STUDENT NAME: _____

TEACHER NAME: _____

CANDIDATE SESSION NUMBER

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EXAMINATION CODE

8	8	2	4	-	7	1	1	2

ST. JOSEPH'S INSTITUTION
YEAR 6 PRELIMINARY EXAMINATION 2024

MATHEMATICS: ANALYSIS AND APPROACHES

HIGHER LEVEL

PAPER 2

Tuesday

20 August 2024

2 hours

1300 – 1500 hrs

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Write your name and your teacher's name in the spaces provided.
- Do not open this examination paper until instructed to do so.
- **Section A**: Answer all questions showing working and answers in the spaces provided in the exam paper.
- **Section B**: Answer all questions using the writing paper provided.
- The use of a scientific or examination graphical calculator is permitted in this paper.
- TI-Nspire calculators must be in Press-to-Test mode and cleared of all previous data.
- TI-84+ graphical calculators must only have permitted apps and be ram cleared.
- A clean copy of the **Mathematics: Analysis and Approaches formula booklet** is required for this paper.
- Unless otherwise stated in the question, all numerical answers are to be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[110 marks]**.
- This question paper consists of **14** printed pages including the Cover Sheet.
- Sections A and B are to be submitted **separately**.

FOR MARKER USE ONLY:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	TOTAL
											/110

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are advised to show all working.

SECTION A (55 marks)

Answer **all** questions in the spaces provided.

1 [Maximum mark: 6]

The weights, W, of newborn babies in Singapore are normally distributed with a mean of 3.24 kg and standard deviation 375 g. A newborn baby is considered a big baby if it weighs more than w kg. 10% of newborn babies are considered big babies.

(a) Find the value of w. [3]

(b) A particular newborn baby is considered a big baby.
Find the probability that the baby weighs at least 4 kg. [3]

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2 [Maximum mark: 4]

Given that **a** and **b** are two non-zero vectors such that $\mathbf{v} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b}$, find the value of $(\mathbf{a}-\mathbf{v})\cdot\mathbf{b}$ and hence show that $|(\mathbf{a}-\mathbf{v})\times\mathbf{b}| = |\mathbf{a}-\mathbf{v}||\mathbf{b}|$.

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3 [Maximum mark: 5]

The Student Council comprises 30 councillors, out of which a 6-member executive committee (EXCO) is formed. The EXCO includes a President and a Vice-President of different genders.

(a) Given that 4 female councillors and 4 male councillors were shortlisted for the President and Vice-President, in how many ways can the EXCO be formed from the 30 councillors? [3]

(b) During an EXCO photo-taking session, the EXCO is seated in a line such that the President and Vice-President are in the centre. In how many ways can the EXCO be arranged for the photo? [2]

4 [Maximum mark: 5]

Consider the polynomial function $P(x) = (x^3 + 3)(x^4 - 6x^3 + x - 10)$.

Its expanded form is given by $P(x) = x^7 + Ax^6 + Bx^5 + Cx^4 + Dx^3 + Ex^2 + Fx + G$ where each coefficient is a constant real number.

(a) Find the value of A. [2]

(b) Find the product of the roots of $P(x) = 0$. [2]

(c) Find the product of the roots of $P(x) = 1$. [1]

5 [Maximum mark: 8]

Let $f(x) = \frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right)$, where $x \geq 0$. The n^{th} **minimum point** on the graph of f has x-coordinate x_n where $n \in \mathbb{Z}^+$.

(a) The graph of $y = \frac{2}{\pi}\sin(x)$ undergoes a horizontal scaling with scale factor k, resulting in the graph of $y = f(x)$. Write down the **exact** value of k. [1]

(b) Given that $x_{n+1} = x_n + a$, find the value of a. [3]

(c) Hence find the least value of N such that $\sum_{n=1}^{N} x_n > 500$. [4]

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6 [Maximum mark: 6]

Let $f(x) = x + \sqrt{x^2 - 7}$ for $x \geq a$ where a is a positive real number.

(a) Find **exactly**, the smallest possible value of a such that the function is defined. [2]

Using the value of a found in part (a), a solid, S, is formed when the region bounded by $y = f(x)$ and the y-axis, between $y = a$ and $y = 7$, is rotated by 2π radians about the y-axis.

(b) **(i)** Show that $x = \dfrac{y^2 + 7}{2y}$.

(ii) Hence, find the volume of S. [4]

7 [Maximum mark: 11]

The following diagram shows a circle, centre O and radius r cm. The circle is divided into 5 equal sectors. One of the sectors is AOB where $\angle AOB = \theta$.

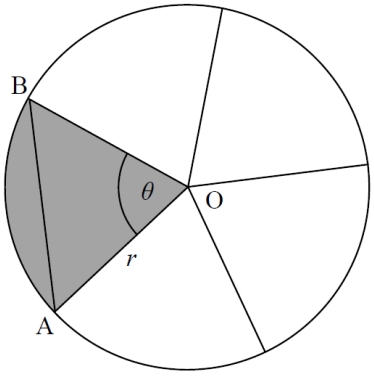

Diagram is not drawn to scale

(a) Write down the **exact** value of θ in radians. [1]

The area of sector AOB is 5π cm^2.

(b) Find the value of r. [3]

(c) The points A and B represent two solutions of the equation $z^5 = \frac{k}{\sqrt{2}}(1+\mathrm{i})$, where $z \in \mathbb{C}$, $\mathrm{i}^2 = -1$, $k \in \mathbb{R}^+$, on an Argand diagram where O is the origin.

(i) Find all the solutions of $z^5 = \frac{k}{\sqrt{2}}(1+\mathrm{i})$ in terms of k, leaving your answer in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$.

(ii) Find the value of k. [7]

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(Question 7 continued)

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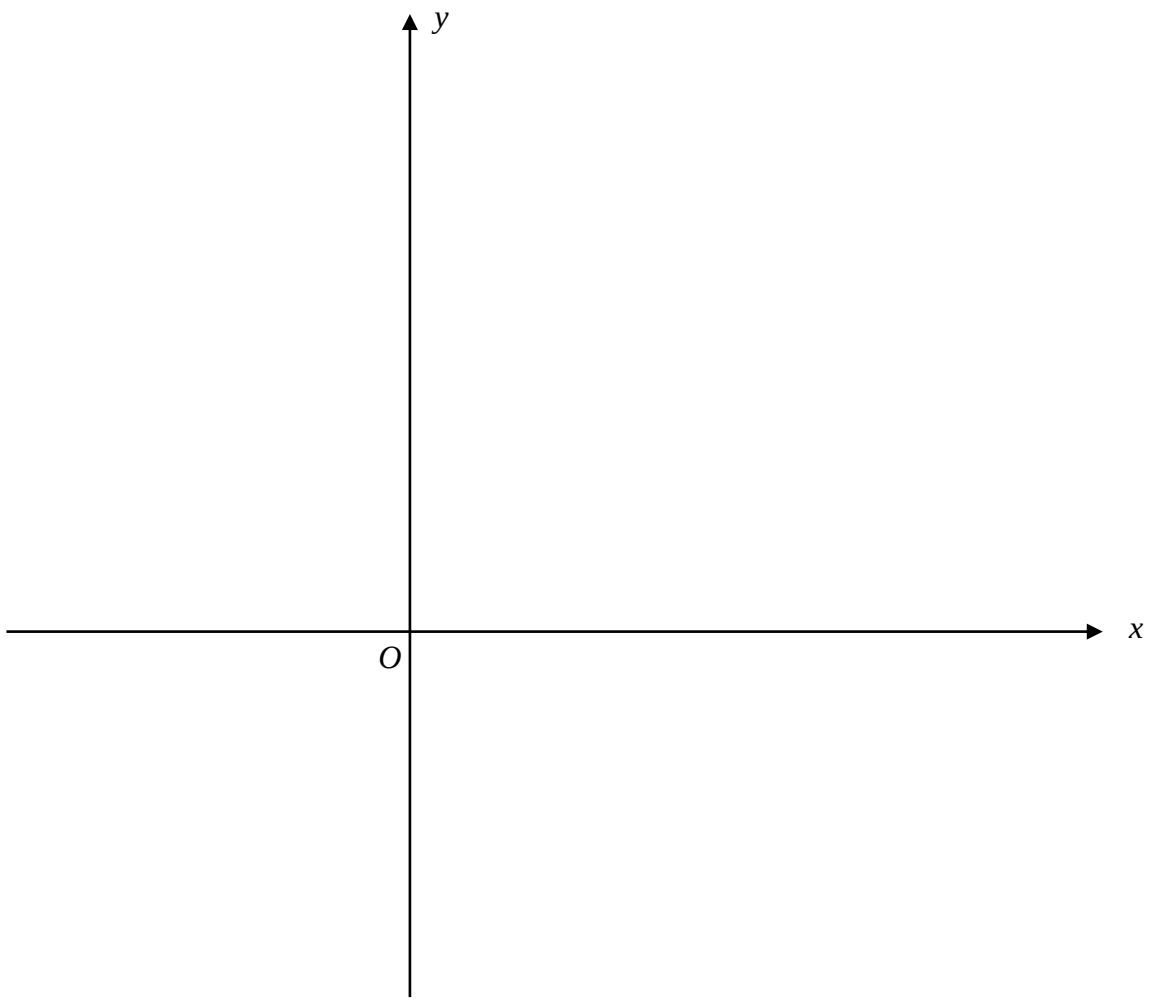
8 [Maximum mark: 10]

(a) On the axes provided, sketch the graph of $y = \frac{2x^2 - 7x + 4}{x - 3}$, clearly labelling the equations of the asymptotes and the coordinates of the turning points of the graph. [6]

(b) Using differentiation, show that the mid-point of the turning points of the graph of $y = \frac{2x^2 - 7x + 4}{x - 3}$ lie on its vertical asymptote. [4]



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(Question 8 continued)

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*Do **NOT** write solutions on this page.*

SECTION B (55 marks)

*Answer **all** questions on the writing paper provided. **Please start each question on a new page.***

9 [Maximum mark: 19]

An unbiased four-sided die with faces labelled 0, 1, 1 and 2 is rolled and the results recorded.

(a) The die is rolled n times. Let X be the number of times '2' is recorded.

(i) Find $E(X)$ in terms of n. [2]

(ii) Find the greatest value of n such that $P(X \geq 3) < 0.5$. [3]

Let S be the result obtained after rolling the die once. The same die is rolled for a second time. Let T be the total of the first and second results obtained.

(b) The probability distribution for T is given in the following table where p_1, p_2 and p_3 are constants. Find the values of p_1, p_2 and p_3. [3]

t	0	1	2	3	4
$P(T = t)$	$\frac{1}{16}$	$\frac{p_1}{16}$	$\frac{p_2}{16}$	$\frac{p_3}{16}$	$\frac{1}{16}$

(c) Find $P(T \geq 2S)$. [2]

(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 9 continued)

Pedro wants to ascertain the fairness of the die. He decides to investigate the correlation between the total number of times he rolls the die (m) and the number of times '2' appears (x). The following table shows the data he collected.

Total number of times the die is rolled (m)	20	30	40	50	60	70	80
Number of times '2' appears (x)	5	6	9	13	14	9	21

Unfortunately, he made an error in recording one value of x.

(d) By considering a scatter plot or otherwise, determine for which value of m did Pedro make the error. [2]

After rectifying his mistake, Pedro found the equation of the linear regression to be

$$x = 0.2834m - 1.741$$

(e) Find the correct value of x for the value of m found in part (**d**). [4]

(f) Hence, find the Pearson's product-moment correlation coefficient. [2]

After conducting his experiment, Pedro concludes that the die is indeed unbiased.

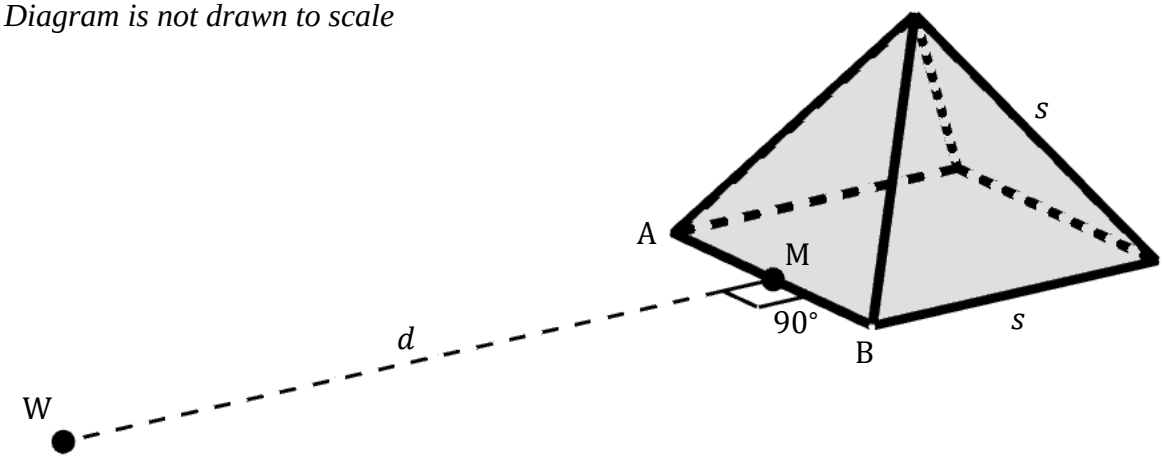
(g) Give a possible reason that would support Pedro's conclusion. [1]

*Do **NOT** write solutions on this page.*

10 [Maximum mark: 19]

A right pyramid with a square base and four equilateral triangular faces is shown in the diagram below. Each edge of the pyramid has a length of s metres. The highest point of the pyramid is called its vertex.

Diagram is not drawn to scale



(a) Show that the height of the pyramid is $\frac{1}{\sqrt{2}}s$ metres. [2]

(b) Hence, or otherwise, find the angle between a triangular face and the base. [3]

W is a point d metres away from M, which is the midpoint of AB. WM and AB are perpendicular as shown above. The height of the pyramid is 140 metres. A woman is standing at point W and the angle of elevation from her to the vertex of the pyramid is $10°$. (Assume the woman's height to be negligible.)

(c) Show that $d \approx 695\,\text{m}$. [4]

This woman pilots a drone that only flies vertically above her. The drone begins its ascent from the ground. Its upwards vertical velocity, v ms^{-1}, at time t seconds is given by $v(t) = 3\sqrt{t}$ for $t \geq 0$.

(d) Find the rate at which the distance between the drone and the woman is increasing when the drone is horizontally levelled with the vertex of the pyramid. [6]

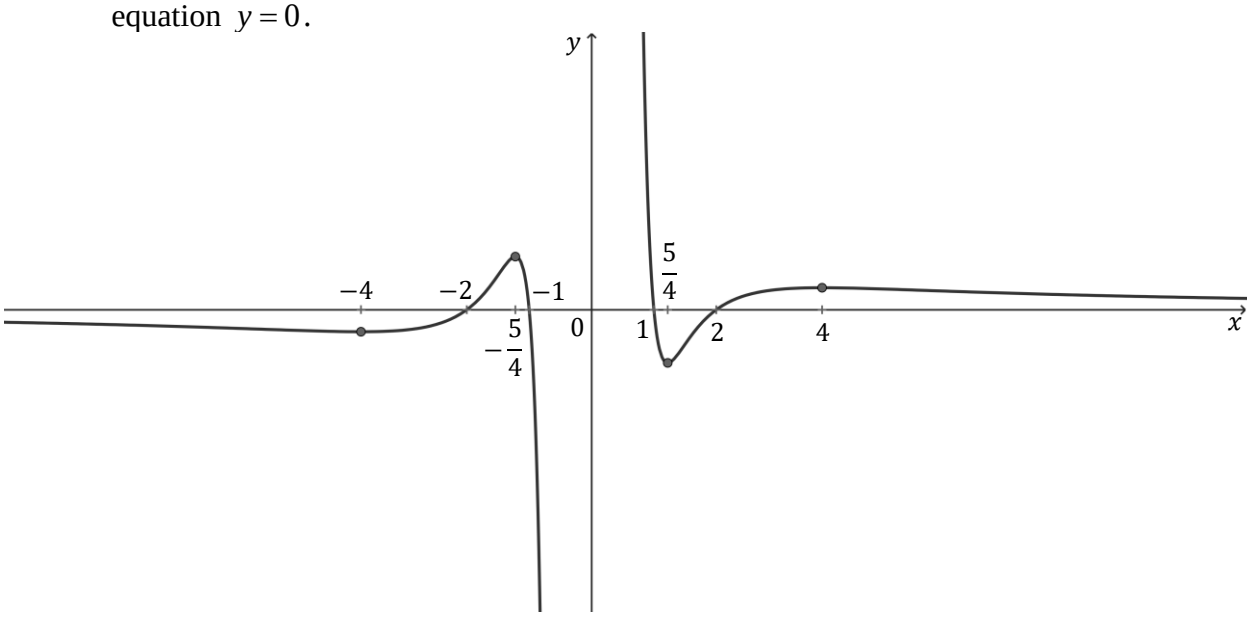
(e) Find the value of t where the angle of depression from the drone to the vertex of the pyramid is increasing the greatest. [4]

*Do **NOT** write solutions on this page.*

11 [Maximum mark: 17]

The graph of $y = f'(x)$ is shown below such that:

- The roots of $f'(x) = 0$ are $-2,\ -1,\ 1$ and 2;
- The stationary points of $y = f'(x)$ are at $x = -4,\ -\frac{5}{4},\ \frac{5}{4}$ and 4;
- $y = f'(x)$ has a vertical asymptote with equation $x = 0$ and a horizontal asymptote with equation $y = 0$.



(a) Sketch a possible graph of $y = f''(x)$, clearly labelling its x-intercepts. [3]

(b) Find the x-coordinates of the stationary points of $y = f(x)$ and determine whether they are a maximum point, a minimum point or neither. [4]

(c) Write down the number of points of inflexion of $y = f(x)$. Justify your answer. [2]

Suppose $g'(x) = f'(x),\ x > 0$, that is, $y = g'(x)$ is the restriction of $y = f'(x)$ over the interval $(0, \infty)$.

(d) Find the intervals where

(i) g is increasing; and, [2]

(ii) g is concave down. [3]

(e) Hence, sketch a possible graph of $y = g(x)$, clearly labelling the x-coordinates of its stationary points and points of inflexion, if any. [3]

End of Paper