

ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS

9758/01

Paper 1
QUESTION PAPER

25 August 2026
3 hours

Additional Materials: Printed Answer Booklet
 List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.



Anglo-Chinese Junior College

[Turn over

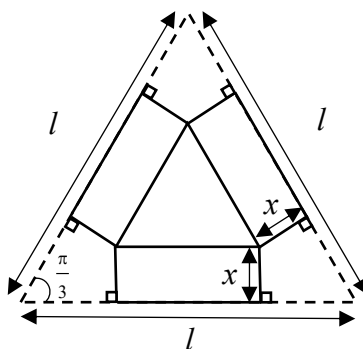
- 1 Use an algebraic method to solve the inequalities

$$\frac{x(5x+9)}{x^2+2x+5} > x \geq 3x-6. \quad [4]$$

- 2 Using standard series from the List of Formulae (MF27), find the Maclaurin's series for $\ln(2+\sin x)$, where x is sufficiently small for terms x^3 and above to be neglected. [3]

Hence deduce the equation of the tangent to the curve $y = \ln(2+\sin x)$ at $x=0$. [1]

- 3 A piece of cardboard has the shape of an equilateral triangle, with fixed side length l cm. Six perpendicular cuts, each of length x cm, are made to the cardboard to remove the corners. The remaining cardboard is then folded to form an open box with an equilateral triangular base.



- (a) Show that the length of the equilateral triangular base is $l - 2x\sqrt{3}$. [1]
- (b) Find the value of x , in terms of l , that gives the maximum volume of the open box. (You need not show that your answer gives a maximum.) [3]
- 4 A curve C has equation $3xy+1=(x+y)^2$.
- (a) Show that $\frac{dy}{dx} = \frac{2x-y}{x-2y}$. [2]
- (b) Hence find the exact x -coordinate of the points on C whose tangent is parallel to the x -axis. [3]
- (c) By finding the gradient of the tangent to C at $(0,1)$, hence find the acute angle between the tangent to C at $(0,1)$ and the x -axis. [2]

- 5 The function f is defined by $f : x \mapsto \frac{kx+2}{2-x}$, for $x \in \mathbb{R}, x \neq 2$.

(a) If the point with coordinates $(-7, 4)$ lies on the curve $y = f^{-1}(x)$, show that $k = 3$. [1]

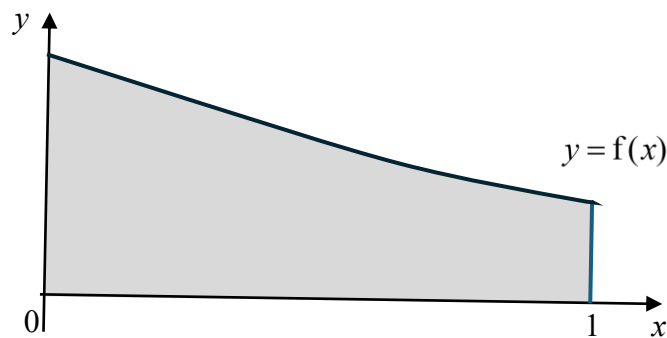
Use $k = 3$ for the rest of this question.

The function g is defined by $g : x \mapsto f(|x|)$, for $x \in \mathbb{R}, x \leq 0$ and $x \neq -2$.

(b) Sketch the graph of $y = g(x)$ stating the axial intercept and the equations of asymptotes. [3]

(c) Find $g^{-1}(x)$, stating its domain. [4]

- 6 The diagram shows a sketch of the curve $y = f(x)$. The shaded region under the curve between $x = 0$ and $x = 1$, as shown in the diagram, is A . This region is split into n vertical strips of equal width, h .



(a) State the value of h . [1]

(b) Hence show, using a sketch, that $\sum_{r=1}^n hf(rh)$ is less than the area of A . [2]

You are now given that $f(x) = \frac{e^{-2x}}{1 + e^{-2x}}$.

(c) Hence find the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n hf(rh)$. [3]

7 It is given that $f: x \mapsto (x-4)^2 - |x-4|$, $0 \leq x \leq \lambda$, where λ is a constant.

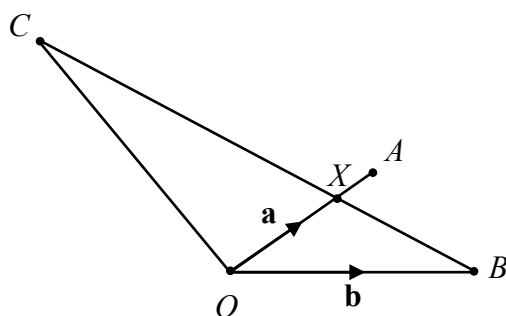
- (a) Find the greatest value of λ for which the function f^{-1} exist. [1]

The function $g(x)$ is defined by

$$g(x) = \begin{cases} x+2 & \text{for } -2 \leq x \leq 0, \\ -x-2 & \text{for } 0 < x \leq 13. \end{cases}$$

- (b) Sketch the graph of $y = g(x)$, stating the coordinates of the endpoints. [2]
- (c) Considering the result in part (a), show that the composite function gf exists. [1]
- (d) Find the range of function gf . [1]
- (e) Without finding gf and g^{-1} , find the exact value of x for which $gf(x) = -3$. [3]

8



With reference to the origin O , the points A , B and C are such that vectors $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = 2\mathbf{a} - 2\mathbf{b}$.

- (a) Given that $|\mathbf{a}| = 2$, $|\mathbf{b}| = 3$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{6}$, find the area of triangle OBC . [3]

The lines OA and BC intersect at point X .

- (b) Show that $\overrightarrow{OX} = \frac{2}{3}\mathbf{a}$. [3]

The plane p has equation $\mathbf{r} \cdot \mathbf{n} = d$ and contains the point A , where $\mathbf{n}$ is a unit vector.

- (c) Find the shortest distance from point X to plane p , in the form of $\frac{1}{k}|d|$, where k is a positive integer to be determined. [3]

- 9 The energy added to a humanoid robot's backup cell increases by a constant number of units each hour of its operation. The energy added during the 8th hour is 26 units, and the total cumulative energy stored over the first 12 hours is 258 units.

(a) Find the energy added during the 1st hour and determine the minimum number of full hours the robot needs to operate before its cumulative energy stored exceeds 1000 units.

[5]

The robot requires compute tokens to run its onboard AI models. The robot's server wallet starts off with 10 000 tokens. At the beginning of each month starting from the first month, a fixed number of K tokens is added to the wallet. At the end of each month, the total number of tokens in the wallet increases by 2% due to network yields. Immediately after, the robot consumes exactly 2500 tokens for a monthly software update.

Let u_n denote the number of tokens in the wallet at the end of n th month after the monthly software update.

(b) Write down the expressions for u_1 and u_2 in terms of K . [2]

(c) Hence show that $u_n = 1.02^n (10\,000) + \alpha K (1.02^n - 1) - 125\,000 (1.02^n - 1)$, where α is a constant to be determined. [3]

(d) Find the minimum integer value of K to ensure the robot's wallet never runs out of tokens in the long run. [2]

- 10 (a) **Do not use a calculator in answering this question.**

One of the roots of the equation $z^3 + az^2 + bz - 1 = 0$, where a and b are real numbers,

is $-\frac{1}{2} + \frac{i\sqrt{3}}{2}$.

(i) Express $z^3 + az^2 + bz - 1$ as the product of a linear factor and a quadratic factor with real coefficients and find the values of a and b . [4]

(ii) Hence state the other roots of the equation $z^3 + az^2 + bz - 1 = 0$. [1]

(iii) Consider $z = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$ which is a root of the equation $z^3 + az^2 + bz - 1 = 0$ and

using the results in part (i), show that $1 + z^5 + z^7 = 0$. [2]

10 (b) Do not use a calculator in answering this question.

It is given that $z_1 = 1 + i\sqrt{3}$ and $z_2 = \sqrt{3} - i$.

- (i) Sketch on an Argand diagram with origin O , showing the points A , B and C representing z_1 , z_2 and $z_1 + z_2$ respectively. [2]
- (ii) Show that $OACB$ is a square. [2]
- (iii) Hence by considering $\arg(z_1)$ and $\arg(z_1 + z_2)$, show that $\tan \frac{\pi}{12} = 2 - \sqrt{3}$. [3]

- 11** Two scientists are interested in studying the growth of a deer population after its introduction into a national park. They model the number of deer, P , at time t years after the introduction of an initial population of 50 deer. Both P and t are assumed to be continuous variables. Scientist A uses the Gompertz growth model.

$$\frac{dP}{dt} = P(1 - 0.2 \ln P).$$

- (a) Using the substitution $y = \ln P$, show that $\frac{dy}{dt} = (1 - 0.2y)$. [1]
- (b) Solve the differential equation for y in terms of t and show that y can be expressed as $y = 5(1 - Ke^{-0.2t})$, where K is a constant to be determined to 4 decimal places. [4]
- (c) Hence find P in terms of t . [1]
- (d) Explain what will happen to the deer population in the long term. [1]
- (e) Sketch the graph of P against t . [2]

Scientist B uses a model that assumes that, at any time t , the growth rate of the deer population is $\frac{dP}{dt} = 0.2P$.

- (f) Find P in terms of t . [2]
- (g) Comment on whether the model used by Scientist A or Scientist B is more appropriate. [1]

- 12** The curve C is defined by the parametric equations

$$x = a \operatorname{cosec} \theta, \quad y = 2a \cot \theta, \quad \text{for } \frac{\pi}{2} < \theta < \pi, \quad \text{where } a \text{ is a positive constant.}$$

The line D has equation $y = -x$ and intersects curve C once.

- (a) Find, in terms of a , the exact x -coordinate of the point of intersection. [2]
- (b) Show that gradient of C , $\frac{dy}{dx} = \frac{2 \operatorname{cosec} \theta}{\cot \theta}$. [2]
- (c) The normal to C at the point $P(a \operatorname{cosec} p, 2a \cot p)$ cuts the x -axis at point A and the y -axis at point B . The point M is the midpoint of AB . Find a cartesian equation of the curve traced by M as p varies. [5]
- (d) Given that the p is increasing at the rate of 0.1 unit per second, find the rate at which the area of triangle AOB is changing when $p = 2$ and $a = 1$. [3]

BLANK PAGE