Topical Revision Paper 3

Term 1 Unit 3: Equations and Inequalities

Question 1:

(a) Solve the equation $\left| x^2 - 3 \right| = 2x$. [4]

(b) Find the range of values of x for which $x^2 + x < 6$.

Hence, find the range of values of x that satisfy $(2x-3)\left(x^2+x-6\right) > 0$. [6]

Solution:

(a)

Given $\left| x^2 - 3 \right| = 2x \quad \Rightarrow \quad x^2 - 3 = 2x$ or $x^2 - 3 = -2x$

Case 1 : $x^2 - 3 = 2x$

or $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0 \qquad x = 3$ or -1

Case 2 : $x^2 - 3 = -2x$

or $x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or 1

But $\left| x^2 - 3 \right| = 2x \geq 0$

Hence after verifying, the answer is:

$x = 3$ or $x = 1$

(b)

$x^2 + x - 6 < 0$

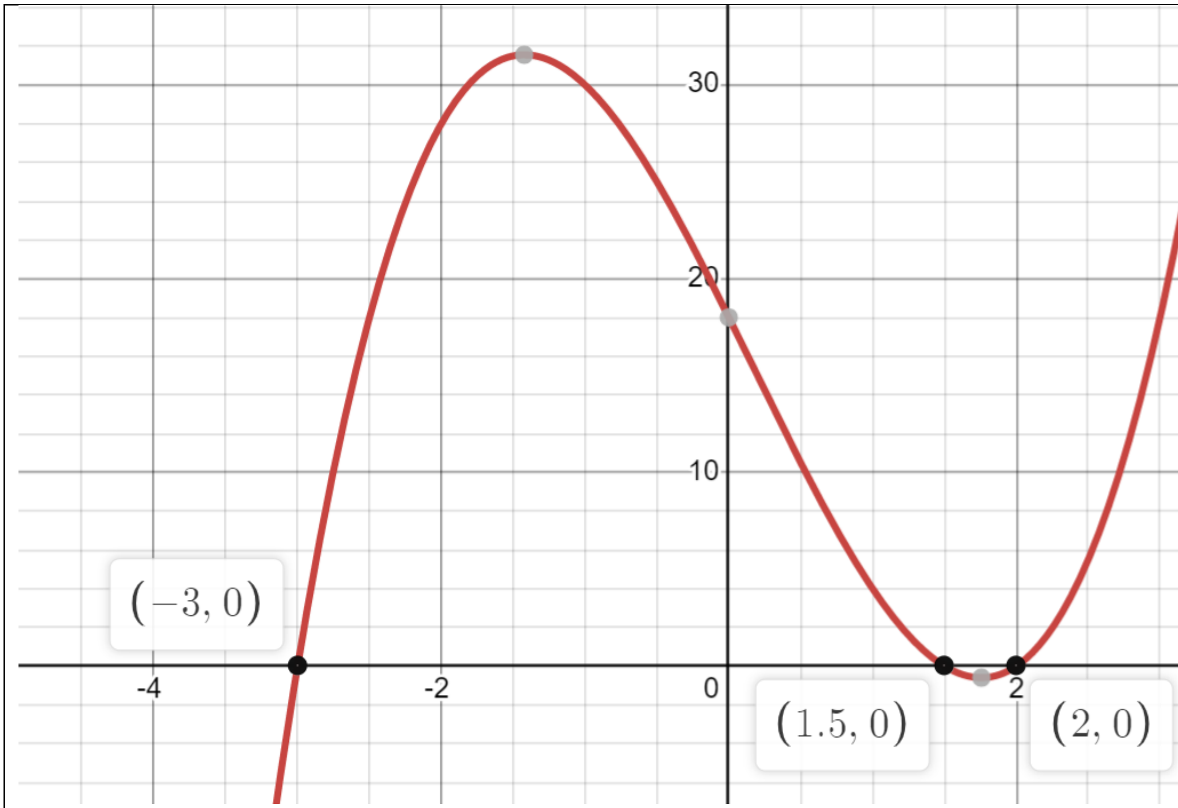
$(x+3)(x-2) < 0$

$-3 < x < 2$

For

$(2x-3)(x^2+x-6) > 0$

$(2x-3)(x+3)(x-2) > 0$



From the sketch $-3 < x < \frac{3}{2}$ or $x > 2$.

Question 2:

Find the possible values of k for which the line $y = kx - 1$ is a tangent to the curve $y = x^2 + 3$.

Hence ,state the range of values of k for which the line will **not** meet the curve. [6]

Solution:

Substitute $y = kx - 1$ into $y = x^2 + 3$

$x^2 - kx + 4 = 0$

$b^2 - 4ac = 0$

$(-k)^2 - 16 = 0$

$(k-4)(k+4) = 0$

$k = 4$ or -4

For $(k-4)(k+4) < 0$

$-4 < k < 4$

Question 3:

Solve the equation $2^{2x+1} = 7(2^{x+1}) + 36$, leaving your answers in two decimal places. [4]

Solution:

$2^{2x+1} = 7(2^{x+1}) + 36$

$2^{2x} \times 2 = 7(2^x \times 2) + 36$

$2 \times \left(2^x\right)^2 - 14\left(2^x\right) - 36 = 0$

Let $y = 2^x \Rightarrow 2y^2 - 14y - 36 = 0$

$(2y+4)(y-9) = 0$

$y = -2$ (rejected) or $y = 9$

$2^x = 9 \Leftrightarrow y = \dfrac{\lg 9}{\lg 2} = 3.17$ (2 decimal places)

Question 4:

Find the range of values of k for which the graph of $y = x^2 + 3kx + \left(2k^2 + \frac{1}{2}k + 2\right)$ intersects the x-axis at two distinct points. [3]

Solution:

$x^2 + 3kx + \left(2k^2 + \frac{1}{2}k + 2\right) = 0$ has two distinct roots

$b^2 - 4ac > 0$

$(3k)^2 - 4(1)\left(2k^2 + \frac{1}{2}k + 2\right) > 0$

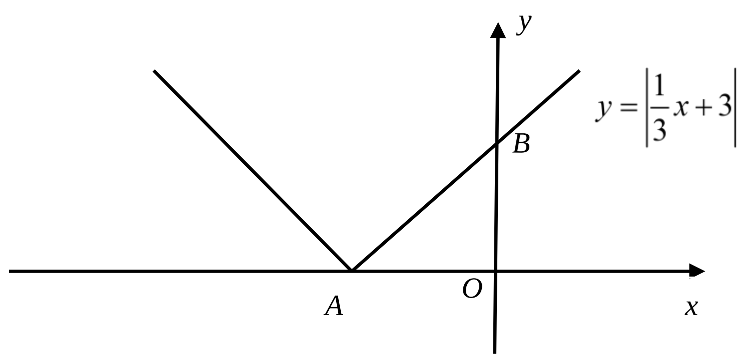
$9k^2 - 8k^2 - 2k - 8 > 0$

$k^2 - 2k - 8 > 0$

$(k-4)(k+2) > 0$

$\therefore k > 4$ or $k < -2$

Question 5:



The diagram above shows part of the graph of $y = \left|\frac{1}{3}x + 3\right|$ with A and B as the x and y intercepts respectively.

(i) State the coordinates of A and B. [2] **(ii)** For each of the cases below, determine the number of intersections of the line $y = mx + c$ with $y = \left\|\frac{1}{3}x + 3\right\|$, justifying your answer. **(a)** $m = \frac{1}{3}$ and $c = 0$, [2] **(b)** $m = \frac{1}{4}$ and $c = 3$. [2]
(i) $A(-9,0)$ and $B(0,3)$
(ii) (a) $y = mx + c$ is parallel and below $y = \left\|\frac{1}{3}x + 3\right\|$ for all x. Hence will not intersect at any point or zero intersection
(iii) (b) $y = mx + c$ will intersect $y = \left\|\frac{1}{3}x + 3\right\|$ at (0,3) and also intersect $y = \left\|\frac{1}{3}x + 3\right\|$ for $x < -9$ once. Hence, two intersections