



**ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION**

Higher 2

FURTHER MATHEMATICS

9649/02

Paper 2

18 September 2025

3 hours

Additional Materials: Printed Answer Booklet
 List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **8** printed pages.



Anglo-Chinese Junior College

[Turn Over

Section A: Pure Mathematics [50 marks]

- 1** A small town has a population that changes every year due to two factors:
- natural decline or growth, where the population becomes a times its previous size, where $a > 0$;
 - migration, where the population changes by k people because of people move in and out of the town.

Let x_n represent the population in year n . The population follows the recurrence relation

$$x_n = ax_{n-1} + k,$$

where $n \geq 2$ and $x_1 = 10000$.

- (i) Solve the recurrence relation to find x_n in terms of n, a and k . [4]
- (ii) Given that $0 < a \leq 1$, find the range of values of k such that the population does not grow indefinitely. [2]
- (iii) If $a = 0.6$ and $k = 100$, find the value at which the population stabilizes after a long period of time. [1]
- 2** Let $f(x) = \tan^{-1}\left(e^x\right) - \frac{1}{x}$. Show that the equation $f(x) = 0$ has only one root α in the interval $[0.7, 1]$. [2]

Two iterative formulae are given below:

$$(I) \quad x_{n+1} = \ln \left(\tan \left(\frac{1}{x_n} \right) \right);$$

$$(II) \quad x_{n+1} = \frac{1}{\tan^{-1}\left(e^{x_n}\right)}.$$

Choose one of the iterative formulae to find an approximation to α , and explain your choice.

Using an initial approximation of $x_0 = 0.75$ for the chosen iterative formula, find an approximation to the root α , correct to three decimal places. [6]

- 3 In an Argand diagram, the point P represents the complex number z that satisfies

$$(z - \sqrt{3} - i)(z^* - \sqrt{3} + i) \leq 4 \quad \text{and} \quad |z| \geq |z - \sqrt{3} - i|.$$

- (i) Sketch the locus of P . [3]

- (ii) Find the exact maximum and minimum values of $|z + \sqrt{3} - i|$. [2]

- (iii) Find the exact complex number z such that $\arg(z + \sqrt{3} - i)$ is

(a) maximum;

(b) minimum,

giving your answer in the form $x + iy$. [5]

- 4 The surface S has equation $z = f(x, y)$ where

$$f(x, y) = \frac{x^2 - y^2}{e^{x^2 - y^2}}.$$

- (a) Find the set of critical points of the surface S . [4]

The point A denotes the point on S where $x = 1$ and $y = 2$.

- (b) Find the equation of the tangent plane of S at A in exact form. [3]

- (c) Find the exact directional derivative of S at A in the direction of $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$. [3]

- 5** A parachute's stabilisation system responds to brief vertical gusts during descent. The small vertical displacement y , in metres, from its steady descent path is modelled by the differential equation

$$\frac{d^2y}{dt^2} + 2a \frac{dy}{dt} + 25y = 5e^{-2t}$$

where t is time in seconds.

- (i)** If $0 < a < 5$ and $a \neq \frac{29}{4}$, find the general solution of the differential equation in terms of a . [6]

- (ii)** At time $t = 0$, the parachute has no vertical displacement and is stationary in the vertical direction. Assume now that $a = 1$. Express y in terms of t . Sketch the solution curve and find the largest vertical displacement. [5]

- (iii)** The actual largest vertical displacement is measured to be 0.12 metres. Does this suggest that the actual value of a is greater than or less than 1? Justify your answer using your result from part **(ii)**. [2]

- (iv)** Suppose the differential equation is now changed to

$$\frac{d^2y}{dt^2} + 10 \frac{dy}{dt} + 25y = 5e^{-5t}.$$

Explain why a particular integral of the form Be^{-5t} or Cte^{-5t} cannot be used to solve the differential equation. [2]

Section B: Statistics [50 marks]

- 6** A company conducted a survey with a random sample of 300 customers to investigate whether the type of advertisement shown to customers is associated with their decision to purchase a product. Three types of advertisements were released for a certain product: a social media ad, an email ad and a billboard ad. Customers were asked to state the type of the first advertisement they saw and whether they purchased the product or not. The results are shown in the table below.

	Purchased	Did not purchase	Total
Social media	53	53	106
Email	20	84	104
Billboard	20	70	90

- (a)** Perform a chi-squared test for independence at the 5% significance level to determine whether there is an association between the type of advertisement and the purchase decision. [5]
- (b)** By referring to the contributions to the test statistic, suggest with a suitable reason which advertisement method the company should invest more to maximise their sales. [2]

- 7 A random sample of 12 participants took part in a study to find out whether drinking regular coffee influences one's reaction time compared to drinking decaffeinated coffee. Each of them did a reaction test after drinking decaffeinated coffee on one day and regular coffee a week later, and their reaction times were measured on both days. The table below shows their reaction times in milliseconds, with smaller times indicating faster reactions.

Participant	Decaffeinated	Regular
A	245	220
B	243	225
C	225	230
D	248	218
E	244	222
F	226	226
G	240	228
H	227	229
I	239	224
J	241	221
K	235	227
L	239	229

- (a) Carry out a suitable Wilcoxon signed rank test at 1% level of significance to investigate whether there is evidence that drinking regular coffee results in faster reaction times compared to decaffeinated coffee. [5]
- (b) Calculate the p -value obtained if a sign test is used instead and state if it would lead to the same conclusion. Give one reason why the Wilcoxon signed rank test is more appropriate than the sign test in this study. [2]

- 8** A manufacturer inspects rolls of fabric for defects. Each roll of fabric is 50 metres long. It is known from the manufacturer's experience that the number of defects on the fabric follows a Poisson distribution with a mean of 2 defects for every 10 metres of fabric.

- (a) State two assumptions needed for the number of defects in a roll of fabric to be well modelled by a Poisson distribution. [2]
- (b) Find the probability that a randomly chosen roll of fabric contains 5 defects. [2]
- (c) By considering the value of $\frac{p_{k+1}}{p_k}$, where p_k is the probability that a randomly chosen roll of fabric has k defects, show algebraically that the most likely numbers of defects that a roll of fabric can have is 9 and 10. [3]
- (d) State the distribution of the length of fabric between consecutive defects and write down the mean length in metres between consecutive defects. [2]

A roll of fabric passes the inspection process if the manufacturer finds no more than 5 defects in the first 10 metres of the fabric. Otherwise, the roll is rejected.

- (e) Find the probability that the 10th roll of fabric inspected is the first one to be rejected. [3]

- 9** The continuous random variable X has probability density function

$$f(x) = \begin{cases} 1 - |x - 1| & \text{for } a \leq x \leq b, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Sketch the graph of $y = 1 - |x - 1|$ and hence state the values of a and b . [2]
- (b) Find $E(X)$ and $\text{Var}(X)$ exactly, showing your working clearly. [6]
- (c) Given that $Y = \cos\left(\frac{\pi X}{2}\right)$, find $E(Y)$ exactly, showing your working clearly. [4]

- 10** A teacher is interested to find out whether students solve math problems more accurately when working on paper or on a tablet. To investigate this, the teacher selects a random sample of 8 students and asks each student to solve a set of math problems on paper and another equivalent set of problems on a tablet. The percentage accuracy for each student under the two conditions are recorded in the table below.

Student	1	2	3	4	5	6	7	8
Paper	82.9	79.6	79.5	85.2	81.6	77.1	83.7	78.1
Tablet	80.2	78.4	76.5	81.9	81.0	75.5	81.0	74.9

- (a) Calculate a 99% confidence interval for the mean difference in accuracy between solving on paper and solving on a tablet, giving the endpoints correct to 4 significant figures. [3]
- (b) State two assumptions about the differences that are necessary in order to carry out a paired sample t -test. [2]
- (c) Suppose the assumptions in part (b) hold. Test at the 1% significance level whether there is evidence that the mean accuracy on paper is higher than on tablet. [5]
- (d) Explain how your confidence interval in part (a) may be used to carry out a test at the 1% significance level to determine if there is any difference in the mean accuracy on paper and on tablet. [2]