JURONG PIONEER JUNIOR COLLEGE
9478 H2 PHYSICS/8867 H1 PHYSICS

MOTION AND FORCES

Content

- Kinematics
- Uniformly accelerated linear motion
- Mass and linear momentum
- Laws of motion

Learning Outcomes

Candidates should be able to:

(a) show an understanding of and use the terms position, distance, displacement, speed, velocity and acceleration

(b) use graphical methods to represent distance, displacement, speed, velocity and acceleration

(c) identify and use the physical quantities from the gradients of position–time or displacement–time graphs and areas under and gradients of velocity–time graphs, including cases of non-uniform acceleration

(d) derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line

(e) solve problems using equations which represent uniformly accelerated motion in a straight line, e.g. for bodies falling vertically without air resistance in a uniform gravitational field

(f) show an understanding that mass is the property of a body which resists change in motion (inertia)

(g) define and use linear momentum as the product of mass and velocity

(h) state and apply each of Newton's laws of motion:

1st law: a body at rest will stay at rest, and a body in motion will continue to move at constant velocity, unless acted on by a resultant external force;

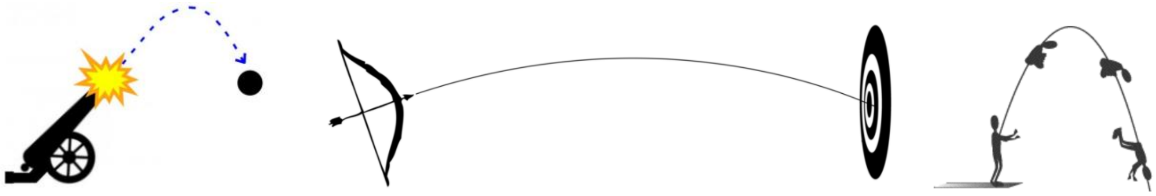
2nd law: the rate of change of momentum of a body is (directly) proportional to the resultant force acting on the body and is in the same direction as the resultant force;

3rd law: the force exerted by one body on a second body is equal in magnitude and opposite in direction to the force simultaneously exerted by the second body on the first body.

(i) recall the relationship resultant force $F = ma$, for a body of constant mass, and use this to solve problems

Part 1: MOTION (KINEMATICS)

Introduction



Introduction

The study of physics often begins with *Newtonian Mechanics* which investigates relationships between force and motion. The study of motion can be divided into two aspects: how objects move **(kinematics)** and why objects move in different ways **(dynamics)**. Models and representations such as motion diagrams, graphs and equations are used to quantify, describe and predict motion. Motion in the real world is rather complex. The study of motion at this level is made simple with the use of assumptions. One valid assumption is to model the moving body as a point with no size where effects such as rotation or change of shape are not considered. It is also appropriate to ignore air resistance in solving most problems.

Key Questions:

1. How do we describe the motion of objects?
2. How can the motion of objects be represented, quantified and predicted?
3. How can we tell if an object is moving with a constant acceleration?
4. For an object falling freely in a gravitational field, how would it move?

1 Kinematics Quantities

(a) *show an understanding of and use the terms position, distance, displacement, speed, velocity and acceleration*

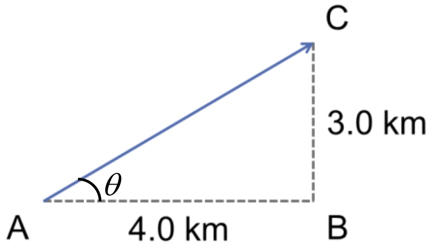
1.1 Distance and Displacement

Distance travelled (scalar) is the total length moved along the path of motion.
Displacement (vector) is the linear distance in a specified direction from a reference point.

Example 1

A man at point A travels 4.0 km due East to point B. He then travels a further 3.0 km due North to point C.

Determine the total distance travelled and his displacement from the original position (point A).



Total distance travelled

$= 4.0 + 3.0 = 7.0$ km

AC is the displacement s

$$s^2 = 4.0^2 + 3.0^2$$

$$s = 5.0 \text{ km}$$

$$\tan\theta = \frac{3.0}{4.0}$$

$$\theta = 37^\circ$$

The displacement from A is 5.0 km in a direction 37° North of East.

1.2 Speed and Velocity

Speed (scalar) is the rate of change of distance moved by an object.

Velocity (vector) is defined as the rate of change of displacement.

Instantaneous speed is the speed of a moving object at a particular instant.

If an object travels along a path with varying speeds, we can always identify the smallest and largest speeds attained during its motion. The average speed lies between these two extremes. Normally when we mention about velocity, we are referring to instantaneous or constant velocity, unless otherwise stated.

	speed	velocity
instantaneous	speed at a particular instant $= \dfrac{\text{small distance travelled}}{\text{short time taken}}$ $= \dfrac{dx}{dt}$	velocity at a particular instant $= \dfrac{\text{small change in displacement}}{\text{short time taken}}$ $= \dfrac{ds}{dt}$
average	average speed between 2 instants $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$ $= \dfrac{\Delta x}{\Delta t}$	average velocity between 2 instants $= \dfrac{\text{change in displacement}}{\text{time taken}}$ $= \dfrac{\Delta s}{\Delta t}$

Example 2

The man in Example 1 took 1 hour to move 4.0 km due East to point B, and another 1 hour to move 3.0 km due North to point C.

Determine the man's average speed and average velocity for the entire journey.

$$\text{Avg speed} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{7.0}{2} = 3.5 \text{ km h}^{-1}$$

$$\text{Avg velocity} = \frac{\text{displacement}}{\text{time taken}} = \frac{5.0}{2} = 2.5 \text{ km h}^{-1}$$

The average velocity is 2.5 km h^{-1} in a direction 37° North of East.

1.3 Acceleration

Acceleration (vector) is defined as the rate of change of velocity.

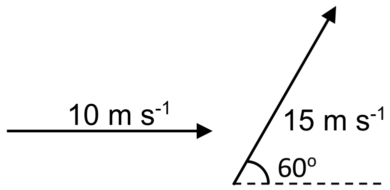
- Instantaneous acceleration, $a = \dfrac{dv}{dt}$

- Average acceleration , $\langle a \rangle = \dfrac{\Delta v}{\Delta t}$ where Δv = change in velocity
  = final velocity – initial velocity
  $= v_f - v_i$
  or $= v - u$

Since velocity is a vector quantity, change in velocity Δv is also a vector with a magnitude and a direction. A change in velocity occurs when either the magnitude or the direction changes, or both the magnitude and direction change.

Example 3

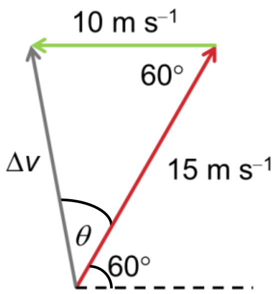
A particle has an initial horizontal velocity v_i of 10 m s^{-1} towards the right. A short time later, its velocity v_f is 15 m s^{-1} at an angle of 60° to the horizontal, as shown.



Calculate the change in velocity that has taken place.

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \vec{v}_f + (-\vec{v}_i)$
$(\Delta v)^2 = 10^2 + 15^2 - 2(10)(15)\cos 60°$
$\Delta v = 13.2$ m s^{-1}

$\dfrac{10}{\sin\theta} = \dfrac{13.2}{\sin 60°} \rightarrow \theta = 41°$



The change in velocity is 13.2 m s^{-1} at an angle of 41° to the left of the final velocity.

physical quantity	symbol	definition	nature	SI base unit
displacement	s	linear distance from a reference point, along a specified direction.	vector	m
speed	v	rate of change of distance travelled.	scalar	m s^{-1}
velocity	v	rate of change of displacement.	vector	m s^{-1}
acceleration	a	rate of change of velocity.	vector	m s^{-2}

1.4 Using + and – to represent directions

For 1-D (straight line) motion, directions of displacement, velocity and acceleration can be represented using '+' and '–' signs. The figure below shows the usual sign convention to denote directions.

displacement with respect to reference point		
+	right	above
–	left	below
velocity and acceleration		
+	towards right	upwards
–	towards left	downwards

Note that the sign convention can be reversed. It is valid as long as it is consistently applied within the same context.

1.5 Acceleration vs Deceleration

Deceleration is a term used to describe an object slowing down (speed decreases). It is not the same as negative acceleration as the negative sign denotes direction only. The sign of the acceleration is not sufficient to provide information on whether the object is moving faster or slower. For example, if upwards is taken as positive, a ball then falls with an acceleration of $-9.81\ \text{m s}^{-2}$. The negative sign denotes the downward direction of acceleration, hence the ball speeds up as it falls.

To determine if an object speeds up or slows down, we need to look at both the velocity and acceleration at the same time.

- When an object's velocity and acceleration are in the same direction, the speed of the object increases with time.
- When an object's velocity and acceleration are in the opposite direction, the speed of the object decreases with time. The object is said to be undergoing deceleration.

1.6 Free fall

An object is said to be in free fall if the only force acting on it is the gravitational force of the Earth.

The downward acceleration of a body undergoing free fall is known as *acceleration of free fall, g,* and has a constant value of $9.81\ \text{m s}^{-2}$. All free-falling objects have this acceleration, independent of their masses and direction of motion.

For a falling object, true free fall only exists in a vacuum where there is no air resistance.

2 Graphical Representation of Motion

(b) use graphical methods to represent distance, displacement, speed, velocity and acceleration

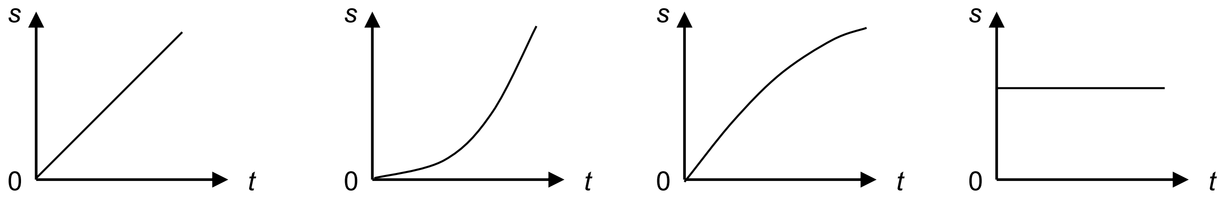
(c) identify and use the physical quantities from the gradients of position–time or displacement–time graphs and areas under and gradients of velocity–time graphs, including cases of non-uniform acceleration

2.1 Displacement-time graph

The motion of a moving object can be represented and analysed using a displacement-time (*s*-*t*) graph, which charts the variation of displacement with time.

- Gradient of a *s*-*t* graph = $\dfrac{ds}{dt}$ = velocity at a particular instant

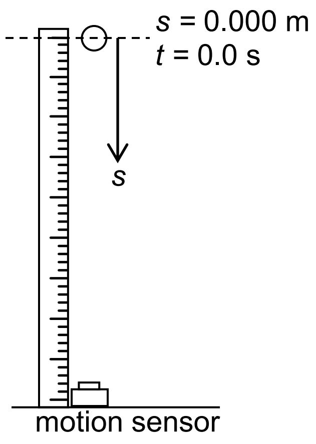
The diagrams below are some common displacement-time graphs:

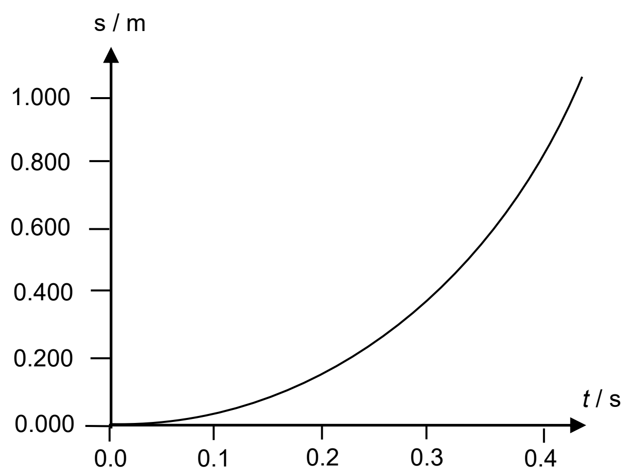


(a) constant velocity (b) increasing velocity (c) decreasing velocity (d) zero velocity

Example 4

A ball placed at the zero mark of a vertical rule is released from rest. A motion sensor records the ball's displacement (with reference to the rule's zeroth mark) and corresponding time. The displacement-time graph is shown below.





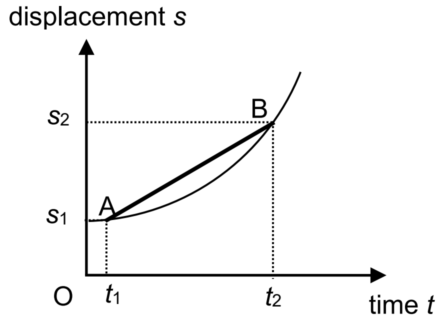
(a) With reference to the graph, briefly describe how the ball's velocity varies with time.
(b) Estimate the velocity of the ball when t = 0.3 s.

(a) The gradient of the graph increases with time. Since the gradient of a displacement-time graph represents the velocity, the ball's velocity increases with time.

(b) When $t = 0.3$ s,

gradient of tangent $=$ velocity $= \dfrac{0.72 - 0.00}{0.40 - 0.15} = 2.9 \text{ m s}^{-1}$

2.2 Graphical interpretation of average velocity and instantaneous velocity



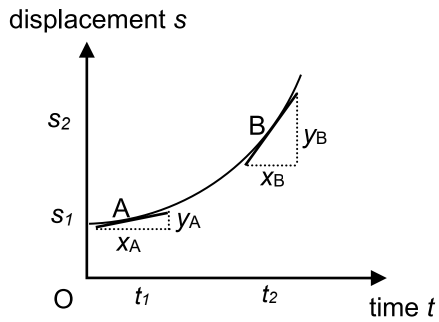
A displacement-time graph to illustrate average velocity

- For a journey taken from point A to point B,

$$\textit{Average velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

- Mathematically, $\langle v \rangle = \dfrac{\Delta s}{\Delta t} = \dfrac{s_2 - s_1}{t_2 - t_1}$

  Graphically, the average velocity between times t_1 and t_2 is derived from the gradient of the line joining these two points.



A displacement-time graph to illustrate instantaneous velocity

- *Instantaneous velocity* refers to velocity at a particular instant.

- Mathematically, $v = \dfrac{ds}{dt}$

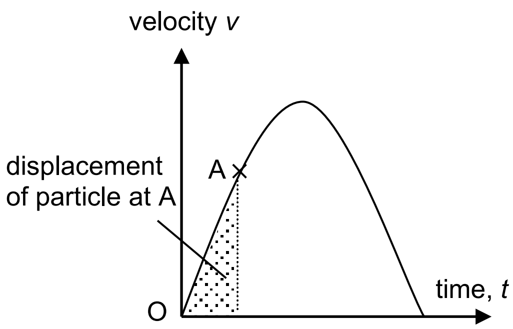
- Graphically, it is determined from the gradient of the tangent at that instant.

  $\Rightarrow$ Instantaneous velocity at A = $\dfrac{y_A}{x_A}$

  $\Rightarrow$ Instantaneous velocity at B = $\dfrac{y_B}{x_B}$

2.3 Velocity-time graph

The motion of a moving object can also be represented and analysed using a velocity-time (v-t) graph, which charts the variation of velocity with time.



A velocity-time graph

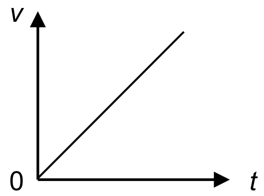
Instantaneous acceleration

$a = \dfrac{dv}{dt}$ is given by the gradient of the v-t graph at that instant.

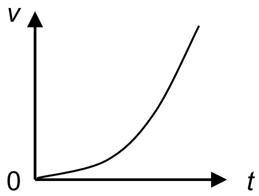
Displacement

$s = \int v\,dt$ is given by the area under the v-t graph.

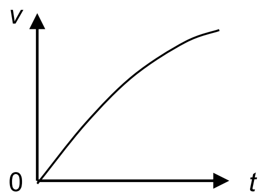
Examples of velocity-time graphs



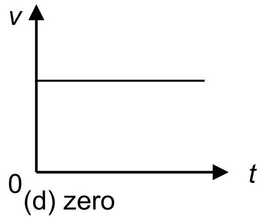
(a) constant acceleration



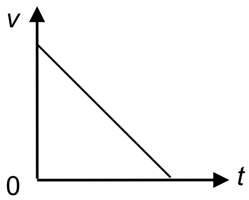
(b) increasing acceleration



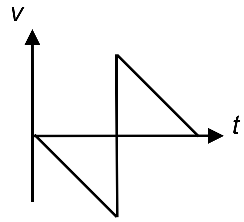
(c) decreasing acceleration



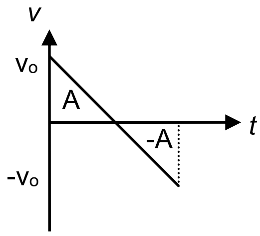
(d) zero acceleration



(e) constant deceleration



(f) Ball 1

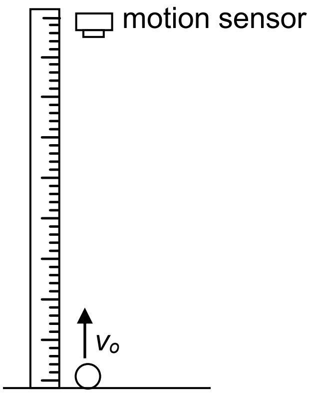


(g) Ball 2

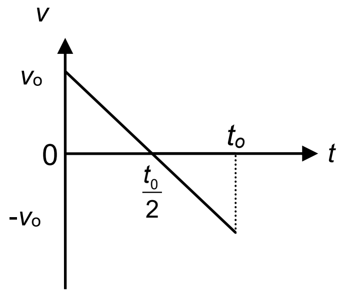
Taking upward to be positive,
Ball 1: released from rest from a height above a horizontal hard surface and rebounds once.
Ball 2: being thrown vertically upwards and falling back to the original position (in free fall).

Example 5

A ball is projected vertically upwards with a speed of v_o. The ball reaches a maximum height and falls down. A motion sensor records the ball's velocity and time, and a velocity-time graph is shown below.



Using the graph,
(a) describe how the ball's acceleration varies with time,
(b) describe how the ball's displacement varies with time,
(c) determine in terms of v_o and t_o,
    (i) the displacement for the whole journey,
    (ii) the distance covered for the whole journey.



(a) The acceleration, which is represented by the gradient of the velocity-time graph, is constant, directed downward (negative gradient).

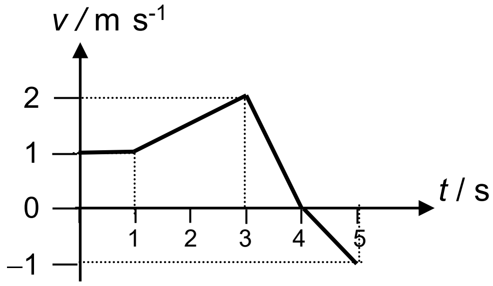
(b) The displacement increases to a maximum value in time $\frac{t_o}{2}$ and then falls back to zero in the same amount of time $\frac{t_o}{2}$.

(c) (i) displacement = 0

(ii) distance $= 2\left(\frac{1}{2}\right)v_o\left(\frac{t_o}{2}\right) = \frac{v_o t_o}{2}$ (total area of the two triangles)

Example 6

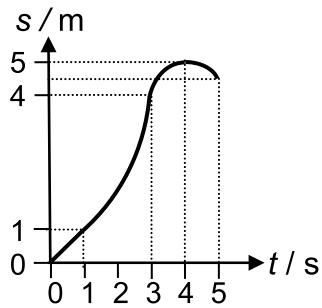
A toy car is moving along a straight path, and its motion is represented by a velocity-time graph as shown.

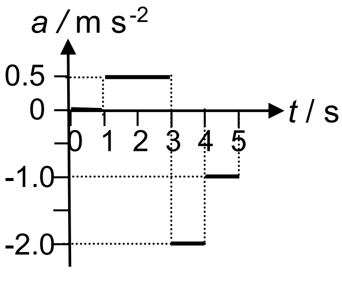


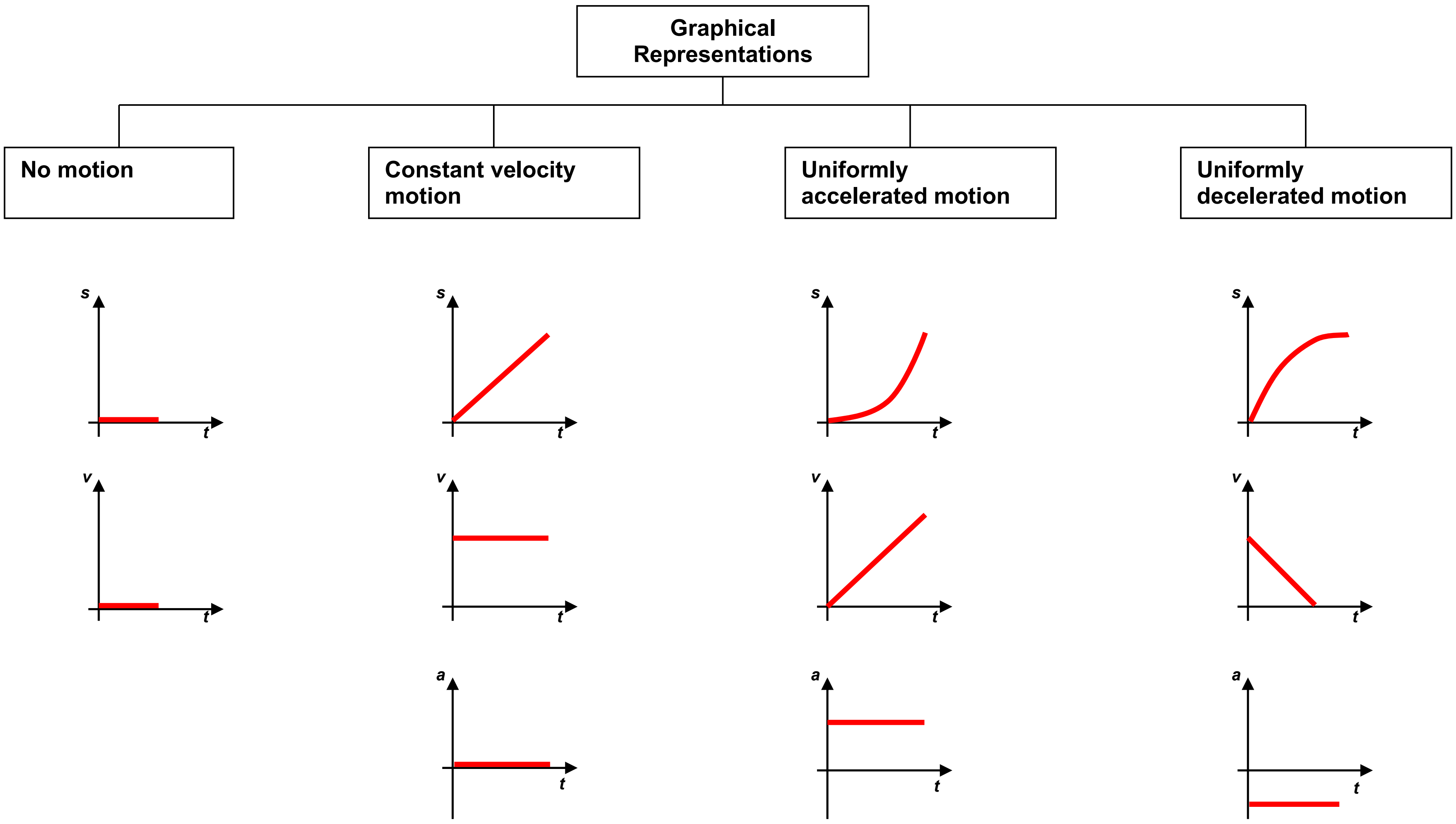
(a) Describe the motion of the toy car.
(b) Sketch the displacement-time and acceleration-time graphs.

(a) For the first 1 s, the car is travelling at a constant velocity of 1 m s^{-1}.
In the next 2 s, it accelerates uniformly with an acceleration of 0.5 m s^{-2} to 2 m s^{-1}.
From t = 3 s to t = 4 s, it then decelerates uniformly at 2 m s^{-2} to rest momentarily.
In the last 1 s, it moves in the opposite direction and accelerates at 1 m s^{-2} until it reaches a velocity of –1 m s^{-1}.

(b)







This figure illustrates that acceleration-time graphs and velocity-time graphs can be deduced from the displacement-time graphs.
Please note that this is by no means an exhaustive list of graphs.

(d) derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line

3.1 Uniformly accelerated motion

When an object's velocity changes at a constant rate, its acceleration is uniform. Two examples of uniformly accelerated motion are:

- a ball thrown up in the air (air resistance is ignored)
- a toy car sliding down a frictionless slope.

Uniformly accelerated motion can be analysed using Equations of Motion.

3.2 Derivation of Equations of Motion from definitions

Consider a body moving with a **constant** acceleration a. Equations used to describe its motion can be derived from first principles:

Acceleration is defined as the rate of change of velocity.

In equation form, $a = \frac{v-u}{t}$

where a = acceleration
u = initial velocity
v = final velocity
t = time taken

Making v the subject,

$\boxed{v = u + at}$ - - - - (1)

Average velocity can be calculated by the ratio of total displacement s to the total time taken.

In equation form, $\langle v \rangle = \frac{s}{t}$

→ $s = \langle v \rangle t$

Since the acceleration is constant, the average velocity is midway between u and v.

i.e. $\langle v \rangle = \frac{u+v}{2}$

Equating and making s the subject,

$\boxed{s = \frac{1}{2}(u+v)t}$ - - - - (2)

Substituting (1) into (2):

$s = \frac{1}{2}\left[u + (u + at)\right]t$

Making s the subject,

$\boxed{s = ut + \frac{1}{2}at^2}$ - - - - (3)

From (1), $t = \frac{v-u}{a}$

Substituting this expression for t into (2):

$$s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right)$$
$$= \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$
$$= \frac{v^2-u^2}{2a}$$

Making v the subject,

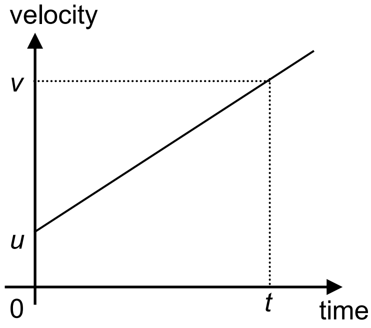
$\boxed{v^2 = u^2 + 2as}$ - - - - (4)

Equations of Motion (1), (2), (3) and (4) only apply under the following two conditions:

- acceleration must be uniform;
- the vectors act along the same line (i.e. motion is rectilinear).

3.3 Derivation of Equations of Motion from graphs

For kinematics graphs, (1) gradient of velocity-time graph = acceleration a
and (2) area under velocity-time graph = displacement s



A velocity-time graph illustrating uniform acceleration

For the velocity-time graph shown,

gradient = $a = \dfrac{v-u}{t-0}$

area = $s = \dfrac{1}{2}(v+u)t$

Hence, $a = \dfrac{v-u}{t} \Rightarrow \; v = u + at$ - - - - (1)

$s = \dfrac{1}{2}(v+u)t$ - - - - (2)

Substituing (1) into (2) by replacing v, $s = ut + \dfrac{1}{2}at^2$ - - - - (3)

Substituting (1) into (2) by replacing t, $v^2 = u^2 + 2as$ - - - - (4)

3.4 Sign Convention

Since u, v, a and s are vector quantities, their directions must be taken into account when solving problems. Appropriate signs must be assigned to them when using these equations. Once determined, the sign convention must apply for the equation, for all the vectors.

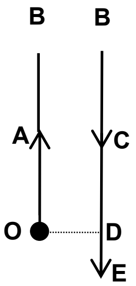
If, say, upward direction is taken to be positive, then

- displacement is negative for points below the initial position
- velocity of body moving downward is negative
- acceleration directed downward is negative

Example 7

A ball is projected vertically upwards from point O. Taking the upward direction as positive, identify the sign of the displacement s, velocity v and acceleration a, during its flight from O, through A, B, C, D to E. (Ignore air resistance)

The trajectory of a ball being thrown upwards:



upward motion: O, A, B

downward motion: B, C, D, E

	s	v	a
O	0	+	–
A	+	+	–
B	+	0	–
C	+	–	–
D	0	–	–
E	–	–	–

(e) *solve problems using equations which represent uniformly accelerated motion in a straight line, e.g. for bodies falling vertically without air resistance in a uniform gravitational field*

4.1 Techniques in problem solving

In solving kinematics questions, the step-by-step process is to:

1. Draw a diagram to show the object's path.
2. State the sign convention.
3. List the known quantities and identify the unknown quantities.
4. Select the appropriate equations to apply.
5. Make the unknown quantities the subject of the equations and solve them.

Example 8

A car is travelling with uniform acceleration along a straight road. The road has marker posts every 100 m. When the car passes the first post, it has a speed of 10 m s^{-1}. When it passes the second post, its speed is 20 m s^{-1}.
(a) Calculate the car's acceleration.
(b) Calculate the car's velocity when it passes the third post?

(a) For the motion between the first and the second posts:

u = 10
v = 20
a = ?
s = 100
t = ?

$$v^2 = u^2 + 2as$$
$$20^2 = 10^2 + 2a(100)$$
$$a = 1.5 \text{ m s}^{-2}$$

(b) For the motion between the second and the third posts:

u = 20
v = ?
a = 1.5
s = 100
t = ?

$$v^2 = u^2 + 2as$$
$$v^2 = 20^2 + 2(1.5)(100)$$
$$v = 26.5 \text{ m s}^{-1}$$

4.2 Necessary information not made explicit in question

There are occasions where deductions must be made so that a problem can be solved.

For example, for vertical motion,

- $a = -g = -9.81$ m s^{-2} when an object is under free fall, taking upwards as positive
- $a = g = 9.81$ m s^{-2} when an object is under free fall, taking downwards as positive
- $v = 0$ m s^{-1} when an object thrown vertically upwards is at maximum height
- $u = 0$ m s^{-1} when an object is released from rest
- $s = 0$ m when an object thrown vertically upwards falls back to its initial position

Example 9

A man standing at the edge of a cliff throws a ball vertically upwards with a velocity of 20 m s^{-1}.

(a) Determine the maximum height above the cliff reached by the ball.

(b) If the cliff is 30 m high, determine

(i) the time taken for the ball to reach the base of the cliff;

(ii) the ball's speed just before it hits the base of the cliff.

Taking upwards as positive:

(a) $u = 20$ $\quad v^2 = u^2 + 2as$

$v = 0$ $\quad 0 = 20^2 + 2(-9.81)h$

$a = -g = -9.81$ $\quad h = 20.4$ m

$s = ?$

(b)(i) $u = 20$ $\quad s = ut + \frac{1}{2}at^2$

$a = -9.81$ $\quad -30 = 20t + \frac{1}{2}(-9.81)t^2$

$s = -30$ $\quad 4.905t^2 - 20t - 30 = 0$

$t = ?$ $\quad t = \dfrac{20 \pm \sqrt{(-20)^2 - 4(4.905)(-30)}}{2(4.905)} = 5.24$ s

(ii) $u = 20$ $\quad v^2 = u^2 + 2as$ $\quad$ OR $\quad v = u + at$

$v = ?$ $\quad v^2 = 20^2 + 2(-9.81)(-30)$ $\quad v = 20 + (-9.81)(5.24)$

$a = -9.81$ $\quad v = \pm 31.4$ $\quad = -31.4$ m s^{-1}

$s = -30$ $\quad v = -31.4$ m s^{-1}

$t = 5.24$

Example 10

A parachuter falls from a hovering helicopter at an altitude of 1.5 km. For the first 10 s, he falls freely, and then he pulls the ripcord. The parachute opens and he falls with an upward acceleration of 20 m s^{-2} until the downward speed is 5.0 m s^{-1}. Thereafter, he falls at a constant velocity. How long does the entire trip take?

The trip may be divided into 3 stages. Taking downward as positive:

For stage 1 (first 10 s, free fall), $\quad u_1 = 0$ $\quad s = ut + \frac{1}{2}at^2$

$v_1 = ?$ $\quad s_1 = 0 + \frac{1}{2}(9.81)(10)^2 = 490.5$ m

$a_1 = 9.81$ $\quad v = u + at$

$s_1 = ?$ $\quad v_1 = 0 + (9.81)(10) = 98.1$ m s^{-1}

$t_1 = 10$

For stage 2 (upward *a*), $\quad u_2 = 98.1$ $\quad v = u + at$

$v_2 = 5.0$ $\quad 5 = 98.1 + (-20)t_2$

$a_2 = -20$ $\quad t_2 = 4.655$ s

$s_2 = ?$ $\quad v^2 = u^2 + 2as$

$t_2 = ?$ $\quad 5^2 = 98.1^2 + 2(-20)s_2$

$s_2 = 240.0$ m

For stage 3 (constant *v*), $\quad t_3 = \dfrac{1500 - 490.5 - 240.0}{5.0} = 153.9$ s

Total time = 10 + 4.655 + 153.9 = 169 s

Part 2: FORCES (DYNAMICS)

Content

- Mass and linear momentum
- Laws of motion

Learning Outcomes

Candidates should be able to:

(f) show an understanding that mass is the property of a body which resists change in motion (inertia)

(g) define and use linear momentum as the product of mass and velocity

(h) state and apply each of Newton's laws of motion:

1st law: a body at rest will stay at rest, and a body in motion will continue to move at constant velocity, unless acted on by a resultant external force;

2nd law: the rate of change of momentum of a body is (directly) proportional to the resultant force acting on the body and is in the same direction as the resultant force;

3rd law: the force exerted by one body on a second body is equal in magnitude and opposite in direction to the force simultaneously exerted by the second body on the first body.

(i) recall the relationship resultant force $F = ma$, for a body of constant mass, and use this to solve problems

Introduction

If you see the velocity of an object change in either magnitude or direction, you know that something must have caused that change (or acceleration). An interaction that can cause an acceleration of a body is called a force, which is loosely speaking, a *push* or a *pull* on the body.

The relationship between a force and the acceleration it causes was first understood by Isaac Newton (1643 – 1727). The study of the relationship is called Newtonian mechanics. We shall focus on its 3 primary laws of motion.

Sir Isaac Newton

Newtonian mechanics does not apply in the following situations:

- When the speeds of the interacting bodies are very large (i.e. approaching the speed of light). In this case, Einstein's special theory of relativity replaces Newtonian mechanics.
- When the interacting bodies are on the atomic scale. In this case, Quantum mechanics replaces Newtonian mechanics.

Physicists now view Newtonian mechanics as a special case of these two more comprehensive theories.

Linking Kinematics and Dynamics

In Kinematics, when we look at ***describing*** the motion of objects, we are concerned only with how the **displacement, velocity** and **acceleration** of the objects change with **time**.

We then try to answer questions like:

a) How far has the object moved from the starting/reference point?

b) How long does the object take to move from point A to point B?

c) How fast must the object move to travel from point A to point B within a certain time interval?

In Dynamics, we will discuss the ***cause*** of motion using two quantities: **force** and **mass**, and study their relationships to the motion of objects. We then try to answer questions like:

a) **Why** does something move?

b) **Why** does something CONTINUE to move?

c) **What causes** something to STOP once it is moving?

1 Newton's laws of motion

(f) show an understanding that mass is the property of a body which resists change in motion (inertia)

(h) state and apply each of Newton's laws of motion

1.1 Newton's first law of motion

- Suppose you send a book sliding across a carpet by applying a horizontal force to it with your hand. After you stop pushing, the book slows down and comes to rest soon after. If you want it to continue sliding, you will have to keep pushing it across the carpet. How about if you now give a push to the book on a frozen lake of ice? In this case, the book would probably slide much further on its own although eventually it will still come to rest.

- What is it that causes the book to come to rest in these two instances? It is the friction between the book and the surface; the friction between the book and the carpet is much higher than that between the book and the frozen lake.

- If we can eliminate friction completely, **the book will never slow down, and we would need no force at all for the book to keep moving with constant velocity**.

- Most moving bodies on Earth visible to us tend to come to rest in the absence of an applied force. This is because moving bodies on Earth are continuously subjected to effects of resistive forces, be it from the ground, air or even between mechanical parts.

- Therefore it is a common <u>misconception</u> that a <u>force must always be applied</u> to keep a body moving at <u>constant</u> velocity.

- **Newton's first law of motion** states that:

A body at rest will stay at rest, and a body in motion will continue to move at constant velocity, unless acted on by a resultant external force

- Motion that is uniform in a straight line implies that **<u>velocity is constant</u>**. In other words, there is **<u>no acceleration</u>**.
- <u>Newton's first law of motion tells the effects of what a force does. A force when applied on an object causes it to accelerate (change in velocity).</u>

- We may have more than one force acting on the object. As such, we consider the effect of the **<u>resultant force</u>** acting on the object.

- Hence, if an object is <u>at rest</u> or <u>moving with constant velocity</u>, **EITHER** no force acts on it, **OR** the resultant force acting on it is zero.

- <u>Newton's first law of motion is often called the law of inertia.</u>

  <u>The inertia of a body is the reluctance of a body to change its state of rest or motion. The mass of a body is a measure of its inertia. The larger the mass, the greater the inertia. It is more difficult to kick a large rock and expect it to move compared to doing the same to a small pebble.</u>

- <u>Mass is the property of a body which resists change in motion (inertia).</u>

- To change the <u>state of motion</u> (i.e. velocity) of an object, a force (push or pull) must be applied to the object. However, the state of motion may remain unchanged even when a force is applied to the object.

- <u>Inertia can be used to explain why a force is needed to:</u>

  - <u>move a stationary object,</u>
  - <u>stop a moving object,</u>
  - <u>change the direction of an object moving in a straight line.</u>

Topic 5: Projectile Motion Learning Outcome (a) describe and use the concept of weight as the force experienced by a mass in a gravitational field

- When an object is brought from place to place, its mass remains the same. However, its weight may vary considerably from place to place.

  While it is true that you will weigh less if you have less mass, this cannot account fully for the difference. For example, you will weigh six times lighter on the Moon than on the Earth. This is not because you have lost mass. You still have the same mass. The difference in weight arises because of the difference in the **gravitational field strength** of the Earth and that of the Moon.

The **weight** of a body is the gravitational force exerted on it by a gravitational field.

weight = mass x acceleration of free fall   or $w = mg$

- SI Unit : newton (N)
- Mass is a scalar quantity, while weight is a vector quantity.
- The direction of weight is always in the direction of the gravitational field strength. In the context of Earth, weight will always point towards the centre of the Earth.

1.2 Newton's second law of motion

(g) define and use linear momentum as the product of mass and velocity

(i) recall the relationship resultant force F = ma, for a body of constant mass, and use this to solve problems

1.2.1 Linear momentum

- It gives a measure of how an object will respond to an externally applied force, e.g. how much force is required to get the object to stop.

The linear momentum p of a body is defined as the product of its mass m and its velocity v.

Mathematically:

$$\boxed{p = m\,v}$$

Linear momentum is a vector quantity and its unit is kg m s^{-1} or N s.

Direction of momentum of body is in the direction of its velocity.

1.2.2 Defining Newton's second law of motion

- **Newton's second law of motion** states that:

The rate of change of momentum of a body is (directly) proportional to the resultant force acting on the body and is in the same direction as the resultant force.

- Mathematically:

$$\boxed{\frac{\mathrm{d}p}{\mathrm{d}t} \propto F}$$

$$\Rightarrow F = k\frac{\mathrm{d}p}{\mathrm{d}t}$$

where k is the constant of proportionality. k is taken to be 1 when the quantities used in computation are expressed in SI base units.

Hence:

$$\boxed{F = \frac{\mathrm{d}p}{\mathrm{d}t}}$$

where

F is the resultant force (or net force) acting on the body; and
$\frac{\mathrm{d}p}{\mathrm{d}t}$ is the rate of change of momentum of the body.

- If the effect of forces acting on a body is analysed over a certain time interval Δt, the following equation can be used:

$$\boxed{\langle F \rangle = \frac{\Delta p}{\Delta t}}$$

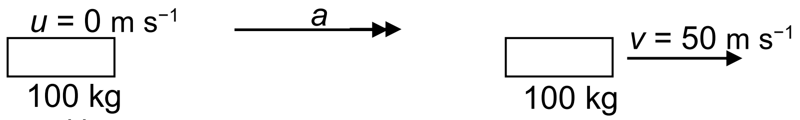
where
$\langle F \rangle$ is the *average* resultant force (or *average* net force) acting on the body; and
Δp is the change in momentum of the body over the time interval Δt.

Example 1

An object of mass 100 kg accelerates uniformly towards the right from rest to 50 m s^{-1} over a period of 2.0 s.

Calculate
(i) the change in the object's momentum, and
(ii) the net force causing the acceleration.

Solution



Take right as positive:
(i) Change in momentum $\Delta p = mv - mu$

$= 100 (50) – 100 (0)$
$= 5000$ kg m s^{-1} (directed towards the right)

(ii) Net force causing the acceleration $= \dfrac{\Delta p}{\Delta t}$

$= \dfrac{5000}{2.0}$

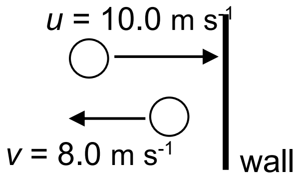
$= 2500$ N (directed towards the right)

Example 2

A ball of mass 0.10 kg hits a smooth vertical wall normally with a speed of 10.0 m s^{-1} and bounces off the wall normally with a speed of 8.0 m s^{-1} after 0.20 s.

Calculate the net force exerted on the ball during the bounce.

Solution



Take left (away from wall) as positive:

Change in momentum of the ball $= mv - mu$

$= 0.10(8.0) - 0.10(-10.0)$

$= 1.8 \text{ kg m s}^{-1}$

From Newton's second law,

net force exerted on the ball during the bounce $= \dfrac{\Delta p}{\Delta t} = \dfrac{1.8}{0.20} = 9.0 \text{ N}$

Direction: (away from wall)

Note: The net force acting on the ball is the impact force that the wall exerts on the ball.

1.2.3 Applying Newtons' second law on a <u>body of constant mass</u>

- From Newton's second law of motion,

  resultant force, $F = \dfrac{\text{momentum change}}{\text{time}}$

  $= \text{mass} \times \dfrac{\text{velocity change}}{\text{time}}$

  $= \text{mass} \times \text{acceleration}$

  Therefore $\boxed{F = m\,a}$

  where
  m is the mass of the body; and
  a is the acceleration of the body.

- The <u>magnitude</u> of the resultant force is obtained from $F = m\,a$.

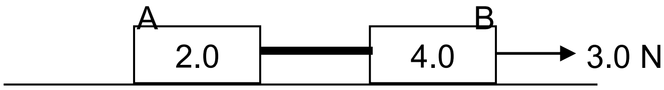
- The <u>direction</u> of the resultant force is the same as that of the acceleration (or momentum change)

- **Steps for determining the acceleration of a <u>constant mass</u>:**

  - Draw a force diagram that shows all the forces acting <u>on</u> the body.
  - Determine the resultant force acting on the body.
  - Use $a = \dfrac{F}{m}$ to determine the acceleration of the body.

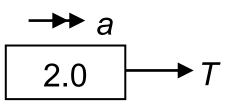
<u>Example 3</u>

Block A of mass 2.0 kg and B of mass 4.0 kg connected by a rope of negligible mass are at rest on a frictionless surface as shown below. If a force of 3.0 N is applied on B, determine the acceleration of A and B.

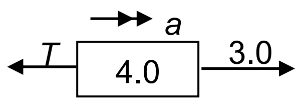


<u>Solution</u>

Force diagram for A,



Force diagram for B,



Take rightwards to be positive:

using $F = ma$

$T = 2.0\,a$ --------- (1)

using $F = ma$

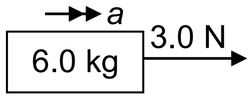
$3.0 - T = 4.0\,a$ --------- (2)

where T is the tension in the rope.

Solving (1) and (2), $T = 1.0$ N, $a = 0.50$ m s^{-2}

Alternatively,

Treating A and B as one body

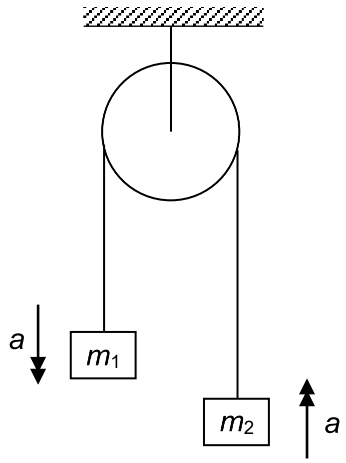


using $F = ma$

$3.0 = (4.0 + 2.0)\,a$
$a = 0.50 \text{ m s}^{-2}$

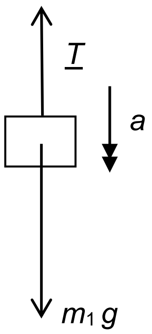
Example 4

For the pulley system shown, determine the acceleration a of the masses m_1 and m_2 ($m_1 > m_2$) and the tension T in the cable in terms of m_1, m_2 and the acceleration of free fall g.

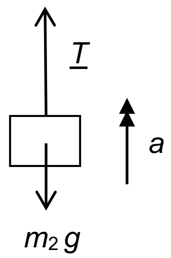


Solution

Force diagram for mass m_1:



Force diagram for mass m_2:



$m_1g - T = m_1a$ ---------- (1)

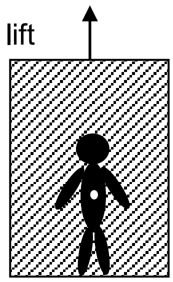
$T - m_2g = m_2a$ ---------- (2)

(1) + (2), $$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

Therefore, $$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

Example 5

Determine the force that the floor of a lift exerts on an 80.0 kg man when the lift

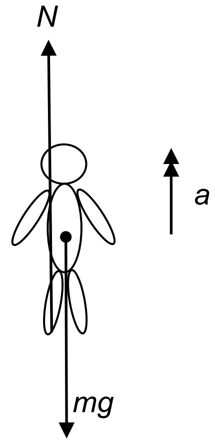


(a) is at rest;
(b) rises with a constant velocity of 2.00 m s^{-1};
(c) rises with a constant acceleration of 2.00 m s^{-2}; and
(d) descends with a constant acceleration of 2.00 m s^{-2}.

(Take g = 9.80 m s^{-2})

Solution

Let N be the force exerted by the lift on the man.
N is also known as the **apparent weight** of the man.



Let a be the acceleration of the system (lift and man).

Taking upwards as positive and by Newton's second law,

$$N - mg = ma$$
$$N = m(a + g)$$

(a) At rest, $a = 0$

$$N = m(0 + g) = mg = (80.0)(9.80) = 784 \text{ N}$$

(b) At constant velocity upwards, $a = 0$

$$\therefore N = mg = 784 \text{ N}$$

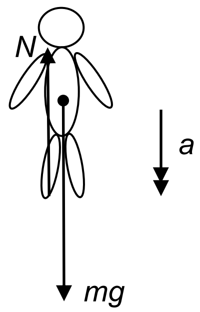
(c) At constant acceleration upwards, a = 2.00 m s^{-2}

$$\therefore N = m(a + g) = (80.0)(2.00 + 9.80) = 944 \text{ N}$$

Note:
- Apparent weight N is larger than the true weight mg.
- The man feels 'heavier' as the lift is pushing up on him with a larger force than his true weight.

(d) Taking downwards as positive:

$$mg - N = ma$$
$$N = m(g - a)$$



At constant acceleration downwards, a = 2.00 m s^{-2}

$$\therefore N = m(g - a) = (80.0)(9.80 - 2.00) = 624 \text{ N}$$

Note:
- Apparent weight N is smaller than the true weight mg.
- The man feels 'lighter' as the lift is pushing up on him with a smaller force than his true weight.

1.2.4 Applying Newtons' second law on a system of changing mass

- There are situations where forces are acting on a system that has a changing mass with respect to time and a velocity change by the system due to the application of these forces.

- Examples of **changing mass systems**: fluids; system of particles.

- From Newton's second law of motion,

resultant force, $F = \dfrac{\text{momentum change}}{\text{time}}$

$= \dfrac{\text{mass}}{\text{time}} \times \text{velocity change}$

$$\boxed{F = \Delta v \frac{dm}{dt}}$$

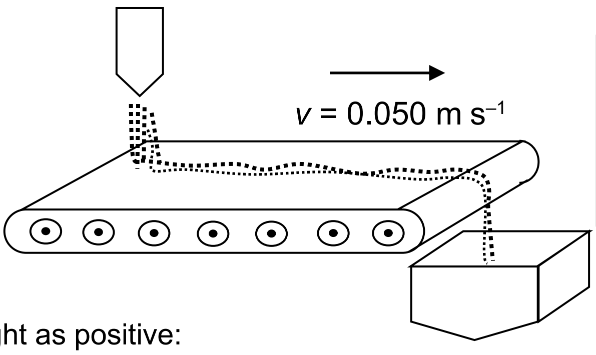
where

Δv is the velocity change; and

$\dfrac{dm}{dt}$ is the rate of change of mass of the system (unit: kg s^{-1}).

- Suppose sand is allowed to fall vertically at a steady rate of 0.10 kg s^{-1} onto a horizontal conveyor belt moving at a constant velocity of 0.050 m s^{-1} as shown below.

The initial horizontal velocity of the sand is zero.
The final horizontal velocity of the sand is 0.050 m s^{-1}.



The belt provides the horizontal force needed for the momentum increase per second of the sand.

Take right as positive:

For every 1 second,

- mass of sand that has fallen onto the conveyor belt = 0.10 kg
- velocity change Δv of the sand = 0.050 m s^{-1} (horizontally, rightwards)
- momentum change Δp of the sand = $m\Delta v = 0.10 \times 0.050 = 0.0050 \text{ kg m s}^{-1}$ (horizontally rightwards)
- resultant force acting on the sand $= \dfrac{\Delta p}{\Delta t} = \dfrac{0.0050 \text{ kg m s}^{-1}}{1 \text{ s}}$

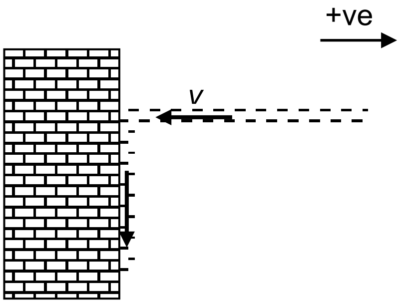
  $= 0.0050 \text{ kg m s}^{-2}$

  $= 0.0050 \text{ N}$ (horizontally rightwards)
- The belt provides this extra horizontal force needed for the momentum increase per second of the sand.
- This is an example where the mass changes with time and the velocity gained is constant.

To apply $F = \Delta v \dfrac{dm}{dt}$:

Resultant force acting on the sand = $F = \Delta v \dfrac{dm}{dt} = (0.050 - 0)(0.10) = 0.0050\,\text{N}$

- Suppose a horizontal stream of water hits a vertical wall and loses all its horizontal momentum.



Change in horizontal velocity of water, $\Delta v = 0 - (-v) = v$

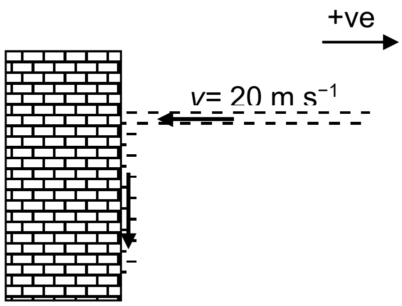
Horizontal force F acting on the water = $\Delta v \dfrac{dm}{dt} = v \dfrac{dm}{dt}$ (in the positive direction)

where $\dfrac{dm}{dt}$ is the mass flow rate of the water (in kg s^{-1}).

Example 6

A hose directs a horizontal jet of water with velocity 20 m s^{-1} onto a vertical wall. The cross-sectional area of the jet is 5×10^{-4} m^2.

If the density of water is 1000 kg m^{-3}, calculate the force acting on the water, assuming the water is brought to rest at the wall.



Solution

Mass flow rate of the water $\dfrac{dm}{dt} = \dfrac{d}{dt}(\rho V) = \rho \dfrac{d}{dt}(As) = \rho A \dfrac{ds}{dt} = \rho A v$

$= (1000)(5 \times 10^{-4})(20) = 10$ kg s^{-1}

Force acting on the water $= \Delta v \dfrac{dm}{dt} = [0 - (-20)](10) = 200$ N (acting to the right).

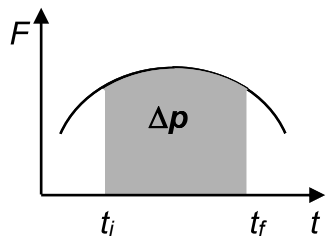
Topic 6: Collisions
Learning Outcome (a) recall that impulse is given by the area under the force–time graph for a body and use this to solve problems

1.2.5 Impulse

- **Impulse is defined as the product of force and time of impact.**

- Recall Newton's second law:

resultant force $F = \dfrac{dp}{dt}$



Integrating both sides give: $dp = F\,dt$

$$\int_{p_i}^{p_f} dp = \int_{t_i}^{t_f} F\,dt$$

$$\boxed{\Delta p = \int_{t_i}^{t_f} F\,dt}$$

where
Δp is the change in momentum of the body over a time interval $t_f - t_i$; and
$\int_{t_i}^{t_f} F\,dt$ is the impulse of the resultant force F acting over the time interval $t_f - t_i$.

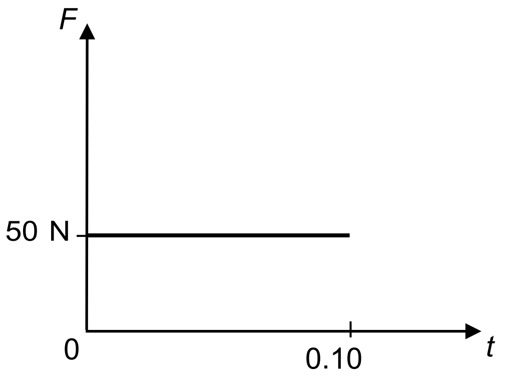
- Impulse is calculated as the integral of force over the time interval during which the force acts and is represented by the **<u>area beneath a force-time graph</u>**. Hence, impulse = $\int_{t_i}^{t_f} F\,dt$. It is also the product of the average net force and the time of impact.

- The impulse acting on an object that is free to move is equal to its **<u>change in momentum.</u>**

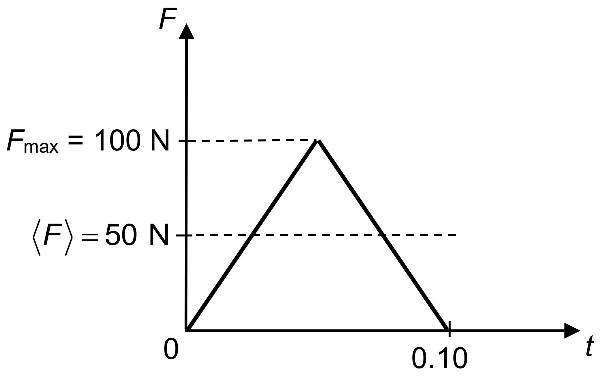
- The unit of impulse is N s or kg m s^{-1}.

- The $F - t$ graphs below show different ways that a resultant force can be applied on a body over a time interval $\Delta t = 0.10$ s.

Graph A shows a constant force of 50 N.
Graph B shows a force increasing from zero to 100 N and then decreasing to zero.



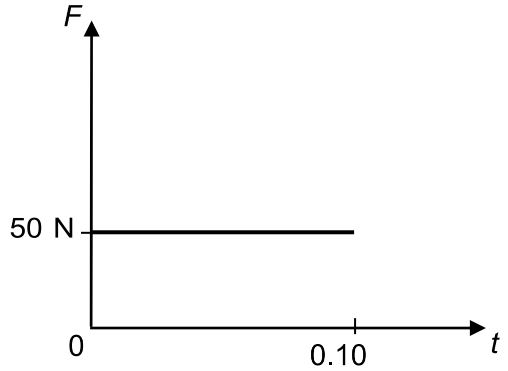
Graph A



Graph B

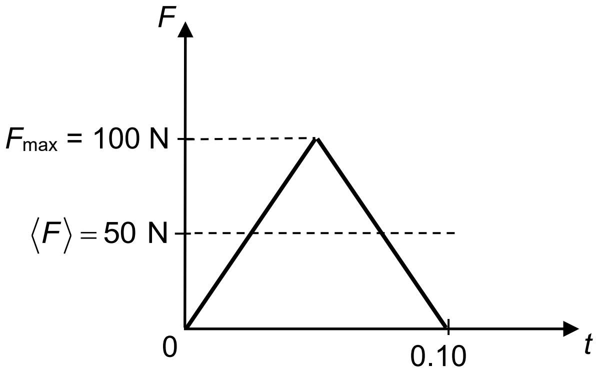
- The <u>area under the graph is equal</u> for both graphs. Therefore, the <u>change in momentum of the body is the same</u> for both cases.

- For Graph A, the change in momentum Δp of the body $= 50 \times 0.10 = 5.0 \text{ kg m s}^{-1}$



- For Graph B, the change in momentum Δp of the body

$= \frac{1}{2}(100)(0.10) = 5.0 \text{ kg m s}^{-1}$ OR $\langle F \rangle \Delta t = 50 \times 0.10 = 5.0 \text{ kg m s}^{-1}$



Therefore, the change in momentum of a body can be expressed as

$$\Delta p = \langle F \rangle \Delta t$$

where

$\langle F \rangle$ is the *average* resultant force acting on the body; and

Δt is the time interval over which the average resultant force is applied.

$\langle F \rangle \Delta t$ is the impulse of the average resultant force $\langle F \rangle$ acting over the time interval Δt.

Example 7

A car of mass 1200 kg is accelerated by a uniform resultant force of 3000 N for a time of 5.0 s. What is the gain in the momentum of the car?

Solution

The resultant force of 3000 N acting on the car is uniform. Therefore the average resultant force $\langle F \rangle$ acting on the car is 3000 N.

Change in momentum of the car $\Delta p = \langle F \rangle \Delta t$

$= 3000 \times 5.0$

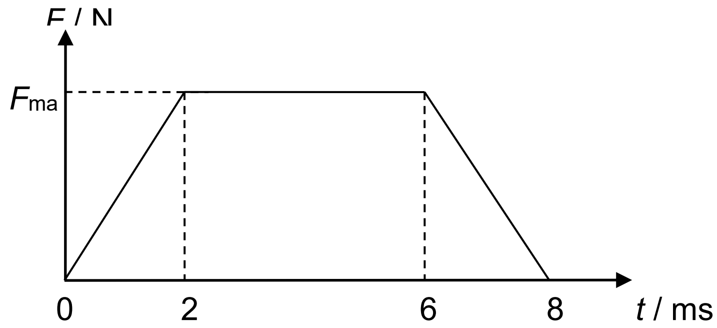
$= 1.5 \times 10^4$ kg m s^{-1}

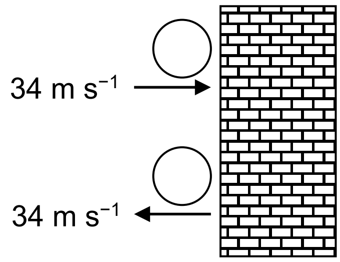
in the same direction as the resultant force

Example 8

The graph shows the variation with time of contact force during the collision of a 58 g tennis ball with a wall. The initial velocity of the ball is 34 m s^{-1} perpendicular to the wall; it rebounds with the same speed, also perpendicular to the wall.

What is F_{max}, the maximum value of the contact force during the collision?





Solution

Taking the direction away from the wall to be the positive direction $\xleftarrow{+ve}$

Change in momentum of ball $\Delta p = m(v_f - v_i) = (0.058)[(34 - (-34)] = 3.94$ kg m s^{-1}

For the F - t graph, $\Delta p = \int F\, dt$

$$3.94 = \frac{1}{2}\left[8 + (6-2)\right] \times 10^{-3} (F_{max})$$

$$3.94 = 0.006 F_{max}$$

$$F_{max} = 657 \text{ N (i.e. away from the wall)}$$

1.3 Newton's third law of motion

- Forces come in pairs. If a hammer exerts a force on a nail, the nail exerts an equal but opposite force on the hammer. If you lean against a brick wall, the wall pushes back on you.

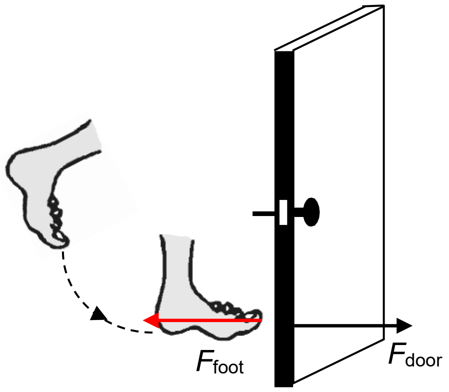
- **Newton's third law of motion** states that:

The force exerted by one body on a second body is equal in magnitude and opposite in direction to the force simultaneously exerted by the second body on the first body.

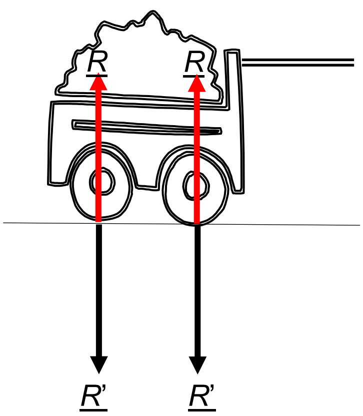
- These two forces are often referred to as an **action-reaction pair.**

  1. They act on two different objects.
  2. They must be of the same type.
     (Same type means that if one is a gravitational force, the other must also be gravitational, or if one is a tension, the other force must also be tensional.)
  3. They have the same magnitude.
  4. They act in opposite directions.
  5. They always exist in pairs, regardless of whether the objects are stationary or moving.

- Examples of action-reaction pairs:



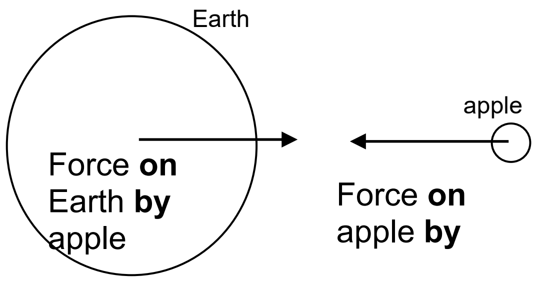
- The force F_{door}, exerted by the boy on the door accelerates the door (it flies open).
- At the same time, the door exerts an equal but opposite force F_{foot}, on the boy, which decelerates the boy (his foot loses forward velocity).
- The boy will be painfully aware of the 'reaction' force to his 'action', particularly if his foot is bare.



R is the force acting on wheel, by ground.
R' is the force acting on ground, by wheel.

R and *R'* is an action-reaction pair.

i) an apple placed in space near the Earth



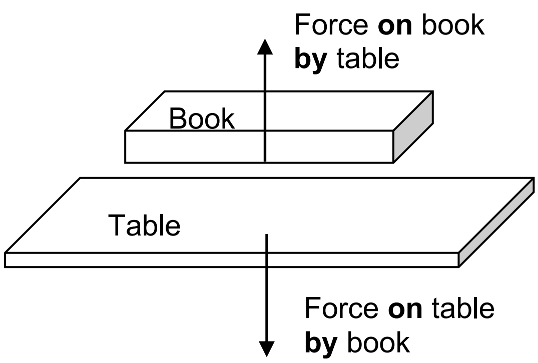
Notes:

The **force on apple by Earth** is the **weight** of the apple.

The apple is actually also attracting the Earth towards itself (**Force on Earth by apple**).

i) a stationary book on a table

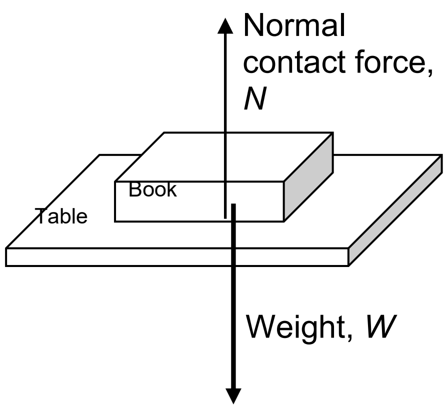
Free body diagram



Notes:

- By replacing the name of the force using ***"force on___ by ___"***, it will be clear which 2 forces are the action-reaction pair.
- The force **on** book **by** table is the force "felt" by the book.
- The force **on** table **by** book is the force "felt" by the table.

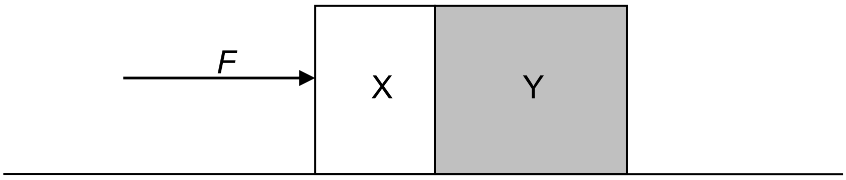
- For a book resting on a table, discuss if the normal contact force, *N*, and the weight, *W*, are Newton's 3rd law paired forces.



Are *N* and *W* action-reaction pair?

Example 9

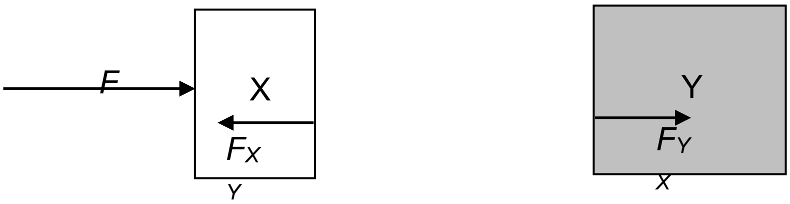
Two blocks X and Y, of masses m and $3m$ respectively, are accelerated along a smooth horizontal surface by a force F applied to block X as shown.



Determine the force exerted by block X on block Y during this acceleration in terms of F?

Solution

Drawing separate force diagrams,



Let F_{XY} be the force exerted by Y on X.
Let F_{YX} be the force exerted by X on Y.

Taking right to be positive direction and applying Newton's second law to X gives:

$$F - F_{XY} = ma \quad \text{-------------- (1)}$$

Applying Newton's second law to Y gives:

$$F_{YX} = 3\,ma \quad \text{-------------- (2)}$$

where a is the acceleration of both blocks.

Dividing (2) by (1) yields $\dfrac{F_{YX}}{F - F_{XY}} = 3$

$$F_{YX} = 3F - 3F_{XY}$$

By Newton's third law, $|F_{XY}| = |F_{YX}|$.

$$\text{Hence } 4F_{YX} = 3F$$

$$F_{YX} = 0.75\,F$$