c*. [3]

SOLUTIONS:

Maximum value = 5 and minimum value = -1

$$c = \frac{5+(-1)}{2} = 2$$

Amplitude = $\dfrac{5-(-1)}{2} = 3$

Since $a < 0$, $a = -3$.

The period is the horizontal distance between adjacent minimum points, 4π.

$$\frac{2\pi}{b} = 4\pi$$

$$b = \frac{1}{2}$$

Overall Performance

Most students were able to determine the values of the unknowns correctly.

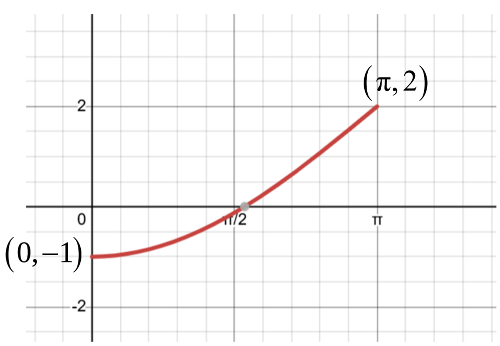
Common Mistakes

The most common error is the value of *a* which many students assumed to be 3 instead of –3.

And there were also a handful who made computational value for *b*.

(b) Hence, sketch the graph of $y = a\cos(bx) + c$ for $0 \le x \le \pi$. [2]

SOLUTIONS:



Overall Performance

Many students were able to sketch the graph correctly if they correctly identified the values in (a)

Common Mistakes

If in (a), the value of a is incorrect, then the graph is usually wrong. A small number of students did not draw the endpoints correctly.

6 **(a)** Without using a calculator, show that $\tan\frac{\pi}{12} = 2-\sqrt{3}$. [2]

SOLUTIONS:

$$\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$$
$$= \frac{\tan\frac{\pi}{4}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{6}}$$
$$= \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$
$$= \frac{\left(\sqrt{3}-1\right)^2}{3-1}$$
$$= \frac{3-2\sqrt{3}+1}{2}$$
$$= 2-\sqrt{3} \quad \text{(shown)}$$

Overall Performance

Most students were able to derive the value correctly but quite a number used alternative solution where $\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$.

Common Mistakes

There were a small number of errors in rationalizing the surds.

(b) Show that $\tan\left(\frac{\pi}{4}-\theta\right) = \frac{1-\tan\theta}{1+\tan\theta}$. [2]

SOLUTIONS:

$$\tan\left(\frac{\pi}{4}-\theta\right) = \frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$$
$$= \frac{1-\tan\theta}{1+\tan\theta} = S \quad \text{(shown)}$$

Overall Performance

Most students were able to apply the addition formula correctly.

Common Mistakes

There were a small number of errors in the $\pm$ sign.

A rotating stage turns through an angle θ radians, where $0 \le \theta \le \frac{\pi}{4}$.

A sensor signal S is given by $S = \frac{1-\tan\theta}{1+\tan\theta}$. The sensor is aligned when $S \le 2-\sqrt{3}$.

(c) Using the results in **(a)** and **(b)**, find the range of values of θ for which the sensor is aligned. [3]

SOLUTIONS:

$$S \le 2-\sqrt{3}$$

$$S \le \tan\frac{\pi}{12}$$

$$\tan\left(\frac{\pi}{4}-\theta\right) \le \tan\frac{\pi}{12}$$

$$\frac{\pi}{4}-\theta \le \frac{\pi}{12}$$

$$\theta \ge \frac{\pi}{6}$$

Since $0 \le \theta \le \frac{\pi}{4}$,

range of values of θ: $\frac{\pi}{6} \le \theta \le \frac{\pi}{4}$

Overall Performance

Most students were able to show that $\theta \ge \frac{\pi}{6}$ but a number of students forgot to consider the given restriction where $0 \le \theta \le \frac{\pi}{4}$.

(d) The rotating stage moves uniformly from $\theta = 0$ to $\theta = \frac{\pi}{4}$ in 24 seconds.

The locking mechanism is activated only if the sensor remains aligned continuously for more than 7.5 seconds.

Determine whether the locking mechanism is activated. Justify your answer. [2]

SOLUTIONS:

Aligned angular interval = $\frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{12}$

Total angular movement = $\frac{\pi}{4}$

Fraction of time aligned = $\dfrac{\frac{\pi}{12}}{\frac{\pi}{4}} = \dfrac{1}{3}$

Time aligned = $\dfrac{1}{3} \times 24 = 8$ seconds

Since 8 > 7.5, the locking mechanism is activated.

Overall Performance

Not many students were able to show that the time is 8 seconds and hence unable to make correct conclusion.

7 The points $A(-3,4)$, $B(5,0)$ and $C(1,-2)$ are joined to form triangle ABC. The perpendicular from C to AB meets AB at D. The point E lies on AB produced, such that $CE = CA$.

(a) Find the coordinates of point D. [4]

SOLUTIONS:

Gradient of AB = $\dfrac{0-4}{5-(-3)} = -\dfrac{1}{2}$

Equation of AB:

$y-4 = -\dfrac{1}{2}(x+3)$

$x+2y-5=0$ ----- (1)

Equation of CD:

$y+2 = 2(x-1)$

$y = 2x-4$ ----- (2)

Solving (1) and (2), $x = \dfrac{13}{5} = 2.6$ and $y = \dfrac{6}{5} = 1.2$.

D is $(2.6, 1.2)$.

Overall Performance

Most students were able to determine the equations of lines and find the coordinates of D. But there were also quite a number of computational errors committed.

(b) Find the coordinates of point *E*. [3]

SOLUTIONS:

Equation of *AB*:

$$x + 2y - 5 = 0$$
$$x = 5 - 2y$$

Since *E* lies on *AB* produced, let *E* be $(5-2y, y)$.

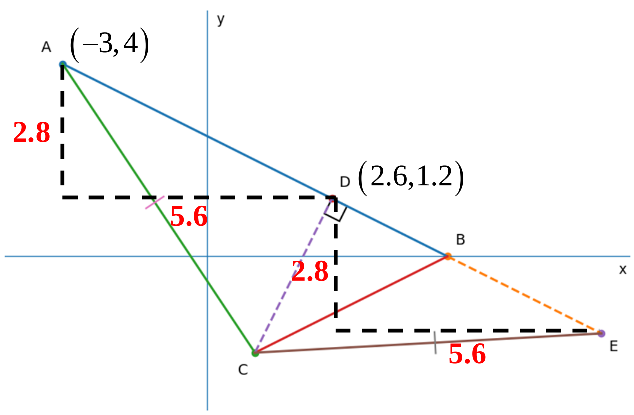
Since $CE = CA$,

$$CE^2 = CA^2$$
$$(5-2y-1)^2 + (y+2)^2 = (1+3)^2 + (-2-4)^2$$
$$(4-2y)^2 + (y+2)^2 = 16 + 36$$
$$16 - 16y + 4y^2 + y^2 + 4y + 4 = 52$$
$$5y^2 - 12y + 20 = 52$$
$$5y^2 - 12y - 32 = 0$$
$$(5y+8)(y-4) = 0$$
$$y = -\frac{8}{5} = -1.6 \quad \text{or} \quad y = 4$$

Overall Performance
Most students were able to determine the length of *CA* given the coordinates but were unable to apply the equation of line to represent the coordinates of *E* in the computation for its coordinates. There were only a handful of students who applied the mid-point theorem.

Since *E* lies on *AB* produced, $y < 0$, $y = -1.6$.
Point *E* is $(8.2, -1.6)$.

ALTERNATIVE SOLUTION:



Since *CA* = *CE*, *CD* ⊥ *AB* and that *E* lies on *AB* produced, *AD* = *DE*.

$$x_E = 2.6 + 5.6 = 8.2$$
$$y_E = 1.2 - 2.8 = -1.6$$

Point E is $(8.2, -1.6)$.

(c) Find the area of triangle *ACE*. [2]

SOLUTIONS:

Area of triangle *ACE*

$$= \frac{1}{2}\begin{vmatrix} -3 & 1 & 8.2 & -3 \\ 4 & -2 & -1.6 & 4 \end{vmatrix}$$

$$= \frac{1}{2}(6 - 1.6 + 32.8 - 4 + 16.4 - 4.8)$$

$$= \frac{1}{2}(44.8)$$

$$= \frac{112}{5} = 22.4 \quad \text{sq. units}$$

Overall Performance

Most students were able to apply the shoe-lace method, though there were many others who tried to determine area using length of line segment.

Common Mistake

The most common mistake is error in the coordinates of *E*, carried forward from earlier.

8 The curve C has equation $y = x^3 + ax^2 + bx + 4$, where a and b are constants. It is given that C has a stationary point when $x = 1$. The tangent to C at $x = 2$ is parallel to the line $y = -3x + 7$.

(a) Find the values of a and b. [4]

SOLUTIONS:

$y = x^3 + ax^2 + bx + 4$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2ax + b$

Since C has a stationary point when $x = 1$,

$$3 + 2a + b = 0.$$
$$2a + b = -3. \qquad (1)$$

The tangent at $x = 2$ is parallel to

$$y = -3x + 7,$$

so the gradient is -3.

$$12 + 4a + b = -3$$
$$4a + b = -15. \qquad (2)$$

Solving equations (1) and (2) simultaneously,

$$\boxed{a = -6, b = 9}$$

Overall Performance

Most students were able to determine $\frac{dy}{dx}$ correctly, as well formulating the two linear equations and solve simultaneously.

Common Mistake

The most common mistake is careless mistakes in formulating the linear equations when students choose to skip steps.

(b) Find the coordinates of all the stationary points of C, and determine their nature. [5]

SOLUTIONS:

$y = x^3 - 6x^2 + 9x + 4.$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x-1)(x-3).$

For stationary points,

$3(x-1)(x-3) = 0.$

$x = 1$ or $x = 3.$

So one stationary point is $(1,8)$.

The other stationary point is $(3,4)$.

$\frac{d^2y}{dx^2} = 6x - 12.$

At $x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 12 = -6 < 0$,

so $(1,8)$ is a maximum point.

At $x = 3$, $\frac{d^2y}{dx^2} = 6(3) - 12 = 6 > 0$,

so $(3,4)$ is a minimum point.

Overall Performance

Many students were able to determine the coordinates of the stationary points and determine its nature by either first or second derivative tests.

Common Mistake

The most common mistake is carried forward from earlier part in determining the values of *a* and *b*. Other mistakes include applying first derivative test without showing exact values used and showing substitution/computation of values, not finding the exact coordinates of the stationary points.

9 The radius, r cm, of a circular chemical stain t seconds after it starts spreading is modelled by $r = 0.04t^2 + 0.6t$ and $0 \le t \le 15$.

(a) Find $\frac{\mathrm{d}A}{\mathrm{d}t}$ in terms of t, where A is the area of the stain. [3]

SOLUTIONS:

$r = 0.04t^2 + 0.6t$
$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$
$\frac{dr}{dt} = 0.08t + 0.6$

Using the chain rule,

$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$
$= 2\pi r(0.08t + 0.6)$.

Substitute

$r = 0.04t^2 + 0.6t$.
$\frac{dA}{dt} = 2\pi(0.04t^2 + 0.6t)(0.08t + 0.6)$.

Overall Performance

Most students were able to determine $\frac{dA}{dt}$ correctly.

Common Mistake

The most common mistake is computational errors in calculating the coefficients of each term.

(b) Find the rate of increase of the area of the stain when $t = 10$. [2]

SOLUTIONS:

When $t = 10, r = 0.04(10)^2 + 0.6(10)$
$= 4 + 6 = 10$.
$\frac{dr}{dt} = 0.08(10) + 0.6 = 1.4$.

$$\frac{dA}{dt} = 2\pi(10)(1.4)$$
$$= 28\pi \quad = 88.0 \text{ cm}^2/\text{s}$$

Overall Performance

Most students were able to determine the value of $\frac{dA}{dt}$ correctly.

Common Mistake

Common mistakes include computational error and error in writing the formula from (a).

(c) For safety reasons, the laboratory has an automatic alert system. The system is activated if the area of the chemical stain is increasing at a rate greater than 100 cm^2/s.

This is because the cleaning team needs to respond immediately before the stain spreads beyond the containment mat.

Determine whether the alert system would already have been activated by the time $t = 14$. Justify your answer clearly. [3]

SOLUTIONS:

When $t = 14$,

$$r = 0.04(14)^2 + 0.6(14)$$
$$= 7.84 + 8.4$$
$$= 16.24$$

$$\frac{dr}{dt} = 0.08(14) + 0.6$$
$$= 1.12 + 0.6$$
$$= 1.72.$$

$$\frac{dA}{dt} = 2\pi(16.24)(1.72)$$
$$= 55.8656\pi$$
$$\approx 175.5 \text{ cm}^2/\text{s}.$$

Since $175.5 > 100$, the area of the stain is increasing at a rate greater than $100 \text{ cm}^2/\text{s}$. Therefore, the alert system would have been activated by the time $t = 14$.

Overall Performance

Most students were able to determine the value of $\frac{dA}{dt}$ correctly and make proper conclusion.

Common Mistake

Common mistakes include computational error and error in writing the formula from (a).

10 A clinic uses an e-bike courier to deliver temperature-sensitive medicine to a care centre 12 km away.

The courier travels against a steady headwind. When riding at a speed of v km/h relative to the air, where $v > 5$, the actual ground speed is $v - 5$ km/h.

The total energy used for the journey, E watt-hours (Wh) consists of two parts.

- Motor energy: $\frac{v^2}{80}$ Wh for every kilometre travelled
- Cooling box energy: 112.5 Wh for every hour of the journey

(a) Show that $E = \frac{3v^2}{20} + \frac{1350}{v-5}$. [3]

SOLUTIONS:

The actual ground speed is $v - 5$ km/h.

Time taken for the journey: $\frac{12}{v-5}$ hours

Motor energy used to overcome air resistance:
$12\left(\frac{v^2}{80}\right) = \frac{3v^2}{20}$

Cooling box energy:
$112.5\left(\frac{12}{v-5}\right) = \frac{1350}{v-5}$.

Hence,
$E = \frac{3v^2}{20} + \frac{1350}{v-5}$ (shown)

Overall Performance
Most students were able to show the formulation of total energy, however, most students' presentation was too brief.

(b) Find the value of v for which E is stationary. [4]

SOLUTIONS:

$$E = \frac{3v^2}{20} + \frac{1350}{v-5}.$$

$$\frac{dE}{dv} = \frac{3v}{10} - \frac{1350}{(v-5)^2}$$

For stationary value,

$$\frac{dE}{dv} = 0.$$

$$\frac{3v}{10} - \frac{1350}{(v-5)^2} = 0$$

$$\frac{3v}{10} = \frac{1350}{(v-5)^2}.$$

$$3v(v-5)^2 = 13500$$

$$v(v-5)^2 = 4500.$$

$$v^3 - 10v^2 + 25v - 4500 = 0.$$

$$(v-20)(v^2 + 10v + 225) = 0.$$

For $v^2 + 10v + 225 = 0$,

$$\text{Discriminant} = 10^2 - 4(225)$$
$$= -800 < 0$$

(no real roots)

Therefore, $v = 20$ km/h

Overall Performance

Most students were able to determine the derivative of total energy, and show how it equates to zero for stationary value.

Common Mistake

Most students did not show how they derive at the solution of v = 20 and/or not showing the discriminant value.

(c) Determine with explanation whether this stationary value of E is a minimum. [3]

SOLUTIONS:

$$\frac{d^2E}{dv^2} = \frac{3}{10} + \frac{2700}{(v-5)^3}.$$

Since
$v = 20$,
$\frac{d^2E}{dv^2} = \frac{3}{10} + \frac{2700}{(20-5)^3} = 1.1 > 0.$*

Therefore, the stationary value of E is a minimum.

Overall Performance
Most students were able to determine the second derivative of the total energy.
Common Mistake
The common mistakes include error in the +/- in the second derivative or not showing the substitution and computation of values when applying the 1st derivative test.

(d) For safety reasons, the courier is only allowed to choose a speed in the range

$$12 \le v \le 18$$

The battery can supply at most 155 Wh for this journey.

Determine the speed the courier should choose, and whether the battery limit is satisfied. Justify your answer. [2]

SOLUTIONS:

The stationary-energy speed is $v = 20$.

However, the allowed range is $12 \le v \le 18$.

Since $20 > 18$, the stationary-energy speed is not allowed.

From part (c), the minimum occurs at $v = 20$. Hence, for $12 \le v \le 18$, E decreases as v increases, so the least energy in the allowed range occurs when $v = 18$.

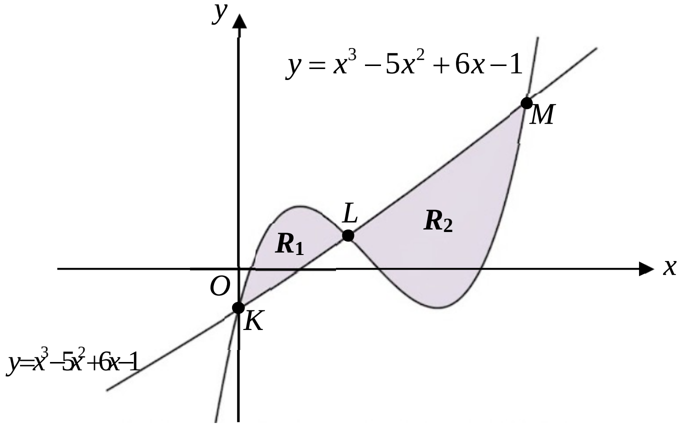
At $v = 18$,
$$E = \frac{3(18)^2}{20} + \frac{1350}{18-5}$$
$$= \frac{972}{20} + \frac{1350}{13}$$
$$= 152.446$$

≈ 152.4 Wh.

Since $152.4 < 155$, the battery limit is satisfied.
Therefore, the courier should choose $v = 18$ km/h and the battery limit is satisfied.

Overall Performance
Not many students were able to justify the value of v = 18.
Common Mistake
The most common mistake is when students assumed E < 155 and compute the value of v as 17.2.

11 The curve C has equation $y = x^3 - 5x^2 + 6x - 1$.
Line l has equation $y = 2x - 1$.
The curve C and the line l intersect at the points K, L and M, as shown in the diagram.
The shaded region consists of the two regions R_1 and R_2.



(a) Find the coordinates of K, L and M. [4]

SOLUTIONS:

At the points of intersection,

$x^3 - 5x^2 + 6x - 1 = 2x - 1.$

$x^3 - 5x^2 + 4x = 0$

$x(x^2 - 5x + 4) = 0$

$x(x-1)(x-4) = 0.$

$x = 0, x = 1, x = 4.$

$\boxed{K(0,-1), L(1,1), M(4,7)}$

Overall Performance
Most students were able to determine the coordinates of K, L and M.
Common Mistake
Common mistakes include computational error when determining the y values of each coordinate.

(b) Find the total shaded area. [6]

SOLUTIONS:

Area of R_1

$$= \int_0^1 \left[\left(x^3 - 5x^2 + 6x - 1\right) - \left(2x - 1\right) \right] \mathrm{d}x$$

$$= \int_0^1 \left(x^3 - 5x^2 + 4x\right) \mathrm{d}x$$

$$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + 2x^2 \right]_0^1$$

$$= \frac{1}{4} - \frac{5}{3} + 2$$

$$= \frac{7}{12}$$

Area of R_2

$$= \int_1^4 \left[\left(2x - 1\right) - \left(x^3 - 5x^2 + 6x - 1\right) \right] \mathrm{d}x$$

$$= \int_0^1 \left(-x^3 + 5x^2 - 4x\right) \mathrm{d}x$$

$$= \left[-\frac{x^4}{4} + \frac{5x^3}{3} - 2x^2 \right]_1^4$$

$$= -\frac{256}{4} + \frac{320}{3} - 32 - \left(-\frac{1}{4} + \frac{5}{3} - 2 \right)$$

$$= \frac{45}{4}$$

Total shaded area

$$= \frac{7}{12} + \frac{45}{4}$$

$$= \frac{71}{6} \text{ square units}$$

Overall Performance

Many students were able to determine the correct value of the area.

Common Mistake

Common mistakes include computational error and error in formulating the equation to find the area by definite integral.

END OF PAPER