Solution to Term 2 CT Practice 3

1 $y + 3x = 1$ ----------- (1)

$x^2 + 2xy + 2y^2 = 25$ ---------- (2)

From (1), $y = 1 - 3x$ ---------- (3)

Substitute (3) into (2).

$$x^2 + 2x(1 - 3x) + 2(1 - 3x)^2 = 25$$

$$x^2 + 2x - 6x^2 + 2(1 - 6x + 9x^2) = 25$$

$$2x - 5x^2 + 2 - 12x + 18x^2 = 25$$

$$13x^2 - 10x + 2 = 25$$

$$13x^2 - 10x - 23 = 0$$

$$(13x - 23)(x + 1) = 0$$

$13x$	-23	$-23x$
x	$+1$	$+13x$
$13x^2$	-23	$-10x$

$13x - 23 = 0$ or $x + 1 = 0$

$13x = 23$ $\quad$ $x = -1$

$$x = 1\frac{10}{13}$$

Substitute $x = 1\frac{10}{13}$ into (3).

$$y = 1 - 3\left(1\frac{10}{13}\right)$$

$$= 1 - 5\frac{4}{13}$$

$$= -4\frac{4}{13}$$

Substitute $x = -1$ into (3).

$$y = 1 - 3(-1)$$

$$= 1 + 3$$

$$= 4$$

The coordinates of A are $\left(1\frac{10}{13}, -4\frac{4}{13}\right)$ and $B = (-1, 4)$.

2 **(i)**

$$
\begin{aligned}
y &= -x^2 - x + 6 \\
&= -(x^2 + x - 6) \\
&= -\left[x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6\right] \\
&= -\left[\left(x + \frac{1}{2}\right)^2 - 6\frac{1}{4}\right] \\
&= -\left(x + \frac{1}{2}\right)^2 + 6\frac{1}{4} \\
&= 6\frac{1}{4} - \left(x + \frac{1}{2}\right)^2
\end{aligned}
$$

(ii) $y = -x^2 - x + 6$

The coefficient of x^2 is -1 and the graph has a maximum point at $\left(-\frac{1}{2}, 6\frac{1}{4}\right)$.

When the curve cuts the x-axis, $y = 0$.

$$
\begin{aligned}
-\left(x + \frac{1}{2}\right)^2 + 6\frac{1}{4} &= 0 \\
\left(x + \frac{1}{2}\right)^2 &= 6\frac{1}{4} \\
x + \frac{1}{2} &= \pm\sqrt{6\frac{1}{4}}
\end{aligned}
$$

$$
x + \frac{1}{2} = \sqrt{6\frac{1}{4}} \quad \text{or} \quad x + \frac{1}{2} = -\sqrt{6\frac{1}{4}}
$$

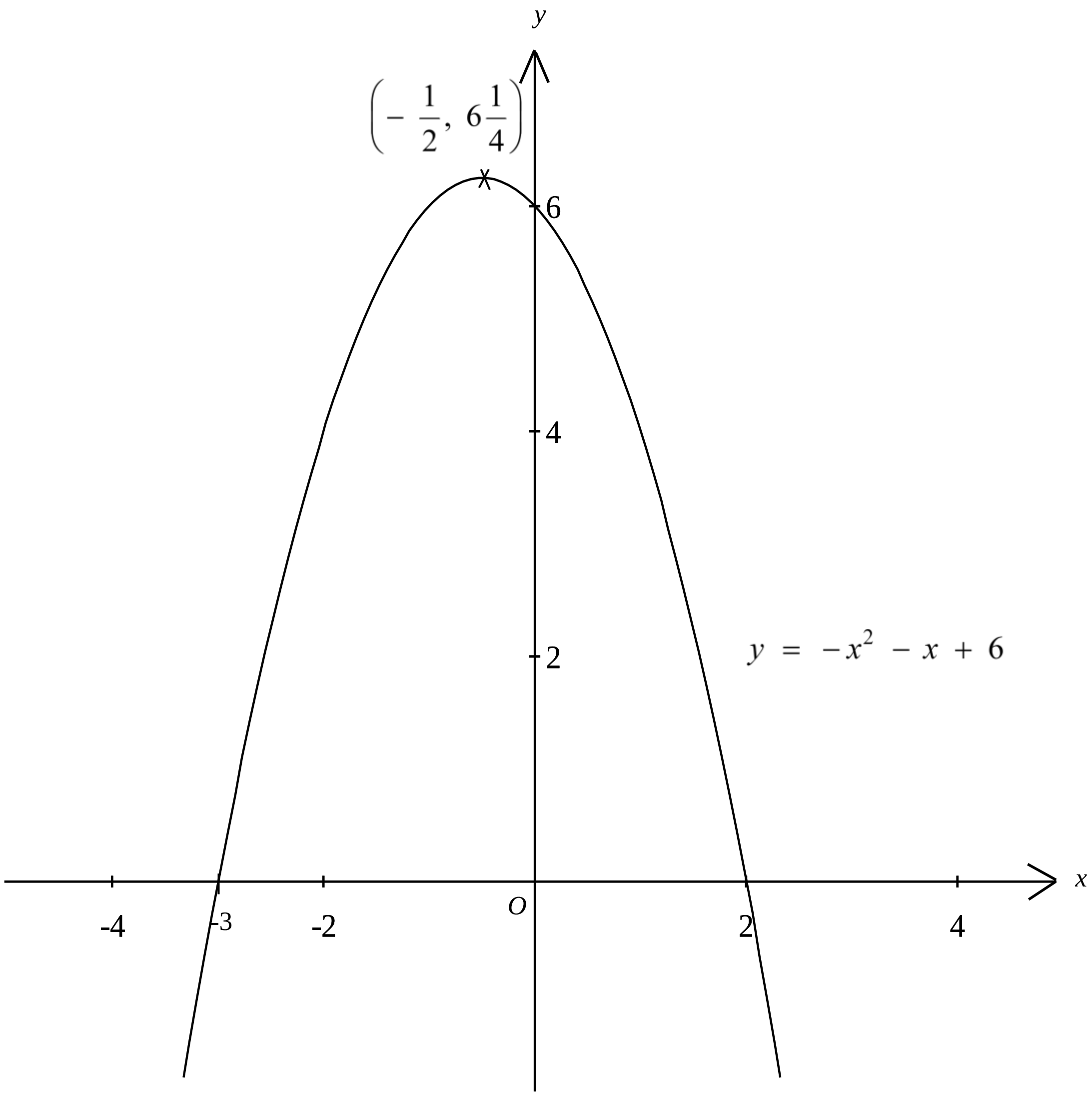
$$
x = 2 \qquad\qquad x = -3
$$

The curve cuts the x-axis at (2, 0) and (–3, 0).

When the curve cuts the y-axis, $x = 0$.

$$
\begin{aligned}
y &= 0^2 - 0 + 6 \\
&= 6
\end{aligned}
$$

The curve cuts the y-axis at (0, 6).



3 $3^{2x+1} - 28(3^x) = -9$

$3^{2x} \times 3 - 28(3^x) + 9 = 0$

Let $u = 3^x$.

$3u^2 - 28u + 9 = 0$

$(3u-1)(u-9) =$

$u = \frac{1}{3}$ or $u = 9$

$3^x = \frac{1}{3}$ $\quad 3^x = 9$

$x = -1$ or $x = 2$

4 **(a)** $\sqrt{x+3}=2+\sqrt{5}$

$x+3=4+4\sqrt{5}+5$ (squaring both sides)

$\therefore x=6+4\sqrt{5}$

Oops.. the question may be more challenging if it is changed to

$$\sqrt{x+3}=2+\sqrt{x}$$

$x+3=4+4\sqrt{x}+x$ (squaring both sides)

$$x+3-4-x=4\sqrt{x}$$

$$-1=4\sqrt{x}$$

$$-\frac{1}{4}=\sqrt{x}$$

$$\therefore x=\frac{1}{16}$$

(b) $x\sqrt{20}+\sqrt{27}=\sqrt{80}-x\sqrt{12}$

$$2x\sqrt{5}+3\sqrt{3}=4\sqrt{5}-2x\sqrt{3}$$

$$2x\left(\sqrt{5}+\sqrt{3}\right)=4\sqrt{5}-3\sqrt{3}$$

$$\therefore x=\frac{4\sqrt{5}-3\sqrt{3}}{2\left(\sqrt{5}+\sqrt{3}\right)}$$

$$=\frac{4\sqrt{5}-3\sqrt{3}}{2\left(\sqrt{5}+\sqrt{3}\right)}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{\left(4\sqrt{5}-3\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}{2\left(5-3\right)}$$

$$=\frac{4\left(5\right)-4\sqrt{15}-3\sqrt{15}+3\left(3\right)}{4}$$

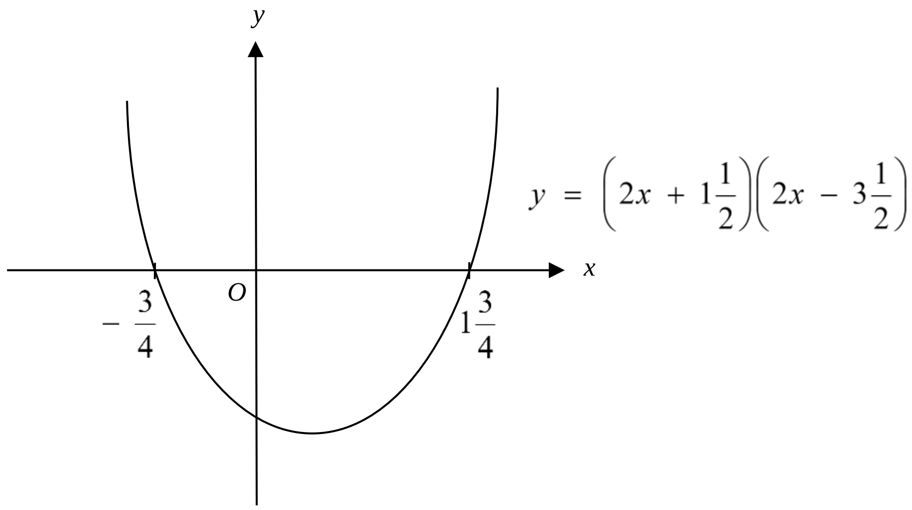
$$=\frac{29-7\sqrt{15}}{4}$$

5 **(a)** $(2x - 1)^2 - 6\frac{1}{4} > 0$

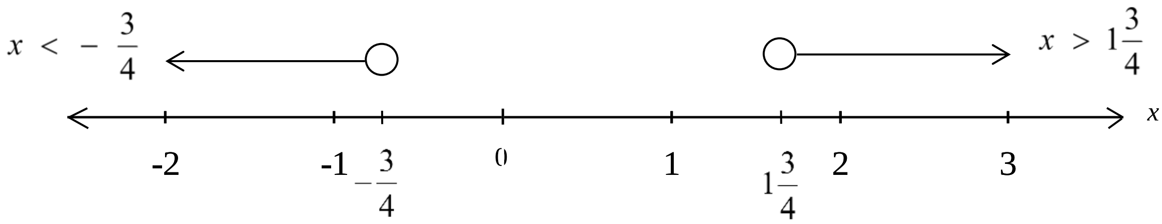
$(2x - 1)^2 - \left(2\frac{1}{2}\right)^2 > 0$

$\left(2x - 1 + 2\frac{1}{2}\right)\left(2x - 1 - 2\frac{1}{2}\right) > 0$

$\left(2x + 1\frac{1}{2}\right)\left(2x - 3\frac{1}{2}\right) > 0$



The range of values of x for which $(2x - 1)^2 - 6\frac{1}{4} > 0$ is $x < -\frac{3}{4}$ or $x > 1\frac{3}{4}$.



(b) $x^2 + 7x - 9 > 8x + c$

$x^2 - x - 9 - c > 0$

For $x^2 - x - 9 - c > 0$, the equation $x^2 - x - 9 - c = 0$ has no real roots.

$$b^2 - 4ac < 0$$

$$(-1)^2 - 4(1)(-9 - c) < 0$$

$$1 + 36 + 4c < 0$$

$$37 + 4c < 0$$

$$4c < -37$$

$$c < -9\frac{1}{4}$$

6 **(i)** $y = (k-3)x^2 - kx + \frac{k}{2}$

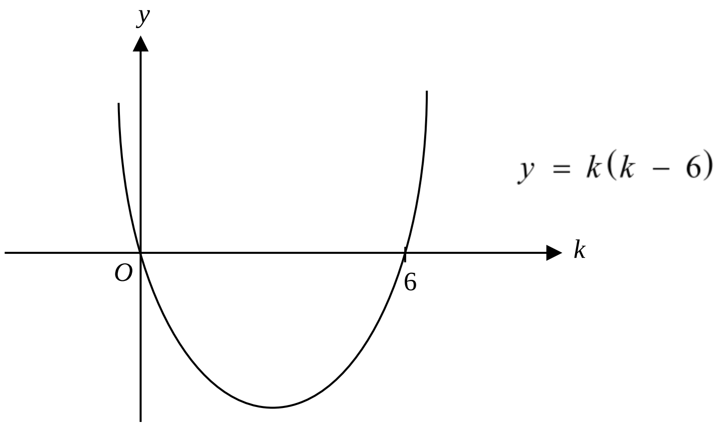
Given that the curve has a minimum point,

$k - 3 > 0$

$k > 3$ ....(1)

Given that the curve cuts the *x*-axis at two distinct points,

$$b^2 - 4ac > 0$$
$$(-k)^2 - 4(k-3)\left(\frac{k}{2}\right) > 0$$
$$k^2 - 2(k-3)(k) > 0$$
$$k^2 - 2k^2 + 6k > 0$$
$$-k^2 + 6k > 0$$
$$k^2 - 6k < 0$$
$$k(k-6) < 0$$



The range of values of *k* is $0 < k < 6$. ……………….(2)

Combining (1) and (2), the range of values of k is $3 < k < 6$.

(ii) $y = (k - 3)x^2 - kx + \dfrac{k}{2}$

When $k = 4$,

$y = (4 - 3)x^2 - 4x + \dfrac{4}{2}$

$y = x^2 - 4x + 2$ ---------- (1)

$y = x + c$ ---------- (2)

Substitute (1) into (2).

$$x^2 - 4x + 2 = x + c$$
$$x^2 - 5x + 2 - c = 0$$

Since the line meets the curve, the above equation has real roots.

$$\begin{aligned} b^2 - 4ac &\geq 0 \\ (-5)^2 - 4(1)(2 - c) &\geq 0 \\ 25 - 8 + 4c &\geq 0 \\ 17 + 4c &\geq 0 \\ 4c &\geq -17 \\ c &\geq -4\frac{1}{4} \end{aligned}$$

7 The sum of their areas $= 84\ \text{cm}^2$

$x^2 + xy = 84$ ---------- (1)

The sum of their perimeter $= 52$ cm

$$\begin{aligned} 4x + 2(x + y) &= 52 \\ 4x + 2x + 2y &= 52 \\ 6x + 2y &= 52 \\ 3x + y &= 26 \\ y &= 26 - 3x \text{ ---------- (2)} \end{aligned}$$

Substitute (2) into (1).

$$\begin{aligned} x^2 + x(26 - 3x) &= 84 \\ x^2 - 3x^2 + 26x &= 84 \\ -2x^2 + 26x - 84 &= 0 \\ x^2 - 13x - 42 &= 0 \\ (x - 6)(x - 7) &= 0 \end{aligned}$$

$x - 6 = 0$ or $x - 7 = 0$

$x = 6$ $\quad\quad$ $x = 7$

x	-6	$-6x$
x	-7	$-7x$
x^2	$+42$	$-13x$

Substitute $x = 6$ into (2).

$$\begin{aligned} y &= 26 - 3(6) \\ &= 26 - 18 \\ &= 8 \end{aligned}$$

Substitute $x = 7$ into (2).

$$\begin{aligned} y &= 26 - 3(7) \\ &= 26 - 21 \\ &= 5 \qquad \text{(rejected } \because y < x \text{)} \end{aligned}$$

The dimensions of the rectangular card is 6 cm by 8 cm.