Chapter 1: Measurement

1. S.I. Units

- The seven **base units** are: metre (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol) and candela (cd).

- **Derived units** are defined in terms of base units. They are expressed as products or quotients of base units.

- An equation is said to be **homogeneous** or **dimensionally consistent** if every term on both sides of the equation has the same base units.

- A homogeneous equation may not be true or correct.

2. Errors and Uncertainties

Item	**Accuracy**	**Precision**
Instrument	Calibration of instrument	Smallest division of instrument
Measurements	Closeness of average to true value	Closeness of measurements to one another

- ***Systematic errors*** result in all readings or measurements being always smaller or always larger than the true value by a fixed amount.

- ***Random errors*** result in readings or measurements being scattered about the true value.

- ***Accuracy*** is the degree of closeness of the average value of the measurements to the true value. It is affected by systematic error.

- ***Precision*** is the degree of agreement between repeated measurements of the same quantity. It is affected by random error.

- Calculations:

  If $Q = aX \pm bY$, then $\Delta Q = |a|\,\Delta X + |b|\,\Delta Y$.

  If $Q = aX^m \times Y^n$ or $Q = a\dfrac{X^m}{Y^n}$, then $\dfrac{\Delta Q}{Q} = |m|\dfrac{\Delta X}{X} + |n|\dfrac{\Delta Y}{Y}$.

  Percentage uncertainty of Q = $\dfrac{\Delta Q}{Q} \times 100\%$.

  - Absolute uncertainty ΔQ should always be expressed to 1 s.f.
  - Quantity Q should be expressed up to the <u>same decimal place</u> as ΔQ.
    Eg: $g \pm \Delta g = (9.81 \pm 0.02)\ \text{m s}^{-1}$

  - Fractional and percentage uncertainty is expressed to 2 s.f.
    Eg: $\dfrac{\Delta g}{g} = \dfrac{0.02}{9.81} = 0.0020\ (2\ \text{s.f.}) = 0.20\ \%\ (2\ \text{s.f.})$

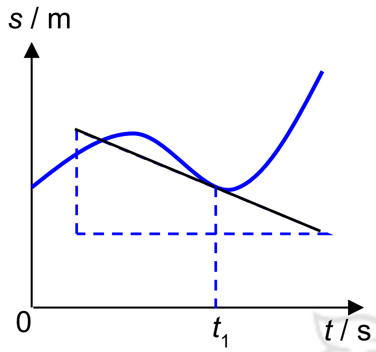
3. Scalars & Vectors

- A **scalar** quantity has a magnitude only.
- A **vector** quantity has both a magnitude and a direction.

Chapter 2: Kinematics

1. Interpretation of Graphs

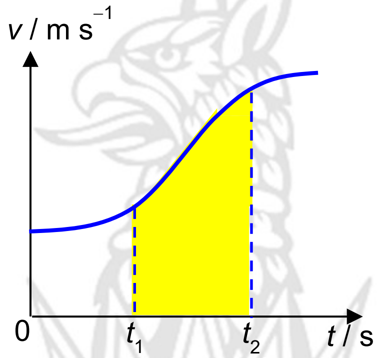
<u>$s - t$ graph</u>



Gradient $= \dfrac{ds}{dt} = v$

$=$ instantaneous velocity

<u>$v - t$ graph</u>



Gradient $= \dfrac{dv}{dt} = a$

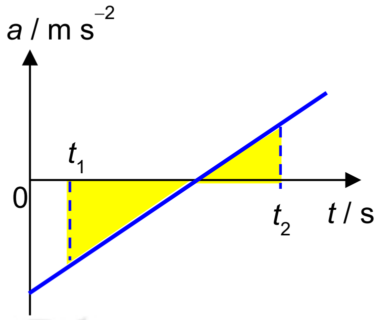
$=$ instantaneous acceleration

Area under the graph

$= \int_{t_1}^{t_2} v \; dt = \Delta s$

$=$ change in displacement

<u>$a - t$ graph</u>

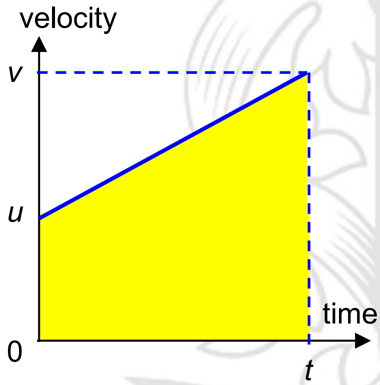


Area under the graph

$= \int_{t_1}^{t_2} a \; dt = \Delta v$

$=$ change in velocity

2. Rectilinear Motion (1-D)



$$\text{gradient} = a = \frac{(v-u)}{t}$$

$$\therefore v = u + at \qquad \cdots(1)$$

$$\text{area} = s = \frac{1}{2}(u+v)t \qquad \cdots(2)$$

substituting (1) into (2),

$$s = ut + \frac{1}{2}at^2 \qquad \cdots(3)$$

$$\text{and } v^2 = u^2 + 2as \qquad \cdots(4)$$

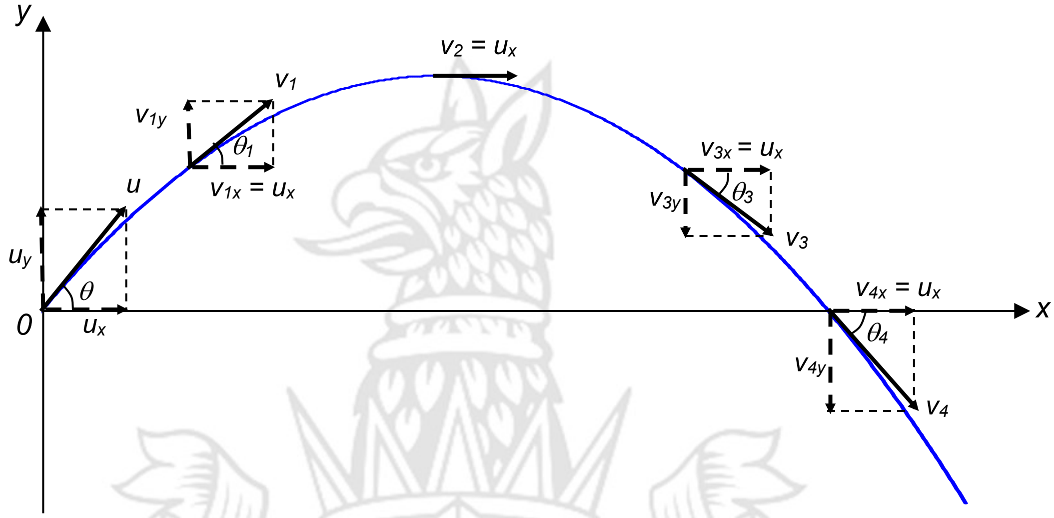
Equations (1) to (4) apply only to motion in a straight line at constant acceleration.

3. Solving kinematics problems

- Draw diagram(s) and transfer data from the question onto the diagram(s).
- Indicate positive displacements along the vertical and horizontal directions.
- Ensure that sign conventions are applied consistently to all quantities.
- Apply the above equations independently along the vertical and horizontal directions.

4. Projectile Motion (2-D)

In the absence of air resistance, the path/trajectory of a projectile is a parabola.
The magnitude of acceleration due to free fall (acting vertically downwards) is g.



		Vertically	Horizontal
Initial velocity	u	$u_y = u \sin \theta$	$u_x = u \cos \theta$
Acceleration	a	$a_y = g$ (downwards)	$a_x = 0$
Final velocity	v	$v_y = u_y + a_y t$ $v_y^2 = u_y^2 + 2a_y s_y$	$v_x = u_x + a_x t$ $\therefore v_x = u_x$
Displacement	s	$s_y = u_y t + \frac{1}{2} a_y t^2$ $s_y = \frac{1}{2}(u_y + v_y)t$ Maximum height (max s_y) is reached when $v_y = 0$	$s_x = u_x t$
Time	t	t	t

Chapter 3: Dynamics

1. Important Laws and Definitions

i. **Newton's First Law** states that every object continues in its state of rest or uniform motion in a straight line unless it is acted upon by a resultant external force.

ii. **Newton's Second Law** states that the rate of change of momentum of a body is proportional to the resultant force acting on it and the change occurs in the direction of the force. That is,

$$F_{net} = \frac{dp}{dt}$$

iii. **Newton's Third Law** states that if a body A exerts a force on body B, then body B exerts an equal but opposite force on body A.

iv. Since the internal forces of a system always add up to zero vectorally, F_{net} = 0 as long as there is no resultant external force acting on the system. When this happens, $\frac{dp_{system}}{dt} = F_{net} = 0$ which means that the total momentum of the system is constant.

The **Principle of Conservation of Momentum** states that when a system of bodies interact, the total momentum of the system remains constant, provided no net external force acts on it.

v. **Impulse** is defined as the product of a force F acting on an object and the time Δt for which the force acts.

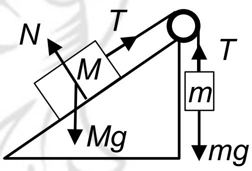
$$\text{Impulse} = F\,\Delta t$$

2. Applications of Newton's Second Law

i. A System of Objects

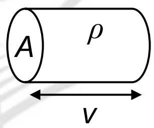
One example is shown in the figure on the right.

1. Identify forces acting on each object.
2. Write down the "$F = ma$" equation for each object.
3. Solve the simultaneous equations to get the answer.



ii. Force due to a Fluid Jet

Consider a continuous stream of fluid of density ρ moving with velocity v. After striking a surface, its velocity becomes v'.



Force experienced by the fluid

$= (\text{mass of fluid flow per unit time}) \times (\text{its change in velocity})$

$= (A\rho v)(v' - v)$

In most questions, v' is either 0 or $-v$. So $F = \rho A v^2$ or $F = 2\rho A v^2$

3. Momentum and its Conservation

i. Momentum is defined as the product of the mass of an object and its velocity.

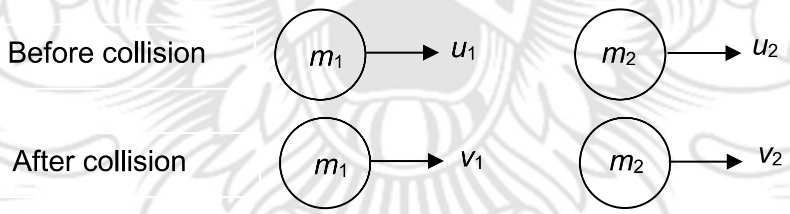
ii. Momentum is a vector quantity and its unit is kg m s^{-1} or N s.

iii. The impulse of a force = $\int F \, dt$ = area under the F−t graph. The impulse exerted on a body is equal to the change in momentum of the body.

iv. In collision problems in which external forces are absent or negligible, the total momentum is always conserved. On the other hand, the kinetic energy of the system is conserved only of the collision is perfectly elastic. We may classify the types of collisions as follows:

Type of collision	**Momentum conserved?**	**K.E. conserved?**
Elastic	✓	✓
Inelastic	✓	×
Completely inelastic (bodies coalesced)	✓	×

v. A head-on collision between two objects is shown below:



Since momentum is always conserved in such a collision, we can always write down:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

It is important to remember that the equation is written down with the figure above in mind, i.e., all the velocities are rightward. If, for example, the question specifies that m_2 is moving at 5 m s^{-1} to the left, then you should substitute u_2 by –5.

If the collision is also perfectly elastic, then one can write down an additional equation

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

This new equation is slightly difficult to solve due to the squared terms. However, we can combine the two equations to obtain a third, linear equation:

$$u_1 - u_2 = v_2 - v_1$$

relative speed of approach = relative speed of separation

Chapter 4: Forces

1. Important Formulae and Definitions

i. Hooke's Law states that the extension x of a spring (or wire) is proportional to the applied force F, if the limit of proportionality is not exceeded.

$$F = kx$$

where k is the force constant.

ii. The weight of a body may be taken as acting at a single point known as its centre of gravity.

iii. Pressure is the normal force acting per unit area, where the force is acting at right angles to the area.

$$p = \frac{F}{A}$$

A scalar quantity.
S.I. units: N m^{-2} or Pascal (Pa)

At a given depth h, pressure *due to the fluid column above* is given by $p = \rho gh$, where ρ is the density of fluid. Note that the total pressure at depth h also includes the atmospheric pressure.

iv. Upthrust is the vertical upward force exerted on a body by a fluid when it is fully or partially submerged in the fluid due to the difference in fluid pressure.

Note: "Due to the difference in fluid pressure" part is important. A fluid exerts other kinds of forces on an object submerged, e.g. the drag force, which is not due to the fluid pressure but rather due to the motion of the object relative to the fluid.

v. Upthrust is equal to the weight of the fluid that is displaced by the body.

$$U = \rho gV$$

where V = volume submerged in the fluid,
ρ = density of fluid,
g = acceleration due to gravity.

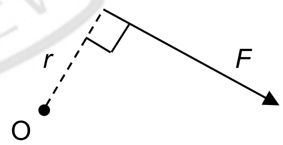
vi. Principle of floatation: For an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object.

2. Moments and Torque

i. Turning effect of a force

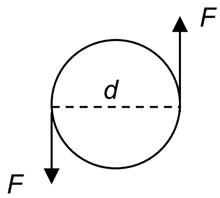
The **moment** (or **torque**) of a force about a point is defined as the product of the force and the perpendicular distance from the point to the line of action of the force.

Moment τ of force F about O, $\tau = F \times r$



A vector quantity (clockwise or anti-clockwise moment).
SI units: N m

ii. Torque of a couple



A couple consists of a pair of equal and opposite parallel forces whose lines of action do not coincide.

Torque of a couple
= magnitude of one force × the perpendicular distance between the two forces.
Torque of the couple shown, $\tau = d \times F$

iii. Principle of moments

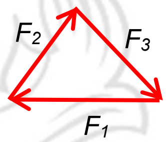
For a body in rotational equilibrium, the sum of all the clockwise moments about any point must equal the sum of all the anticlockwise moments about the same point.

3. Forces in Equilibrium

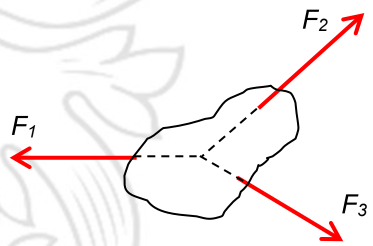
For a body in static or dynamic (constant velocity) equilibrium, the necessary conditions are:

i. The resultant force on the body is zero, $\Sigma F = 0$
ii. Resultant torque on the body about any point is zero, $\Sigma \tau = 0$

For 3 non-parallel forces acting on an object in equilibrium:



- The resultant force on the object is zero, $\Sigma F = 0$, implies that the forces, when taken in order, form a closed triangle.



- The resultant torque on the object about any point is zero, $\Sigma \tau = 0$, implies that their lines of action must all pass through a single common point (if they are not parallel).

4. Solving Problems Involving System in Equilibrium

i. Select a body or system of bodies for analysis.

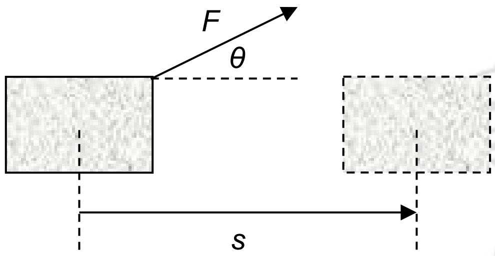
ii. Draw a free-body diagram showing all the forces acting on the body or system. Take care to draw the arrows for the forces starting from their correct points of action, as these will affect the moment calculation.

iii. Select a positive direction each for forces and moments. A wise choice of the point of reference (a.k.a. the pivot) simplifies the calculation for moments tremendously.

iv. Evaluate the type of problem to be solved and form the equations using:
$\Sigma F_x = 0$ and $\Sigma F_y = 0$ and $\Sigma \tau = 0$

Chapter 5: Work, Energy & Power

1. Work done (W) by a constant force is the product of the force and the displacement in the direction of the force.



$$W = Fs\cos\theta$$

Notes:
1. Work done becomes negative if $\theta > 90°$.
2. The above formula applies to a constant force. For a variable force, W.D. is equal to the area under the **force-displacement graph**.

2. The **external** force needed to stretch a spring by an extension x (from its unstretched length) is proportional to the extension x.

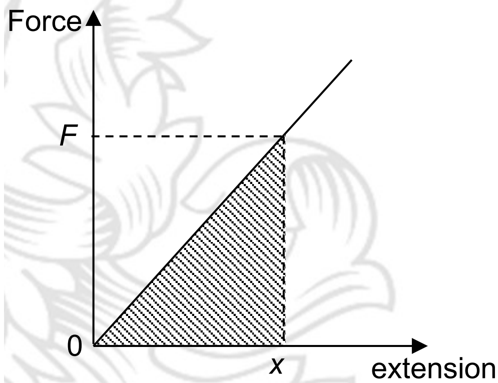
$$F = kx$$

where k is the force constant of the spring.

The work done by such an external force is stored in the spring as its elastic potential energy (U_E). It is given by the area under the **force-extension** graph (see right):

$$U_E = W = \frac{1}{2}kx^2$$

The same formula applies to compression.



3. Work done by a gas expanding against a **constant external** pressure p:

$$W = p\Delta V$$

where ΔV is the increase in volume of the gas. Note that W.D. **by** the gas (on the environment) is the negative of the W.D. **on** the gas by the environment. For example, if the gas contracts, ΔV is negative, W.D. by the gas is negative, and W.D. on the gas is positive.

4. Energy is the capacity to do work. It exists in various forms, including kinetic energy, (electric, gravitational, elastic) potential energy, nuclear energy, chemical potential energy and internal energy. Energy cannot be created or destroyed. It can only be converted from one form to another. This is known as the law of conservation of energy.

5. The sum of kinetic energy (E_k) and the (electric, gravitational, elastic) potential energies (E_p) together is called mechanical energy of a system. In a non-isolated system where there is work done by an external force, W_F, the energy equation is as follows:

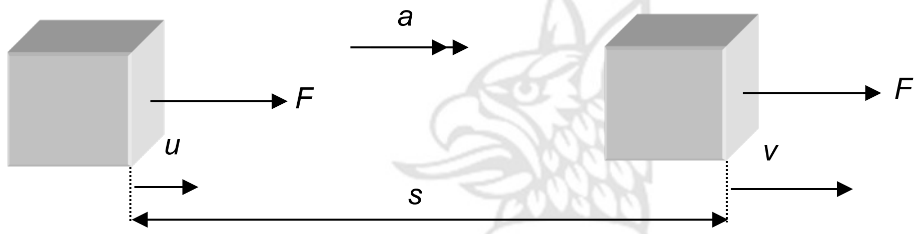
$$(E_p + E_k)_{\text{initial}} + W_F = (E_p + E_k)_{\text{final}}$$

One may understand this equation by considering W_F as the external energy input due to the external force, which becomes part of the final energy of the system. A system with no net external force acting on it is known as a closed system. For such a system, $W_F = 0$, the energy equation becomes

$$(E_p + E_k)_{\text{initial}} = (E_p + E_k)_{\text{final}}$$

6. Derivation of $E_k = \frac{1}{2}mv^2$

Consider a body of mass *m* that is moving with an initial velocity *u*. It is acted on by a **constant resultant force** *F* which is parallel to *u*. The body accelerates with a uniform acceleration *a* to a final velocity *v* over a displacement *s*.



Since the displacement of the body is in the same direction as the applied force,
$W = Fs = (ma)s$ ..... (1)

From $v^2 = u^2 + 2as$ (applicable since *a* is constant),

$s = \frac{v^2 - u^2}{2a}$ ..... (2)

Substituting equation (2) into (1),

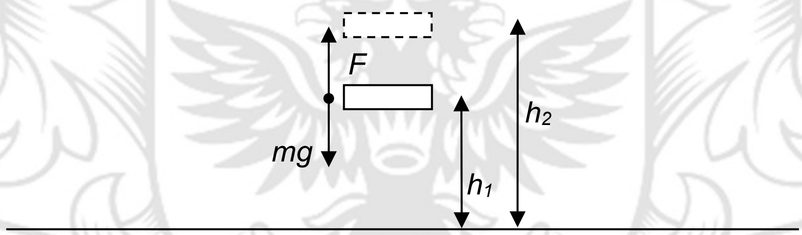
$W = (ma)\left(\frac{v^2 - u^2}{2a}\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ ..... (3)

Thus, the work done *W* is equal to the *change* in the quantity $\frac{1}{2}m \times (\text{velocity})^2$, which is termed the kinetic energy, E_k:

$$E_k = \frac{1}{2}mv^2$$

7. Derivation of gravitational $E_p = mgh$

Consider an object being raised upwards, at a **constant velocity**, from a height of h_1 to a height of h_2 near the Earth's surface. Since the velocity at which the object moves is constant, the **external** force, *F*, required to lift the object must be equal to its weight, *mg*.



Work done by *F* in displacing the center of mass of object vertically upward,

$$W = Fs = mg(h_2 - h_1) = mgh \quad \text{where } h = h_2 - h_1$$

In raising the object, the work done by the **external** force *F* (given by the expression above) is the gain in the gravitational potential energy of the object.

8. For a **field of force** (such as a gravitational field or an electric field), the relationship between the force F and the potential energy U for one-dimensional motion is given by

$$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$$

From the relationship, we can infer:

1. The magnitude of the force at point x is equal to the gradient of the potential energy curve at x;
2. The direction of the force is the direction of decreasing potential energy.

9. Efficiency gives a measure of how much of the total energy may be considered useful and is not "lost". It is given by the expression:

$$\eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

Which form of energy is "useful" depends on the scenario. The difference between total energy input and useful energy output is the energy loss.

10. Power is defined as the rate of work done or energy conversion with respect to time:

$$P = \frac{\mathrm{d}W}{\mathrm{d}t}$$

The above formula yields the instantaneous power. What is frequently useful is the average power, given by

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t}$$

11. If a **constant force** F is applied and does work by moving its point of application a displacement s in time t, the power supplied is given by the following derivation:

$P = \dfrac{\mathrm{d}W}{\mathrm{d}t} = \dfrac{\mathrm{d}(Fs\cos\theta)}{\mathrm{d}t} = F\dfrac{\mathrm{d}s}{\mathrm{d}t}\cos\theta = Fv\cos\theta$, (Note: θ is the angle between F and v or s)

where $v = \dfrac{\mathrm{d}s}{\mathrm{d}t}$ is the velocity of the point of application.

$$P = Fv\cos\theta$$

12. Problems related to mass flow rate $\dfrac{dm}{dt}$ (such as energy conversion in a wind turbine):

1. Calculate the amount of mass passing through per unit time (E.g.,1 second).
2. Hence, calculate the amount of kinetic energy, passing through per unit time.
3. Write down the corresponding Conservation of Energy equations for the unit time.

Chapter 6: Circular Motion

1. Angular displacement: angle an object makes with respect to a reference line.

2. One radian : angle subtended by an arc length equal to the radius of the arc.

$\theta = \dfrac{s}{r}$ where s : arc length; r : the radius of the circle.

3. π radian $\equiv 180°$

4. Angular velocity ω, of an object in circular motion is defined as the rate of change of its angular displacement with respect to time.

$$\omega = \frac{d\theta}{dt} \qquad \text{(units: rad s}^{-1}\text{)}$$

5. Linear velocity $v = r\omega$ (units: m s^{-1})

6. Period $T = \dfrac{2\pi}{\omega} = 2\pi f$ (unit: s)

7. Frequency $f = \dfrac{1}{T}$ (unit: Hz)

8. Centripetal acceleration $a = v\omega = r\omega^2 = \dfrac{v^2}{r}$ (units: m s^{-2})

9. Centripetal force $F = ma = mv\omega = mr\omega^2 = \dfrac{mv^2}{r}$ (units: N)

10. An object in uniform circular motion is accelerating because the direction of its velocity is changing continuously although there is no change in its speed.

Problem-Solving Strategy for **Uniform** Circular Motions:

1. Draw a free-body diagram of the body under consideration. Label **all** forces acting on the body.
2. Identify the centre of the circular motion.
3. Resolve the forces along two perpendicular axes according to each scenario.
4. Find the expression for the resultant force towards the centre of the circular path.
5. Equate it to ma_c, where m is the mass of the body and $a_c = \dfrac{v^2}{r} = r\omega^2$.

Problem-Solving Strategy for **Non-Uniform** Circular Motions:

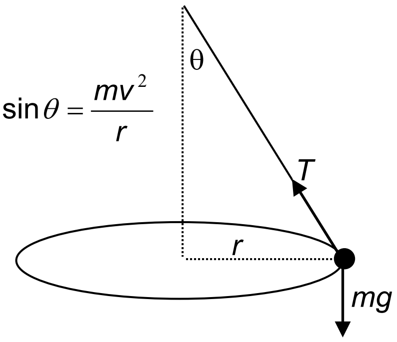
1. Draw a free-body diagram of the body under consideration. Label all forces acting on the body. Usually there are some special conditions such as “just in contact” which implies $N = 0$ or “string is just taut” which implies, $T = 0$.
2. Identify the centre of the circular motion.
3. Resolve the forces along two perpendicular axes according to each scenario.
4. Find the expression for the resultant force towards the centre of the circular path.
5. Equate it to mv^2/r or $m\omega^2 r$.

   (Take note: for Non-Uniform circular motion, the speed v or angular velocity ω is NOT constant through the motion. Hence, when equating to mv^2/r or $m\omega^2 r$, we are finding v or ω only at THAT instant).
6. Using Conservation of Energy, form another equation for the motion.
7. Solve them simultaneously to find the unknowns.

The centripetal force for the following **Uniform** Circular Motions are:

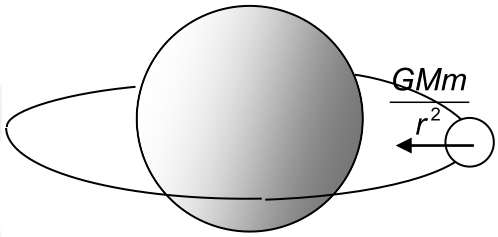
(i) Conical Pendulum:

$$T\sin\theta = \frac{mv^2}{r}$$



Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, the centripetal force is provided by the <u>horizontal component of tension $T\sin\theta$.</u>

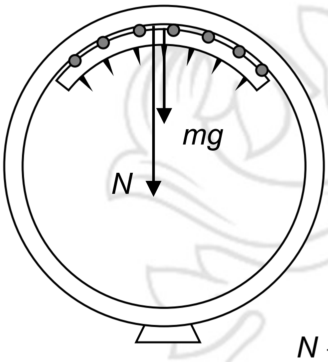
(ii) Celestial Objects:



$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, centripetal force is provided by <u>gravitational force</u>.

(iii) **Roller coaster (top):**



$$N + mg = \frac{mv^2}{r}$$

Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, the centripetal force is the sum of the normal contact force and weight.

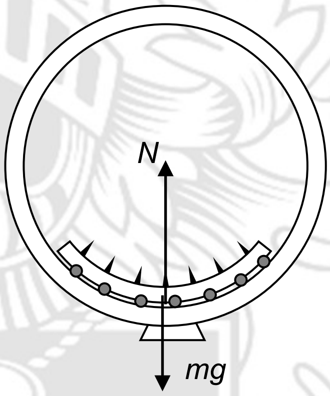
*** Minimum speed to stay in contact at the top**

For the roller coaster to just stay in contact, $N = 0$

$$\text{So } N + mg = \frac{mv^2}{r} \quad \Rightarrow mg = \frac{mv_{\min}^2}{r}$$

$$\therefore\ v_{\min} = \sqrt{rg}$$

(iv) **Roller coaster (bottom):**



$$N - mg = \frac{mv^2}{r}$$

Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, the centripetal force is the <u>normal contact force minus weight.</u>

*** Minimum speed to enter to keep in contact at the top**

By Conservation of Energy,

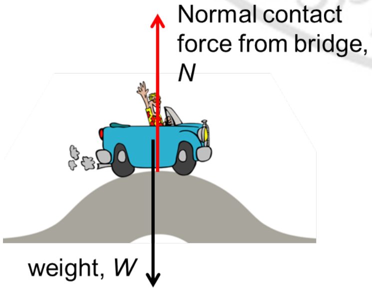
$$E_{k(bottom)} = E_{k(top)} + GPE_{(top)}$$

$$\frac{1}{2}mv_{bottom}^2 = \frac{1}{2}mv_{\min}^2 + mg(2r)$$

$$v_{bottom}^2 = v_{\min}^2 + 2g(2r)$$

$$v_{bottom} = \sqrt{rg + 4rg} = \sqrt{5rg}$$

(v) **A car going over an arch:**



$$mg - N = \frac{mv^2}{r}$$

Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, the centripetal force is the <u>weight minus normal contact force.</u>

Chapter 7: Gravitational Field

1. A **gravitational field** is a region of space in which a mass placed in that region experiences a gravitational force.

2. Definitions

Newton's law of gravitation states that two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

The **gravitational potential energy** of a mass at a point in a gravitational field is defined as the work done by an external force in bringing the mass from infinity to that point.

$$F_G = -\frac{dU}{dr}$$

Vectors:

$$F_G = -\frac{GMm}{r^2} \xrightarrow{\div m} g = -\frac{GM}{r^2}$$

Scalars:

$$U = -\frac{GMm}{r} \xrightarrow{\div m} \phi = -\frac{GM}{r}$$

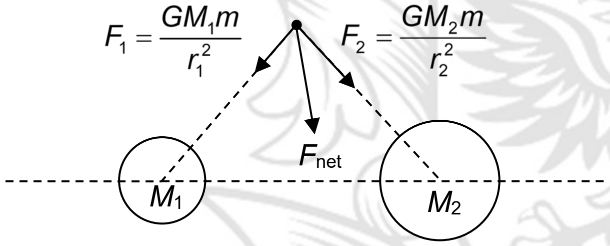
$$g = -\frac{d\phi}{dr}$$

The **gravitational field strength** at a point in space is defined as the gravitational force experienced per unit mass at that point

The **gravitational potential** at a point in a gravitational field is defined as the work done per unit mass by an external force in bringing a small test mass from infinity to that point.

3. Force & Field Strength vs **Potential Energy & Potential**
Consider a point mass of mass m placed near 2 large uniform masses of mass M_1 and M_2 respectively, where $M_1 < M_2$. The distance between the point mass and each of the masses' centre of mass is r_1 and r_2 respectively.

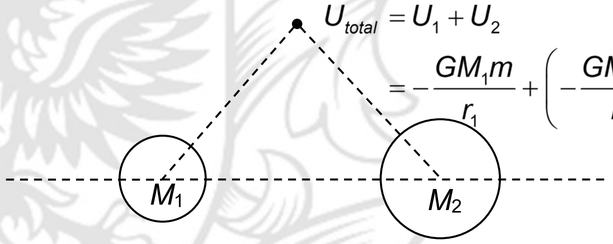
Resultant Force & Field Strength



$$F_1 = \frac{GM_1m}{r_1^2} \qquad F_2 = \frac{GM_2m}{r_2^2}$$

F_{net} is the vector sum of F_1 and F_2

Total Potential Energy & Potential



$$U_{total} = U_1 + U_2 = -\frac{GM_1m}{r_1} + \left(-\frac{GM_2m}{r_2}\right)$$

U_{total} is the scalar addition of U_1 and U_2

4. For motion of a celestial object in orbit, **gravitational force provides centripetal force**.

$$G\frac{Mm}{r^2} = \frac{mv^2}{r} \qquad G\frac{Mm}{r^2} = mr\omega^2$$

$$v = \sqrt{\frac{GM}{r}} \qquad T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

- This is the **orbital speed** of the object
- This is **Keplar's 3rd Law**

Note: for the object to be in orbit, it is bound within the gravitational field and cannot escape.

5. To obtain the minimum speed required for an object to **escape its orbit**,
Principle of Conservation of Energy
Gain in GPE = Loss in KE

$$0-\left(-\frac{GMm}{r}\right)=\frac{1}{2}mv_{esc}^{2}-0 \quad \therefore v_{esc}=\sqrt{\frac{2GM}{R}}$$

6. Energy of an orbiting satellite:

potential energy, $E_\text{p}=-\dfrac{GMm}{r}$

kinetic energy, $E_\text{k}=\dfrac{1}{2}mv_\text{o}^{2}=\dfrac{1}{2}\dfrac{GMm}{r}$

total energy, $E_T=E_P+E_K=-\dfrac{GMm}{r}+\dfrac{1}{2}\dfrac{GMm}{r}=-\dfrac{1}{2}\dfrac{GMm}{r}$

Objects that are bounded (cannot escape the field) have negative total energy.
Objects with $E_T \geq 0$ will be able to reach infinity and escape.

7. A **geostationary satellite** is one that is always above the same point on Earth's surface as it orbits the Earth and satisfies the following conditions:

- Its orbital period is the same as that of the Earth about its axis of rotation (24 hours).
- Its direction of rotation is the same as that of the Earth about its axis of rotation (from west to east).
- Its plane of orbit lies in the same plane as the equator (i.e. the satellite is above the equator).

Chapter 8: Temperature and Ideal Gases

1. When an object gains heat from its surrounding objects at the same rate as it loses heat to them, then there will be no net flow of heat between the object and its surrounding objects. They are all now in a state called **thermal equilibrium**.

2. An **ideal gas** is a gas which obeys the equation of state $pV = nRT$ at all pressures, volumes and temperatures. ($pV = nRT = NKT$)

3. A temperature scale is a system of measuring temperature. Most of them rely on measuring the changes in some physical property of a substance that varies with temperature. This is known as **thermometric property**.
The thermometric property of a gas thermometer is the pressure of a fixed mass of gas at constant volume. Experiments using gas thermometers show that the thermometer readings are nearly independent of the type of gas particularly when pressure is low. This is because the lower the pressure of a real gas, the more linear is the variation of pressure or volume with temperature, as the behaviour of real gases approaches that of an ideal gas.

4. Absolute zero on the thermodynamic scale is the temperature at which the molecules of a substance have the lowest energy and hence, the substance has the minimum internal energy.

5. The **Thermodynamic Temperature Scale** is an absolute scale of temperature that does not depend on the thermometric property of any particular substance.
T / K = θ / °C + 273.15

6. The mole is defined as the amount of matter containing the number of particles (atoms, ions or molecules) equal to the Avogadro's constant (6.02×10^{23}).

7. Assumptions of the kinetic theory of gases:

1. The gas consists of a large number of molecules in random motion.
2. The volume of molecules is negligible compared with the volume occupied by the gas.
3. The molecules exert no intermolecular forces on one another except during collisions.
4. Collisions between molecules and the container are perfectly elastic.
5. The duration of an intermolecular collision is negligible compared with the duration between collisions.

8. Explanation and derivation of $pV = \frac{1}{3}Nm\langle c^2 \rangle$.

- Consider a cubic container of side d with N molecules of an ideal gas.
- Consider a molecule with mass m and velocity c_x in the x-direction.
- For each collision event between a molecule and the wall, the impulse of molecule is $\Delta P = -2mc_x$
- The time between two successive collisions by the molecule is $\Delta t = \frac{2d}{c_x}$
- The average force exerted on the molecule by the wall is given by Newton's Second Law, $F_{\text{on molecule}} = \frac{\Delta P}{\Delta t} = \frac{-2mc_x}{2d/c_x} = -\frac{mc_x{}^2}{d}$

- The average force exerted on the wall is given by Newton's Third Law $F_{\text{on wall}} = -F_{\text{on molecule}} = \dfrac{mc_x^2}{d}$
- The total pressure p on the wall is $P = \dfrac{F_{\text{tot}}}{d^2} = \dfrac{Nm}{d^3}\langle c_x^2\rangle = \dfrac{Nm}{V}\langle c_x^2\rangle$
- By considering all the N number of molecules, the total force acting on the wall, $F_{\text{tot}} = \dfrac{Nm}{d}\langle c_x^2\rangle$, where $\langle c_x^2\rangle = \dfrac{c_{x1}^2 + c_{x2}^2 + c_{x3}^2 + \ldots + c_{xN}^2}{N}$
- Applying the Pythagoras Theorem, $c^2 = c_x^2 + c_y^2 + c_z^2$
- Since N is large and the molecules are moving randomly, with no preference in any direction, $\langle c_x^2\rangle = \langle c_y^2\rangle = \langle c_z^2\rangle$ and $\langle c_x^2\rangle = \dfrac{1}{3}\langle c^2\rangle$
- Hence, $p = \dfrac{Nm}{V}\langle c_x^2\rangle = \dfrac{1}{3}\dfrac{Nm}{V}\langle c^2\rangle$ and $pV = \dfrac{1}{3}Nm\langle c^2\rangle$

9. Mean translation kinetic energy of a molecule of an ideal gas

$$\langle E_K\rangle = \frac{1}{2}m\langle c^2\rangle = \frac{1}{2}mc_{\text{rms}}^2 = \frac{3}{2}kT$$

Chapter 9: First Law of Thermodynamics

1. The numerical value of the specific heat capacity of a substance is the quantity of heat required to raise the temperature of unit mass of the substance by one degree.

$\boldsymbol{Q = C\Delta\theta}$, $\boldsymbol{Q = mc\Delta\theta}$ **(*C*: heat capacity, *c*: specific heat capacity)**

2. The numerical value of the specific latent heat of fusion (or vaporisation) is the quantity of heat required to convert unit mass of solid to liquid (or liquid to vapour) without any change of temperature. $\boldsymbol{Q = m L}$

3. The internal energy of a system is determined by the state of the system and it is the sum of the random distribution of kinetic and potential energies associated with the molecules of the system.

4. A rise in temperature of a body implies an increase in the average kinetic energy of the molecules, which in turn implies an increase in the internal energy of the body.

5. The First Law of Thermodynamics states that the <u>increase</u> in the internal energy of a system is equal to the sum of the heat <u>supplied to</u> the system and the work done <u>on</u> the system, and the internal energy of a system depends only on its state.

$\Delta U = Q + W$ (increase in internal energy = heat supplied + work done *on* gas)

Work done = $\int_{V_1}^{V_2} P\ dV$ (Area under *P*–*V* graph)

Work done *by* gas is positive for expansion, negative for compression

For an ideal gas, internal energy = total KE of molecules

$$= N \times \frac{1}{2} m \langle c^2 \rangle = N \times \frac{3}{2} kT = \frac{3}{2} NkT = = \frac{3}{2} nRT = \frac{3}{2} pV$$

Change in internal energy of ideal gas, $\Delta U = \frac{3}{2} Nk\Delta T = \frac{3}{2} nR\Delta T$

6. Special Processes:

Isobaric	Change in volume occurs at <u>constant pressure</u>
Isovolumetric	Change in pressure occurs <u>without a change in volume</u>
Isothermal	Change in pressure and volume occur <u>without a change in temperature</u>
Adiabatic	Change in pressure and volume occur with <u>no heat supplied to or lost</u> from the system
Cyclic	System goes through a series of processes and ends at its original state

Chapter 10: Oscillations

1. **Amplitude** x_0 is the magnitude of the maximum displacement of the particle from its equilibrium position.
   Period T is the time taken to complete one oscillation.
   Frequency f is the number of oscillations per unit time.
   Angular frequency ω is the rate of change of phase angle of an oscillation.

$$T = \frac{1}{f} \qquad\qquad \omega = \frac{2\pi}{T} = 2\pi f$$

2. **Simple harmonic motion** is defined as the motion of a particle about a fixed point such that its acceleration is proportional to its displacement from the fixed point and is always directed towards the point.

   (Note: the fixed point is the equilibrium point)

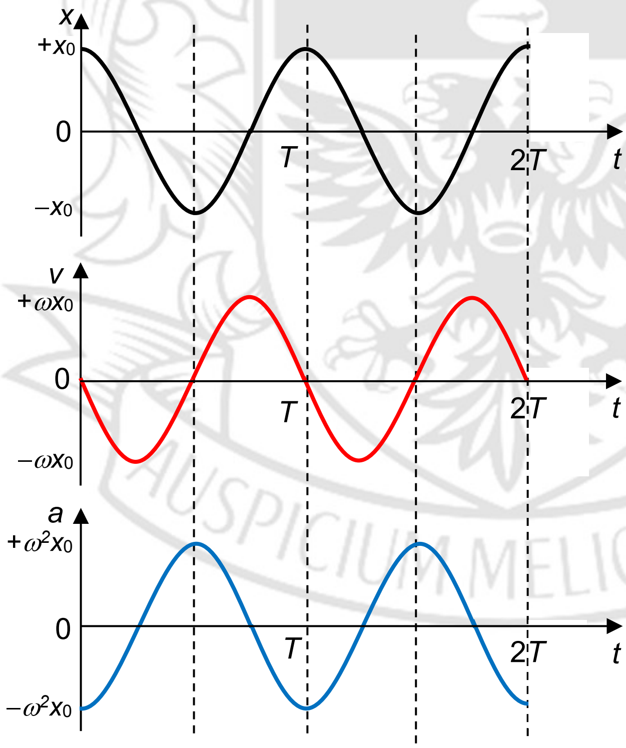
3. The defining equation for a particle undergoing simple harmonic motion is

$$a = -\omega^2 x$$

   This equation tells us that:
   - The magnitude of the acceleration a is directly proportional to the magnitude of the displacement x.
   - The direction of the acceleration is always in the opposite direction to that of the displacement.

4. <u>**Variation of displacement, velocity and acceleration with time:**</u>

   For a particle that begins its oscillation at the maximum displacement, i.e., $x = x_0$ when $t = 0$ s,



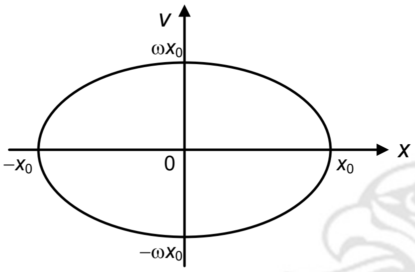
$$x = x_0 \cos \omega t$$

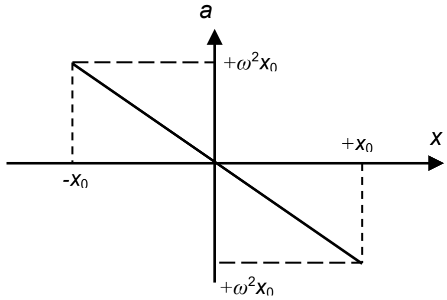
$$v = -\omega x_0 \sin \omega t$$

$$a = -\omega^2 x_0 \cos \omega t$$

5. Variation of velocity and acceleration with displacement:

For any oscillating particle,





$$v = \pm\omega\sqrt{x_0^{\,2} - x^2}$$

$$a = -\omega^2 x$$

6. Mass spring system

The motion of an oscillating mass-spring system (both horizontal and vertical) depends only on the mass m and spring constant k.

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

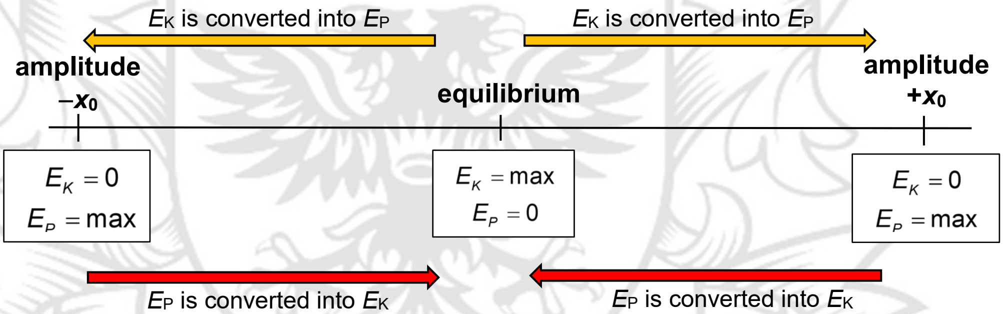
Simple Pendulum system

The motion of a simple pendulum system depends only on the length L and the gravitational acceleration g.

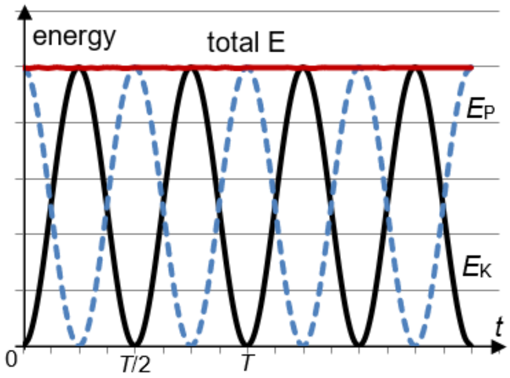
$$\omega = \sqrt{\frac{g}{L}} \qquad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

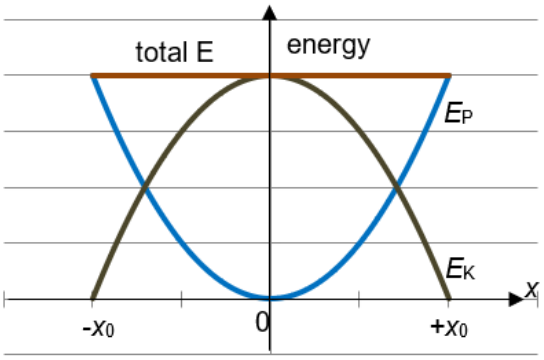
(Note: the natural frequency of the oscillating system can be found from the above equations.)

7. Variation of energy of an oscillating particle



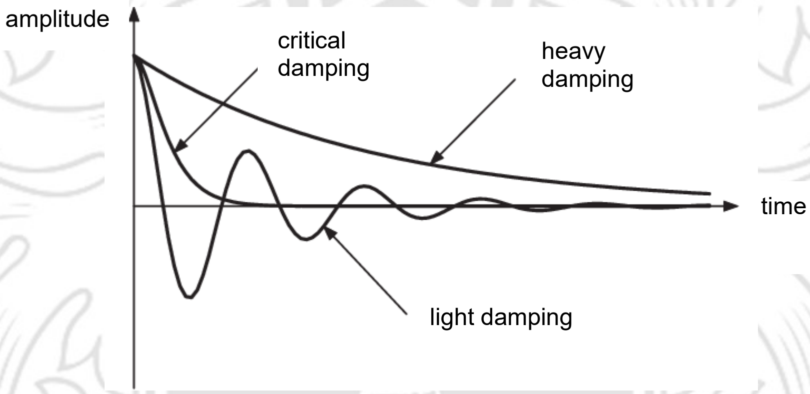
Variation of E with time for a particle that begins its oscillation at the maximum displacement, i.e. $x = x_0$ when $t = 0$ s,	Variation of E with displacement for any oscillating particle





$$\text{Total energy} = E_{K,\,\max} = E_{P,\,\max} = \frac{1}{2}m\omega^2 x_0^{\,2}$$

8. Damping is the process whereby energy is removed from the system.



Light Damping:

- Amplitude of the oscillation decays exponentially with time.
- The object undergoes oscillations until they stop.
- Frequency can be taken to be constant (or decreasing slightly).
- Example: a simple pendulum oscillating under air resistance.

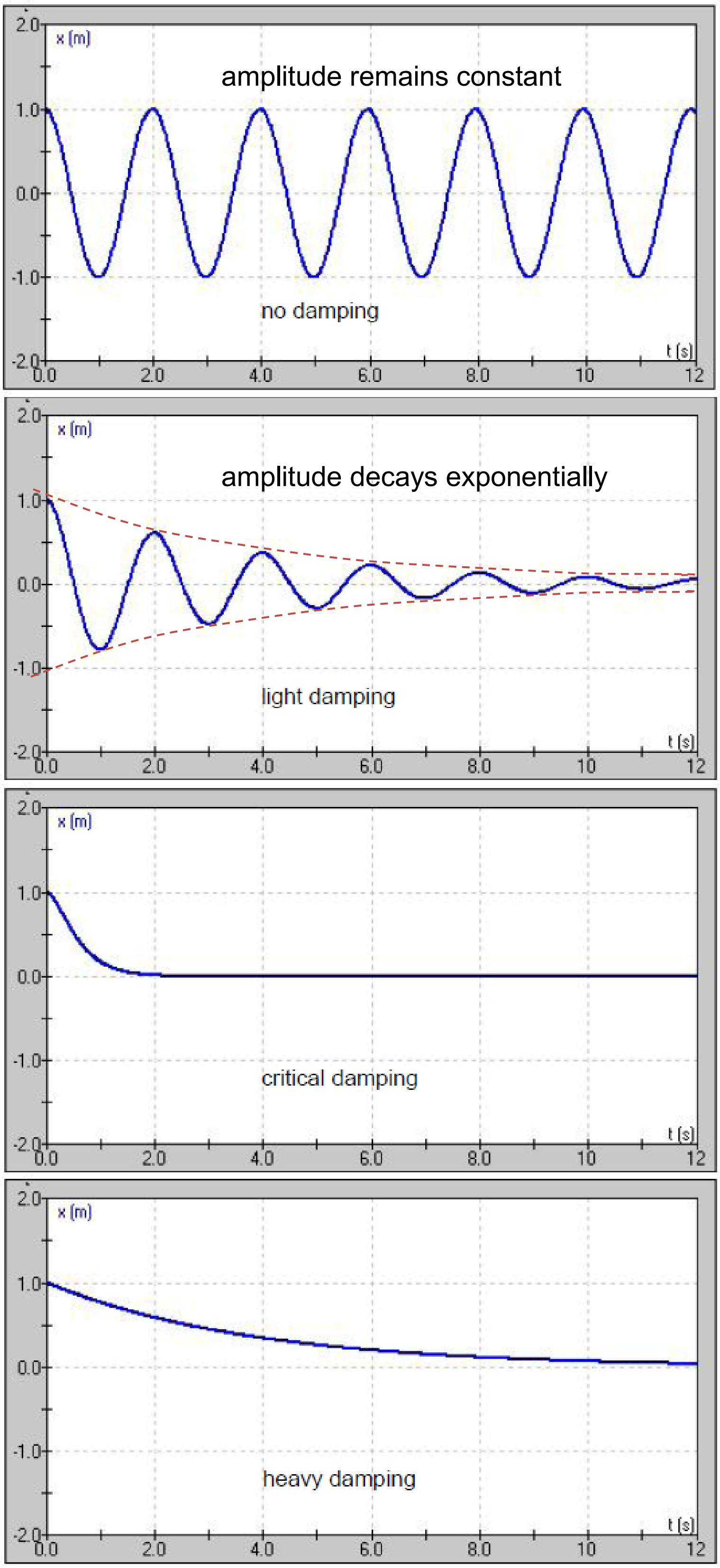
Critical Damping:

- The object does not oscillate.
- It takes the shortest time to return to its equilibrium position.
- Example: car suspension system.

Heavy Damping:

- The object does not oscillate.
- It returns to equilibrium very slowly.
- This occurs because the resistance to its motion is very large.

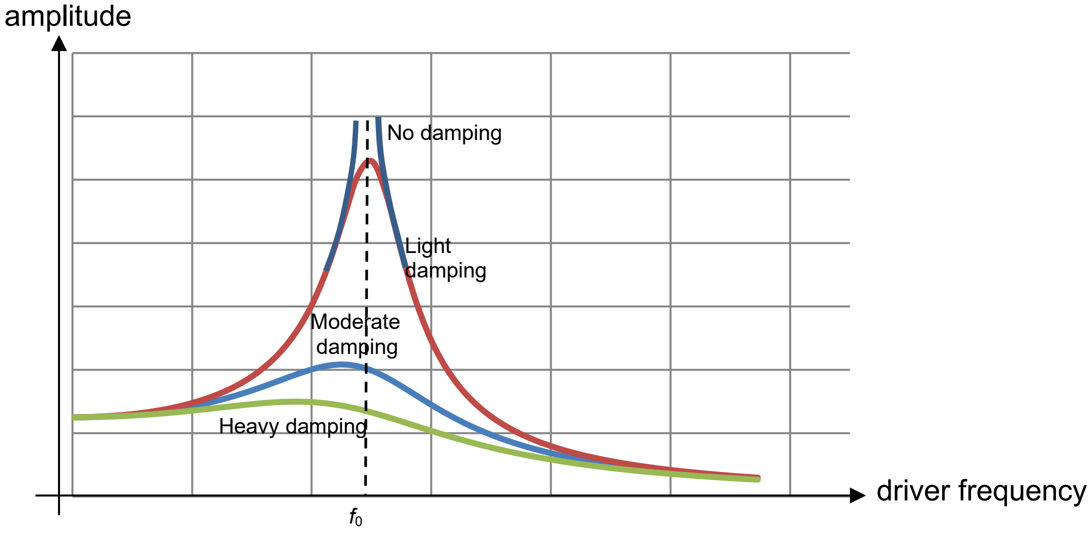
Effects of different degrees of damping



9. A system is under **forced oscillations** if it is caused to oscillate by an external periodic driving force, which supplies energy to the system.

10. **Resonance** occurs when a system responds at maximum amplitude to an external driving force. This occurs when the frequency f of the driving force is equal to the natural frequency f_0 of the driven system.

11. Frequency response curve of forced oscillations



Effect of increased degree of damping:

- Amplitude of oscillations decreases.
- Resonance occurs at a driving frequency lower than the natural frequency.
- The peak of the frequency response curve shifts leftwards and downwards.

12. Useful resonance: radio receivers, magnetic resonance imaging, musical string instruments

Harmful resonance: oscillations in bridges, vibrations in buildings during earthquakes

Chapter 11: Wave Motion

1. Important Terms

Symbol	Description	S.I. Unit
x	Displacement: the distance in a specific direction of a particle of a wave from its equilibrium position.	m
x_0	amplitude: magnitude of maximum displacement of a particle in the wave from its equilibrium position	m
λ	wavelength: distance between two consecutive points which are in phase, such as two successive crests or two successive troughs	m
T	period: time taken for a particle to complete one oscillation	s
f	frequency: number of oscillations made per unit time	Hz
v	speed: speed at which the wave shape moves.	m s^{-1}
$\Delta\phi$	phase difference between two particles in a wave or or between two waves at a point is a measure of the fraction of a cycle which one is ahead of the other.	rad
	Wavefront: line or surface joining points on a wave that are in phase. The wave travels in a direction that is perpendicular to the wavefront.	-

2.

Relationship		
$T = \dfrac{1}{f}$	$\omega = \dfrac{2\pi}{T} = 2\pi f$	$\dfrac{\Delta\phi}{2\pi} = \dfrac{\Delta x}{\lambda} = \dfrac{\Delta t}{T}$

3. In a time of one period T, the waveform moves a distance of one wavelength λ.

Since speed $= \dfrac{\text{distance}}{\text{time}}$, $v = \dfrac{\lambda}{T} \Rightarrow v = f\lambda$ (since $f = \dfrac{1}{T}$)

4. A **progressive wave** is the movement of a disturbance from a source which <u>transfers energy</u> from the source to regions around it without the transfer of matter.

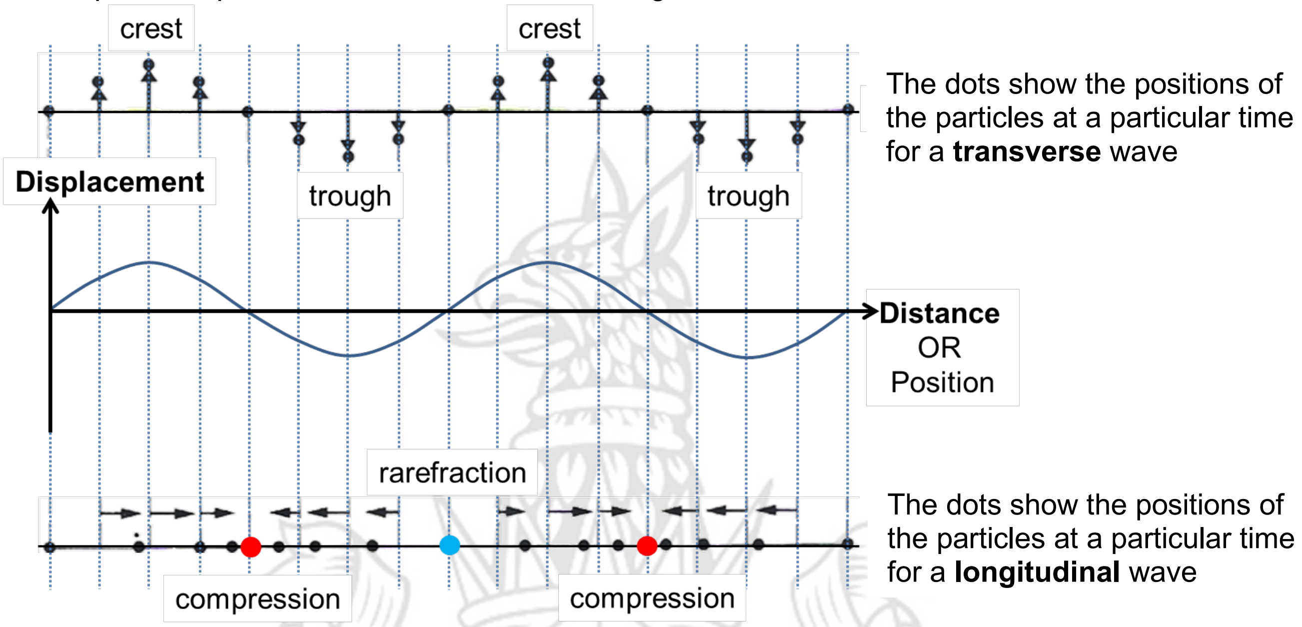
For progressive mechanical waves, the medium itself does not travel through space; its individual particles undergo back-and-forth or up-and-down motions around their equilibrium positions. <u>The overall pattern of the wave disturbance is what travels</u>.

Waves transport energy, but not matter, from one region to another.

5. A **transverse wave** is one in which its <u>particles oscillate</u> in a direction <u>perpendicular</u> to the direction of <u>energy transfer</u>.

6. A **longitudinal wave** is one in which its <u>particles oscillate</u> in the direction <u>parallel</u> to the direction of <u>energy transfer</u>.

7. Graphical Representation of Transverse & Longitudinal Waves



8. The **intensity** I of a wave is defined as the rate of transfer of energy per unit area normal to the direction of propagation of the wave.

$$I = \frac{P}{\text{Area}}$$

The <u>power P delivered</u> by any sinusoidal wave is <u>proportional to the square of the amplitude A</u>. Hence,

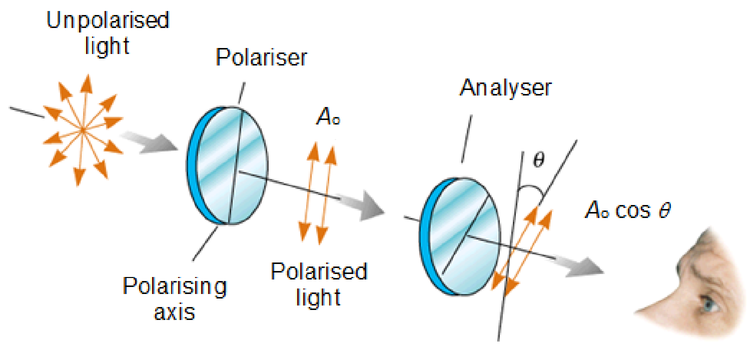
$$I = kA^2$$

If the power P of **a point source** is constant and there is no loss of power when the wave propagates (in all directions), then the intensity decreases as the inverse of the square of the distance from the source r :

$$I = \frac{k}{r^2}$$

9. Polarisation is a phenomenon associated with transverse waves.

- Malus' Law: When polarised light of amplitude A_o is incident on a second polariser (usually called an analyser) placed with its polarising axis at an angle θ to the polarising axis of the first polariser, only the electric field componenet parallel to the polarising axis of the analyser will be transmitted.



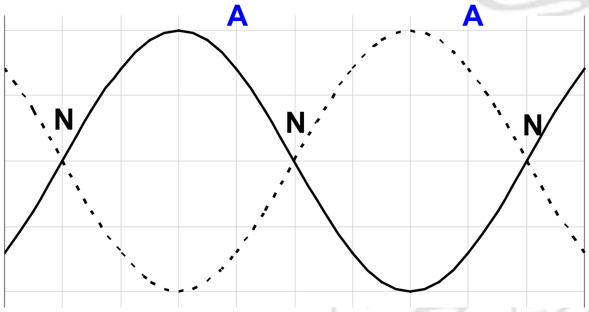
Resultant amplitude A of the transmitted wave is therefore $A = A_0 \cos\theta$

Since $I = kA^2$, resultant intensity I of the transmitted wave is therefore $I = I_0 \cos^2\theta$

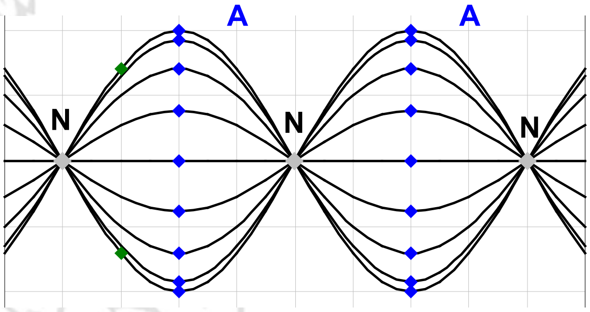
Chapter 12: Superposition

1. The **principle of superposition** states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point.

2. When two progressive waves of the **same type, same amplitude, same frequency and same speed** travel **towards each other** along the same line, they superpose and form **stationary** (or standing) **waves**.



A **graphical representation** of a stationary wave (where the maximum displacement is drawn on a displacement-distance graph)



The waveform (displacement-position) of a stationary wave at equal time interval of $T/16$.

3. The **waveform of a stationary wave** has the following features:

   (i) The wave profile does not propagate.

   (ii) Every particle of the wave merely oscillates about their respective equilibrium positions with the **same frequency**, but **different amplitudes**.

   (iii) An **antinode** is a point in a standing wave where the amplitude is the maximum. Particles at the antinodes vibrate with the greatest amplitude. These are labelled ‘A’.

   (iv) A **node** is a point in a standing wave where the amplitude is zero. They are labelled ‘N’.

   (v) Distance between two adjacent nodes (or antinodes) is ½λ.

   (vi) Within two consecutive nodes, every particle oscillates **in phase**. Particles in neighbouring loops vibrate 180° (or π rad) out of phase with each other.

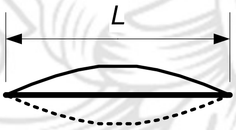
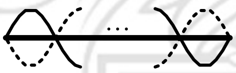
4. Comparing progressive and stationary waves:

Property	Progressive Wave	Stationary Wave
Waveform	The disturbance propagates with the velocity of the wave across space.	The disturbance does not propagate across space.
Energy	Transports energy	Does not transport energy
Amplitude	Every point oscillates with the same amplitude.	Amplitude varies from 0 at the nodes to the maximum at the antinodes.

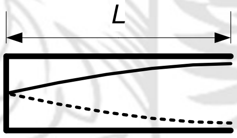
Property	Progressive Wave	Stationary Wave
Phase	All particles within one wavelength have different phases.	All particles between two adjacent nodes have the same phase. Particles in adjacent segments have a phase difference of π rad.
Frequency	All points vibrate in s.h.m. with the frequency of the wave.	Except for the nodes which are at rest, all points vibrate in s.h.m. with the same frequency as the progressive wave that gives rise to it.
Wavelength	Is the distance between adjacent points which have the same phase.	Is equal to twice the distance between a pair of adjacent nodes or antinodes.

5. Stationary waves in strings and pipes:

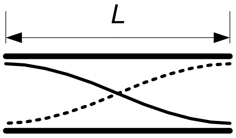
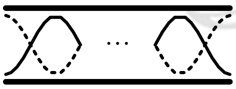
Stationary Waves on a String

Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as…
Fundamental frequency		$L = 1\left(\frac{\lambda_1}{2}\right)$ $\Rightarrow \lambda_1 = 2L$	$f_1 = \frac{v}{2L}$	1st harmonic
$(n-1)^{th}$ overtone		$L = n\left(\frac{\lambda_n}{2}\right)$ $\Rightarrow \lambda_n = \frac{2L}{n}$	$f_n = \boldsymbol{n}\left(\frac{v}{2L}\right)$	n^{th} harmonic

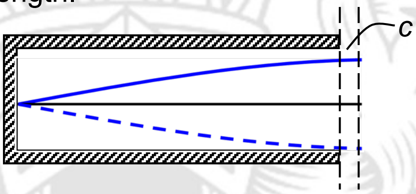
Stationary Waves in Closed Tubes

Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as…
Fundamental frequency		$L = 1\left(\frac{\lambda_1}{4}\right)$ $\Rightarrow \lambda_1 = 4L$	$f_1 = \frac{v}{4L}$	1st harmonic
$(n-1)^{th}$ overtone		$L = (2n-1)\left(\frac{\lambda_{2n-1}}{4}\right)$ $\Rightarrow \lambda_{2n-1} = \frac{4L}{(2n-1)}$	$f_{2n-1} = (\boldsymbol{2n-1})\left(\frac{v}{4L}\right)$	$(2n-1)^{th}$ harmonic

Stationary Waves in Open Tubes

Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as…
Fundamental frequency		$L = 1\left(\frac{\lambda_1}{2}\right)$ $\Rightarrow \lambda_1 = 2L$	$f_1 = \frac{v}{2L}$	1st harmonic
$(n-1)^{th}$ overtone		$L = n\left(\frac{\lambda_n}{2}\right)$ $\Rightarrow \lambda_n = \frac{2L}{n}$	$f_n = \boldsymbol{n}\left(\frac{v}{2L}\right)$	n^{th} harmonic

The displacement antinode at the open ends of the pipes are actually located slightly outside the pipe as shown below. This results in a small *end correction c* to be included in the calculation of the wavelength.



6. **Diffraction** refers to the bending or spreading of waves when they travel through an aperture or when they pass round an obstacle. It is a phenomenon of waves.

7. Waves or sources are said to be **coherent** if they have a **constant phase difference**. This implies that coherent waves or sources must have the same frequency, but the reverse is not true. Velocities of the waves are assumed to be identical.

8. **Constructive interference** takes place when two waves arrive at the same point **in phase** (i.e. their phase difference is zero). A **maximum** is obtained.

9. **Destructive interference** takes place when two waves arrive at the same point with a **phase difference of π rad**. A **minimum** is obtained.

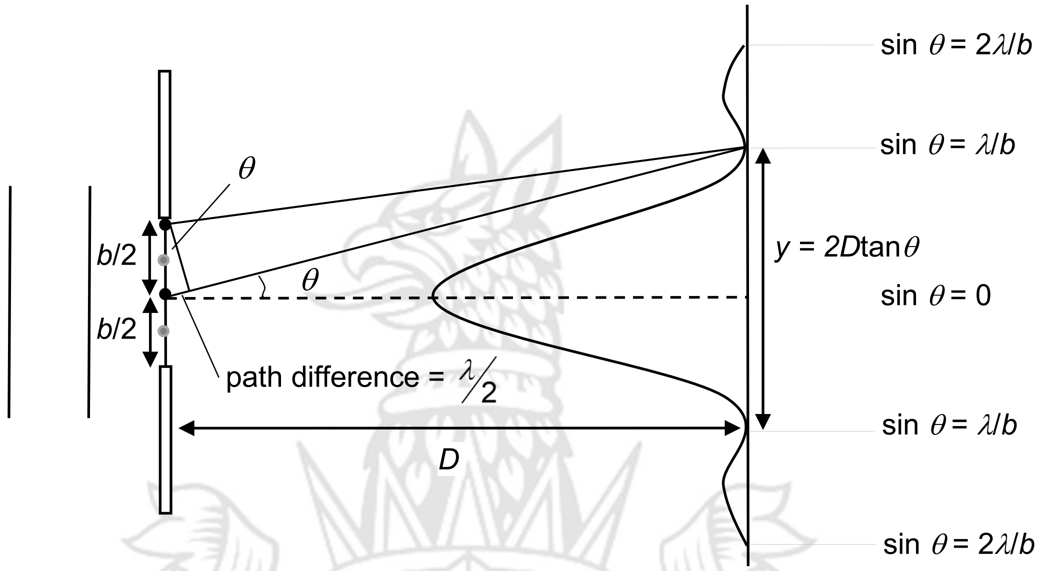
10. Conditions for **observable interference:**

    (i) The waves (or sources) must be **coherent**.
    (ii) The waves must have **similar amplitude**.
    (iii) The waves must be **unpolarised** or **polarised** in the same plane for transverse waves.
    (iv) The waves must **overlap** and be of the **same type** to give regions of **maxima** (constructive interference) and **minima** (destructive interference).
    (This point is very trivial and should be the last point to write in your answer.)

11. Conditions for path difference **Δ*L*** for constructive and destructive interference:

	2 sources in phase	2 sources π rad out of phase
Constructive Interference (maxima)	$\Delta L = n\lambda$	$\Delta L = (n + ½)\,\lambda$
Destructive Interference (minima)	$\Delta L = (n + ½)\,\lambda$	$\Delta L = n\lambda$

12. Single-Slit Diffraction and Interference Pattern



For $b << D$, the positions of the two **first minima** for single slit diffraction are given by

$$b \sin\theta = \lambda$$

Width y of the central bright fringe

$$y = 2D\tan\theta$$

Since θ is usually very small,

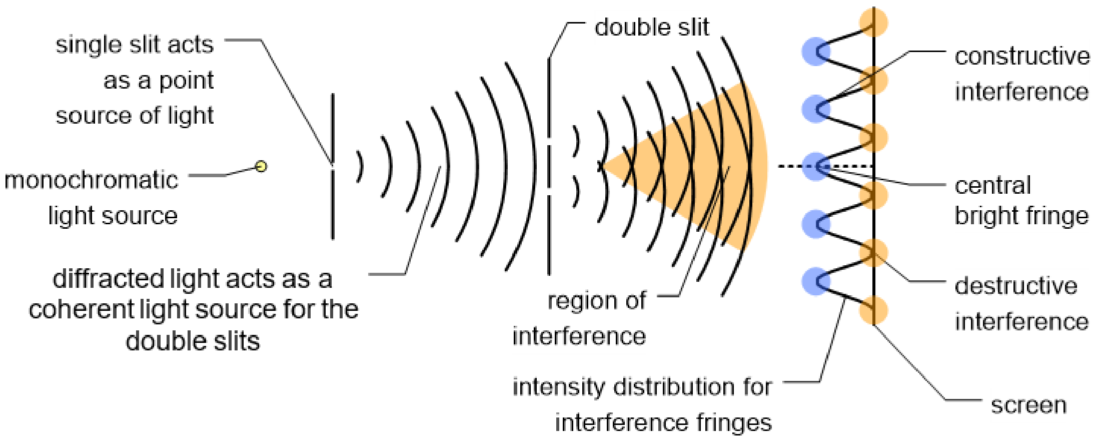
$$y \approx 2D\theta$$

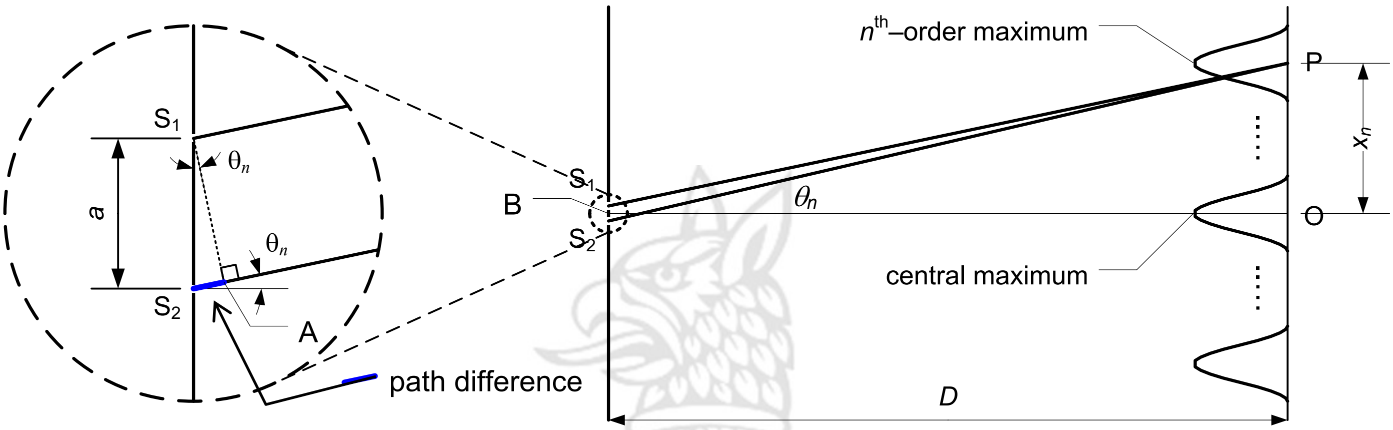
Resolving power of a single slit:
The resolving power of a single slit is the ability to distinguish between closely spaced objects.

Rayleigh's Criterion:
Two images are **just resolved** when the central maximum of one image falls on the first minimum of the other image. The criterion is satisfied when the limiting angle of resolution θ_{min} (or minimum angular separation of the sources) follows

$$\theta_{min} \approx \frac{\lambda}{b} \qquad \text{for } \sin\theta \approx \theta$$

13. Young's Double-Slit Experiment





For $a << D$, **constructive interference (maxima)**,

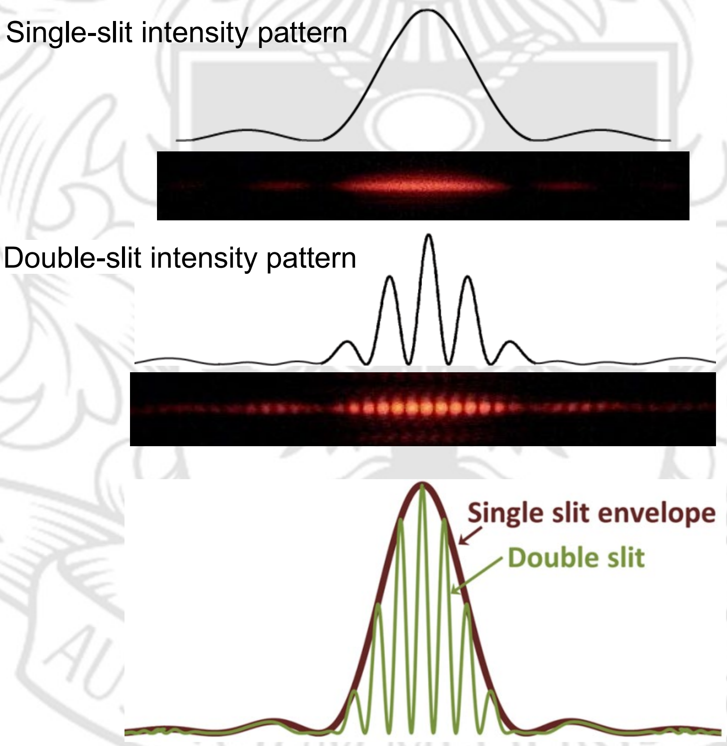
$$a \sin \theta_n = n\lambda \qquad \text{where } n = 0, 1, 2, \ldots$$

Fringe separation,

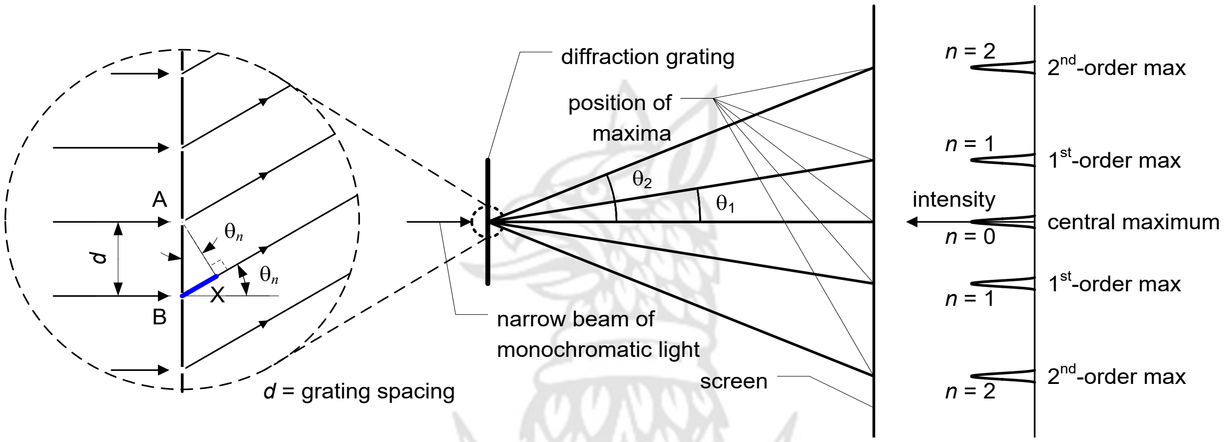
$$x = \frac{\lambda D}{a}$$

The formula $x = \dfrac{\lambda D}{a}$ is applicable only if

- $a << D$ (rays are approximately parallel)
- $a >> \lambda$ (for small angle approximation to be used)



14. Diffraction Grating



For $d << D$, **constructive interference (maxima)**,

$$d \sin\theta_n = n\lambda \qquad \text{where } n = 0, 1, 2, \ldots$$

<u>To determine **maximum order** of bright fringes:</u>

From $\sin\theta_n = \dfrac{n\lambda}{d}$, we see that the larger the value of n, the larger the value of θ.

Since $\theta_n < 90°$, $\sin\theta_n < 1 \;\Rightarrow\; \dfrac{n\lambda}{d} < 1 \;\Rightarrow\; n < \dfrac{d}{\lambda}$

An even easier way to remember or understand the formula $n < \dfrac{d}{\lambda}$ is to recognize that the slit separation d represents the maximum path difference between adjacent rays (when the rays bend by 90° to the original direction).

Chapter 13: Electric Fields

1. Electric force and field strength

$$F_E = qE$$

	Electric field strength, *E*	**Electric force, F_E**
due to point charge Q	$\dfrac{Q}{4\pi\varepsilon_0 r^2}$	$q\left(\dfrac{Q}{4\pi\varepsilon_0 r^2}\right) = \dfrac{qQ}{4\pi\varepsilon_0 r^2}$

Electric field strength and electric force are **vector** quantities.
Tip: when using above equations, substitute numerical values of charges into equation. Direction of force/electric field strength is determined by looking at diagram. Like charges repel, unlike charges attract. The above equations give magnitudes but do not indicate direction.

2. Electric potential energy and potential

$$U = qV$$

	potential	**potential energy**
due to point charge Q	$\dfrac{Q}{4\pi\varepsilon_0 r}$	$q\left(\dfrac{Q}{4\pi\varepsilon_0 r}\right) = \dfrac{qQ}{4\pi\varepsilon_0 r}$

Electric potential and potential energy are **scalar** quantities.
Tip: The signs for *Q* and *q* have to be included, which will determine the sign for *U*.

3. Important relationships:

$$E = -\frac{dV}{dr}$$

The **electric field strength at a point is numerically equal to the potential gradient at that point**. The gradient of a V–r graph gives the magnitude of the electric field strength.

The <u>negative sign</u> indicates that the electric field strength points in the direction of <u>decreasing potential</u>.

$$F = -\frac{dU}{dr}$$

The gradient of a *U*–*r* graph gives the magnitude of the electric force.

The <u>negative sign</u> indicates that the electric force points in the <u>direction of decreasing potential energy</u>.

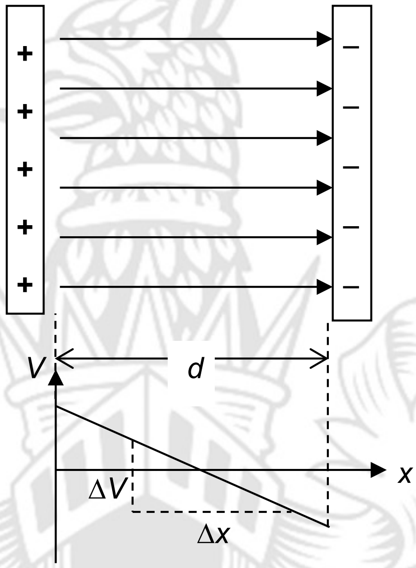
4. Uniform Electric Field

Within a region of uniform electric field:

(a) the **electric field strength** at any point has the **same magnitude and direction**;

(b) the **equipotential lines are parallel and equally spaced**.

Two equal and oppositely charged parallel plates of infinite dimensions can produce a uniform electric field between them.



Since the electric field between the parallel plates is uniform, the potential gradient at any point within the plates is given by

$$\frac{\Delta V}{\Delta x} = -E = \text{constant}$$

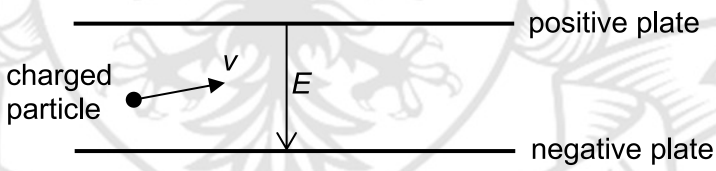
where Δx is the perpendicular distance from the plate with the higher potential.

If we know the potential difference ΔV between the two plates and the separation d of the plates, the **magnitude of the electric field strength E between the parallel plates** is

$$E = \frac{\Delta V}{d}$$

	Electric field strength, E	**Electric force, F_E**
due to parallel plates of p.d. ΔV and separation d	$\frac{\Delta V}{d}$	$q\frac{\Delta V}{d}$

5. Motion of charged particle within the uniform electric field between two parallel plates:



If the electric force F is the only force acting on the charged particle, then the <u>acceleration of the particle is also constant</u> in the direction of the force. Thus, the <u>equations of motion for constant acceleration may be applied.</u>

Perpendicular to electric field: <u>constant velocity</u>
Parallel to electric field: <u>constant acceleration</u> (equations of motion applied in this direction)
Trajectory: <u>parabolic</u>

Motion of charged particle beyond the two parallel plates:
Trajectory: <u>straight line</u>

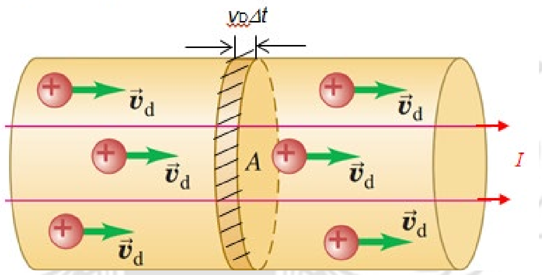
Chapter 14: Current of Electricity

1. Electric current is defined as the **rate of flow of charge**. **Average current** is given by **total charge divided by total time.** $I = \dfrac{Q}{t}$

2. By convention, the **direction of current** is defined as the **direction of flow of positive charge**, regardless of the actual sign of the charged particles in motion.

3. The free charges in a conductor move with an **average** velocity v_D called drift velocity, which is on the order of 10^{-4} m s^{-1}.

4. Consider a conductor with cross-sectional area A (perpendicular to the direction of current flow) and n charge carriers per unit volume (or the number density).



   - In a time interval Δt, each charge carrier, assumed to be positive, moves a distance $v_D(\Delta t)$ to the right. Thus, in Δt, charges within a distance of $v_D \Delta t$ to the left of the cross-section A (the shaded cylinder) will pass through A.
   - The volume of the shaded cylinder is $Av_D(\Delta t)$. The number of charge carriers passing through A in Δt is thus $nAv_D(\Delta t)$.
   - The total charge passing through A in Δt is $nAv_D q(\Delta t)$, where q is the charge carried by each charge carrier.
   - Hence, current $I = \dfrac{\text{Total charge through } A \text{ in } \Delta t}{\text{Time interval } \Delta t} = \dfrac{nAv_D q(\Delta t)}{\Delta t}$

$$I = nAv_D q$$

5. The **potential difference V** between two points in a circuit is defined as the amount of **electrical energy per unit charge** that is converted to **other forms of energy** when charge passes from one point to the other.

6. One **volt** is the potential difference between two points in a circuit when one joule of electrical energy is converted to other forms of energy as one coulomb of charge passes from one point to another ($1\text{ V} = 1\text{ J C}^{-1}$). $V = \dfrac{W}{Q}$

7. The **electromotive force (e.m.f.) of a source** is defined as the amount of **electrical energy that is converted *from* other forms of energy** when charge passes through the source.

8. The **resistance** of a circuit component is defined as the **ratio** of the potential difference across it to the current flowing through it. $R = \dfrac{V}{I}$

9. Ohm's law states that the potential difference across a conductor is **proportional to** the electric current passing through it, provided that its **temperature remains constant**.

10. One **ohm** is the resistance of a conductor when a potential difference of one volt across it causes a current of one ampere to flow through it (1 Ω = 1 V A^{-1}).

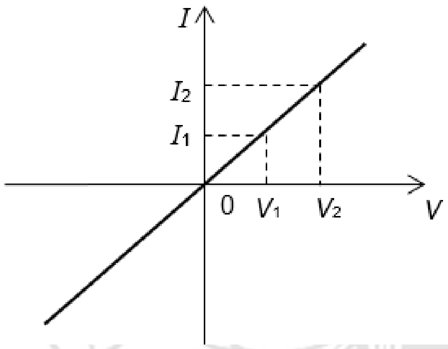
11. Resistance of a uniform conductor (at a given temperature) is inversely proportional to the cross-sectional area A, and directly proportional to the length L:

$$R = \rho \frac{L}{A}$$

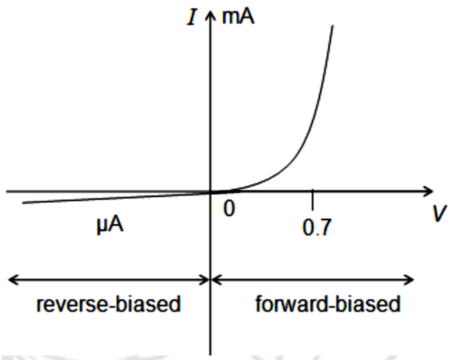
The proportionality constant ρ is called the resistivity.

12. I–V characteristics

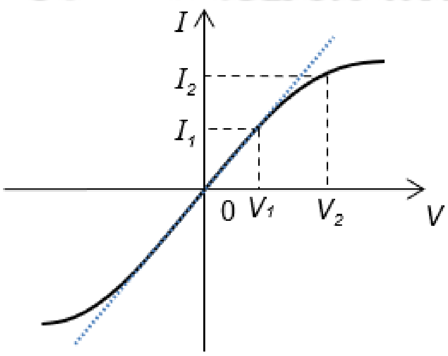
Ohmic conductor



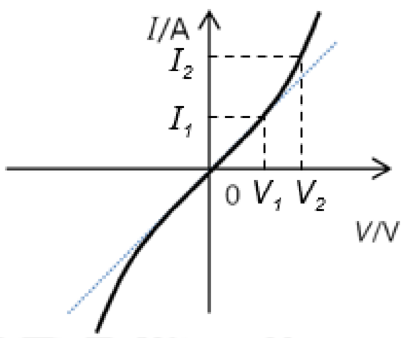
Semiconductor diode



Filament Lamp



NTC thermistor



13. To determine the resistance of an electrical component from the I–V curve, read off the values of V and I required and calculate $R = \frac{V}{I}$. Note that R is not the gradient of any point on the I–V curve.

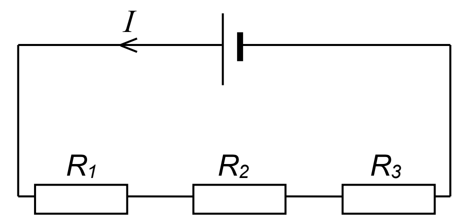
14. Electric power $= IV = I^2R = \frac{V^2}{R}$.

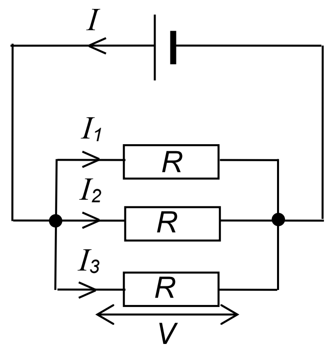
15. A realistic battery can be modeled as an ideal e.m.f. source E in series with a resistance r, which is called the internal resistance of the battery.
If the current flowing through the circuit is I, the p.d. across the battery (which is called the terminal p.d.) is given by

$$\text{terminal p.d.} = E - Ir.$$

Chapter 15: DC Circuits

Equivalent Resistance





Resistors in Series

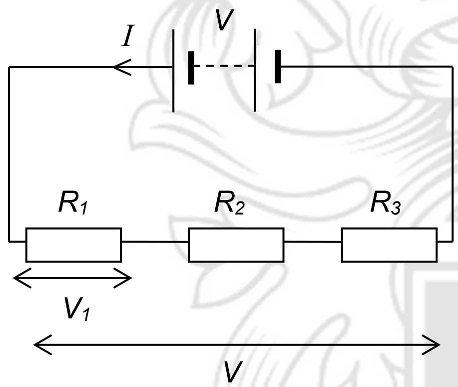
1. Magnitude of current, I flowing through the resistors is the same.

2. $R_{eq} = R_1 + R_2 + \ldots + R_n$

Resistors in Parallel

1. Potential difference, V across the resistors is the same.
2. $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

Potential Divider



1. Resistors are in series.

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} V$$

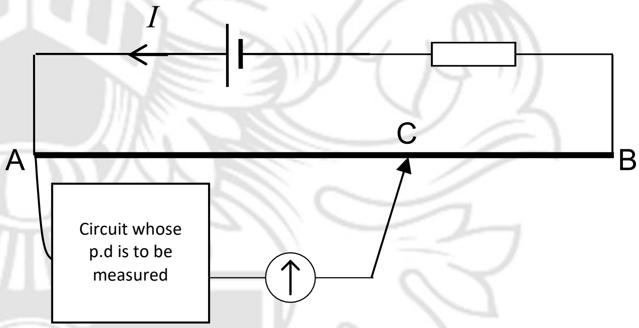
$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} V$$

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} V$$

2. R_1 or R_2 or R_3 may be replaced by other components such as LDR or thermistor.

*Light intensity increase, R_{LDR} decrease.
*Temperature increase, $R_{thermistor}$ decrease.

Potentiometer Principle

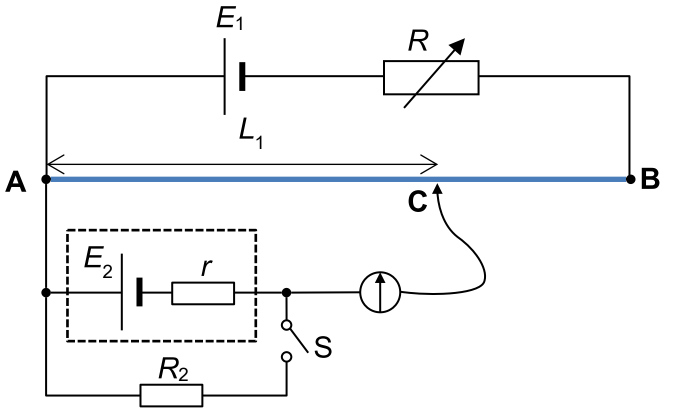


1. Sliding contact is moved along resistance wire AB until a point C is found such that there is <u>no current</u> in the galvanometer.
2. Point C is called the <u>null point</u> or <u>balance point</u>.
3. Length AC is called the <u>balance length</u>.
4. Potential difference across AC is <u>equal</u> to the potential difference across the circuit to be measured.
5. If wire AB has <u>uniform</u> cross-sectional area, potential difference across AB and the length of AB <u>remains constant</u> with time.
6. $$\frac{V_{AC}}{V_{AB}} = \frac{R_{AC}}{R_{AB}} = \frac{L_{AC}}{L_{AB}}$$

Solving Potentiometer Questions

Generally there are 2 scenarios:

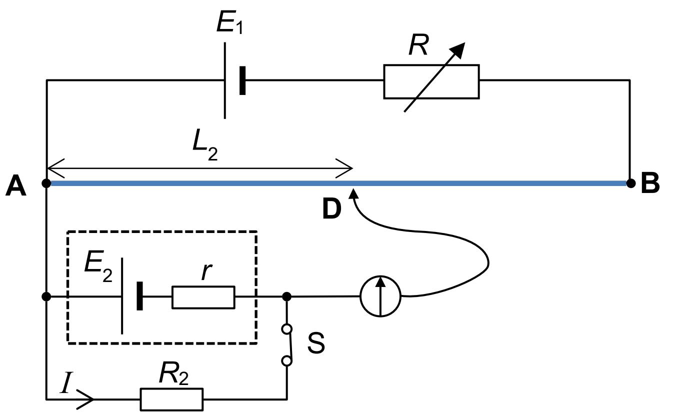
1. When switch S is opened.



With switch S **open**, no current passes through the internal resistance r. Hence, there is no potential drop across r.

$$\therefore V_{AC} = E_2 = \frac{L_1}{L_{AB}} \times V_{AB}$$

2. When switch S is closed.



With switch S **closed**, current passes through internal resistance r and resistor R_2.

The **balance point** is now at point D and the **balance length** is L_2.

$$\therefore V_{AD} = \frac{L_2}{L_{AB}} \times V_{AB}$$

Since battery E_2 and resistor R_2 are all parallel to AD,

$$V_{AD} = E_2 - Ir$$

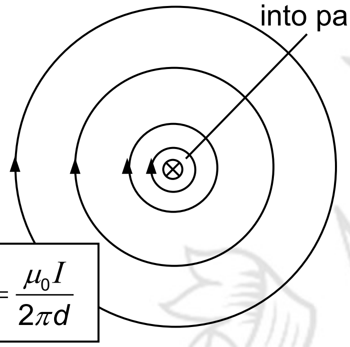
and also $V_{AD} = IR_2$

or $V_{AD} = \dfrac{R_2}{r + R_2} E_2$ (potential divider)

Chapter 16: Electromagnetism

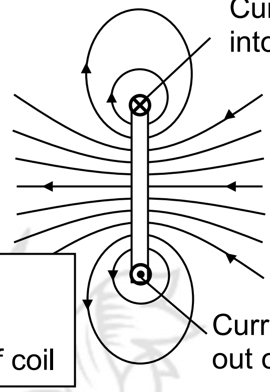
1. A magnetic field is a field of force produced by current-carrying conductors, moving charges or permanent magnets. Magnetic field pattern due to various configurations of current-carrying wires:

(a) a long straight wire



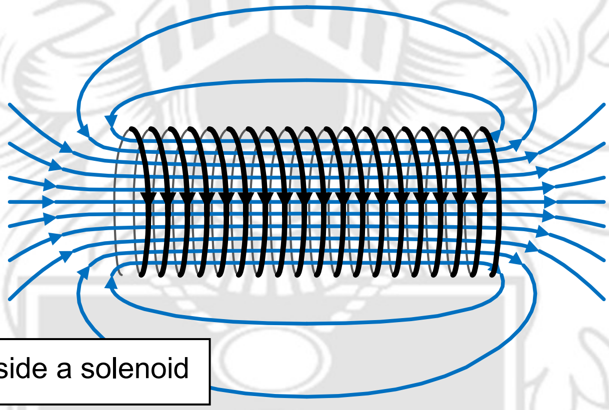
$B = \dfrac{\mu_0 I}{2\pi d}$

(b) a **flat circular** coil



$B = \dfrac{\mu_0 N I}{2r}$: at centre of coil

(c) a long solenoid



$B = \mu_0 n I$: uniform inside a solenoid

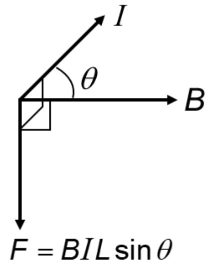
2. **Right-hand grip rule**
The directions of the current and the flux density follow the right-hand grip rule. When using this rule, your right thumb must be straight, whereas the fingers of your right-hand curled.

	thumb points in the direction of	fingers curl in the direction of
straight wire	current	flux density
coil	flux density (at centre of coil)	current
solenoid	flux density	current

3. **Definition of *flux density B***
The **magnetic flux density** of a magnetic field is numerically equal to the force per unit length per unit current acting on a long straight current-carrying conductor placed at right angles to the magnetic field.

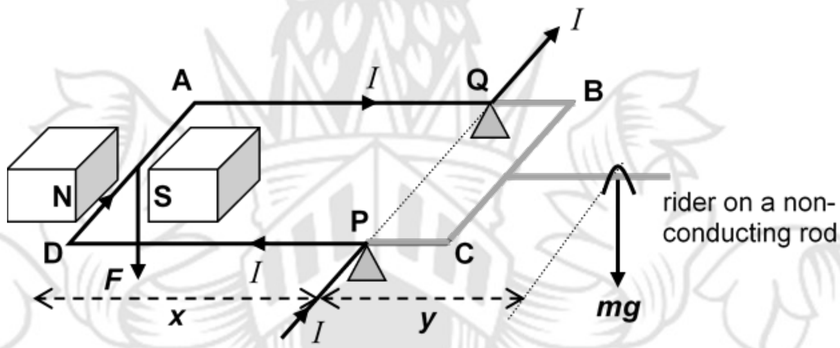
4. **Fleming's Left Hand Rule (FLHR)**
To determine the direction of the magnetic force, FLHR is used. The directions of your left thumb, index and middle fingers are the directions of, respectively, the magnetic Force (F), the B field flux density (B) and the current I.
In the case of a moving positive charge, the direction of the middle finger is the direction of v. If the charge is negative, then the middle finger is opposite in direction to v.

5. The magnitude and direction of force F acting on a conductor carrying a current I placed at an angle θ to the magnetic field of flux density B:



Note: The sin θ factor yields the component of B **perpendicular** to I.

6. **Measuring flux density using a current balance**
A current balance is shown below. With no current through PDAQ and no rider (the small mass) on the right, the loop ABCD is horizontal and balanced.



With current I turned on, side AD experiences a downward magnetic force, which yields an anticlockwise torque about PQ. The clockwise torque due to the weight of the rider on the right is needed to balance the system:

$$\text{Sum of clockwise moments} = \text{Sum of anticlockwise moments}$$

$$mgy = Fx$$

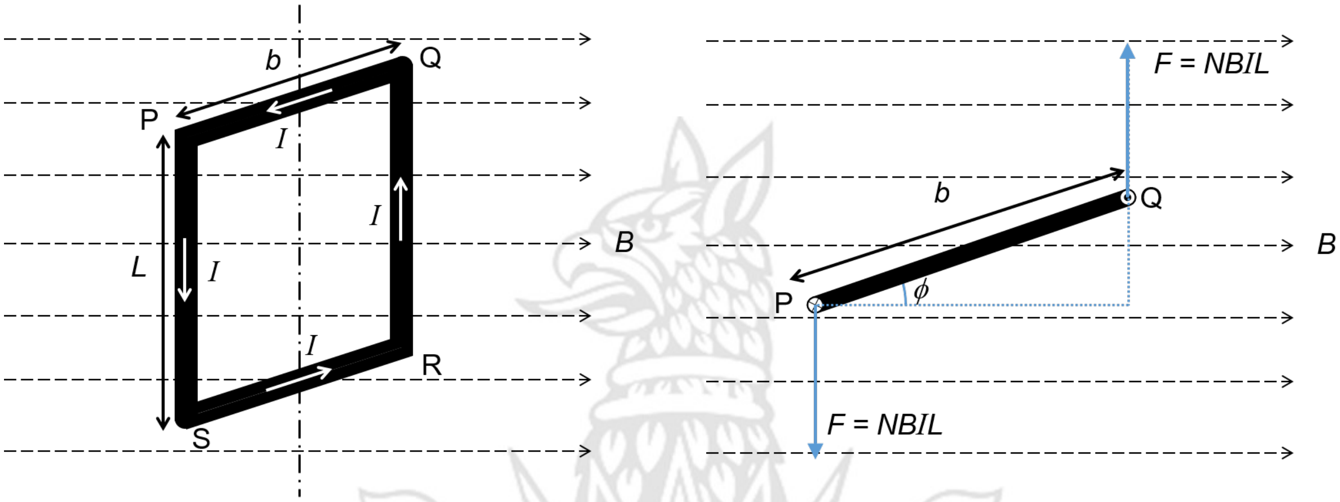
$$mgy = BILx$$

$$B = \frac{mgy}{ILx}$$

Hence, the magnetic flux density B of a magnetic field can be determined with the appropriate quantities known.

7. **Forces between *two long straight current-carrying conductors***:
currents in the same direction: the wires <u>attract</u> each other;
currents in the opposite direction: the wires <u>repel</u> each other

8. Forces on a coil in B field:



(a) side view (b) top view

Torque on the coil due to the couple
$= F \times$ perpendicular distance between line of action of forces
$= F \times b\cos\phi = NBILb\cos\phi = NBIA\cos\phi$

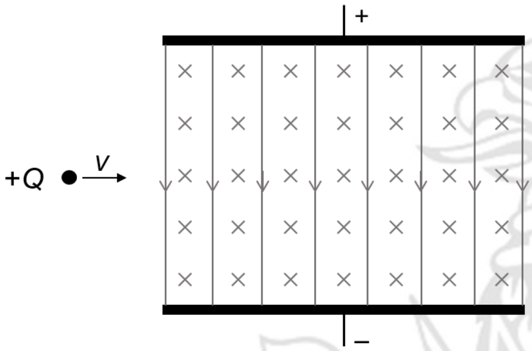
9. The magnitude and direction of the force acting on a particle of charge Q moving at an angle θ to a uniform magnetic field of flux density B.
$F = BQv\sin\theta$

10. When a charged particle is **injected at right angles** into a uniform **magnetic field**, it experiences a force that is perpendicular to the velocity of the particle.
The velocity changes in direction and not in magnitude.
The particle hence moves under the influence of a force that is constant in magnitude (since $F = BQv$ and speed is constant), which is at all times perpendicular to its velocity, and always points towards the same point (centre of the circle). The trajectory is therefore circular.

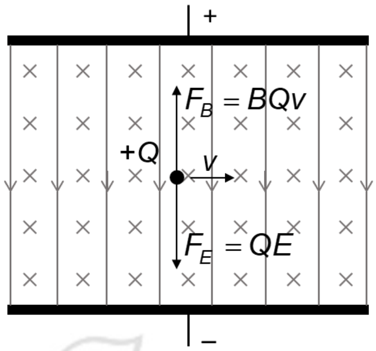
11. When a charged particle is **injected at an angle θ** to a uniform magnetic field such that $0° < \theta < 90°$, its motion is the result of superposing
(i) a uniform circular motion in a plane perpendicular to the direction of B (due to the component of velocity perpendicular to B)
(ii) a steady axial speed along the direction of B.
The trajectory is a helix.

12. A *Velocity Selector* uses an **electric field** E and a **magnetic field** B at **right angles** to each other.

(a) orientation of fields:
B-field acts into the page
E-field acts downwards



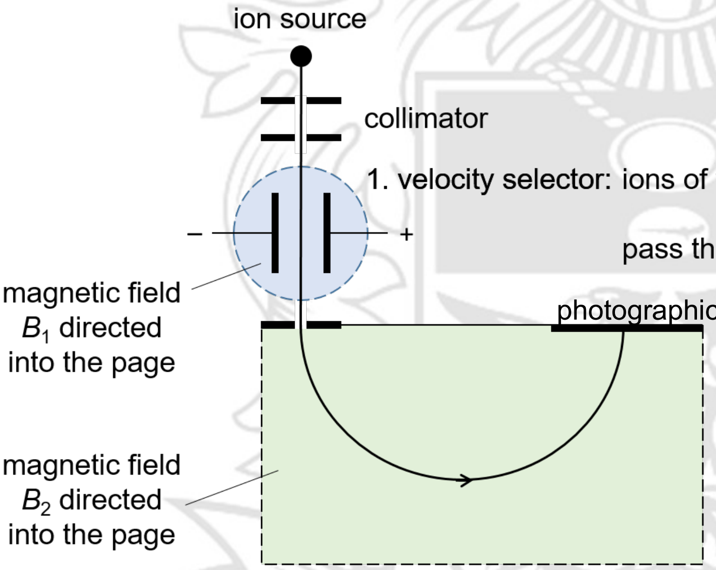
(b) forces acting on a positively-charged particle on entering the cross field:
Magnetic force F_B acts upwards
Electrostatic force F_E acts downwards



(c) For a particle to pass through undeflected:

$$F_B = F_E$$
$$BQv = QE$$
$$v = \frac{E}{B}$$

13. A *mass spectrometer* separates ions according to their mass-to-charge ratio.



1. velocity selector: ions of velocity $v = \frac{E}{B_1}$ pass through undeflected.

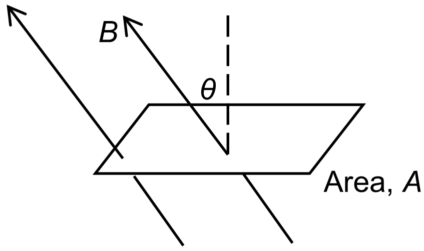
2. ions are then deflected into a semi-circular path in B_2
magnetic force provides the centripetal force

$$F_{B_2} = F_C$$
$$B_2Qv = \frac{mv^2}{r}$$
$$r = \frac{mv}{QB_2} = \frac{m}{QB_2}\left(\frac{E}{B_1}\right) = \frac{M}{Q}\boxed{\left(\frac{E}{B_1B_2}\right)}$$
$$\therefore r \propto \frac{M}{Q}$$

constant!

Chapter 17: Electromagnetic Induction

1. Magnetic Flux and Magnetic Flux Linkage



Magnetic Flux through an area is the product of the area and the magnetic flux density that passes through the area perpendicularly.

$$\phi = BA\cos\theta$$

Note that θ is the angle between the magnetic field and the **normal to the area**.

The magnetic flux linkage of a coil is the magnetic flux passing through each turn of the coil multiplied by the number of turns of the coil.

$$\Phi = NBA\cos\theta$$

Both Φ and ϕ are measured in webers (Wb).

2. Faraday's Law

To find the *magnitude* of the induced e.m.f., we use Faraday's Law:

Faraday's law of electromagnetic induction states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage.

$$\text{Induced e.m.f.} = -\frac{d\Phi}{dt}$$

3. Lenz's Law

Lenz's law states that the direction of the induced e.m.f. is such as to cause effects to oppose the change producing it.

4. From point **2.**, applying Faraday's Law to determine the induced e.m.f. for the following situations, the corresponding expressions for the induced e.m.f. are obtained:

	Situation	Formula for induced e.m.f. *E*
1	Moving rod	Emf = Blv
2	Generator	Emf = $NBA\omega \sin\omega t$
3	Rotating Disc	Emf = BAf = ½ $Br^2\omega$

5. The direction of the induced e.m.f. can be determined by:
- (a) Fleming's Right Hand Rule (in exams, right-hand rule cannot be used to explain the direction of induced emf or induced current)
- (b) Lenz's Law
- (c) Considering the *magnetic* force acting on the individual charge carriers of the conductor.

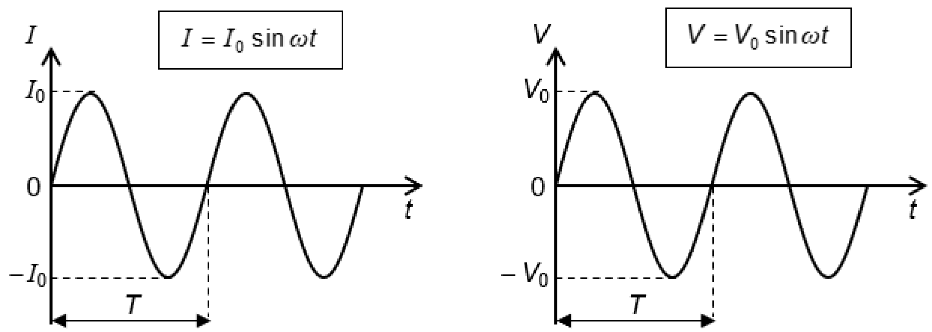
Chapter 17: Alternating Currents

1. Important Terms

Term	**Definition**
Period, T	Time taken for one complete cycle.
Frequency, f	Number of complete cycles per unit time.
Peak current, I_0	Amplitude of the current.
Root-mean-square value	Value of the direct current or voltage that would produce thermal energy at the same rate in a resistor.

2. Mean power for a sinusoidal a.c. $\langle P \rangle = I_{rms} V_{rms} = \left(\dfrac{I_0}{\sqrt{2}}\right)\left(\dfrac{V_0}{\sqrt{2}}\right) = \dfrac{P_0}{2}$

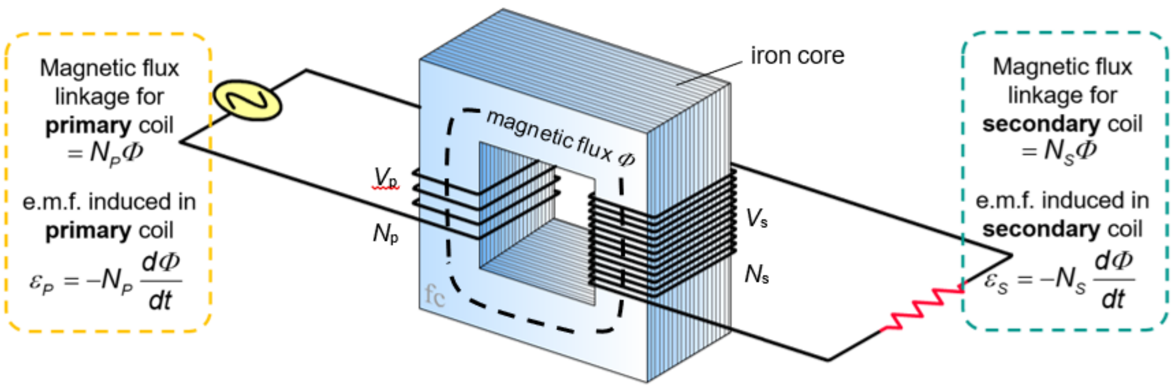
3. The sinusoidal alternating current and alternating voltage can be represented by



4. For a sinusoidal current and sinusoidal voltage: $I_{\text{rms}} = \dfrac{I_0}{\sqrt{2}}$ and $V_{\text{rms}} = \dfrac{V_0}{\sqrt{2}}$

5. A transformer is a device that uses mutual electromagnetic induction to vary (either step up or step down) an alternating voltage.

- The a.c. voltage source (primary voltage) V_P causes an alternating current I_P to flow through the primary coil.
- This sets up a varying magnetic field in the iron core which links the primary coil with the secondary coil.
- In accordance to Faraday's law of electromagnetic induction, the changing magnetic field induces an alternating e.m.f. across each turn of wire in both the primary and secondary coils.

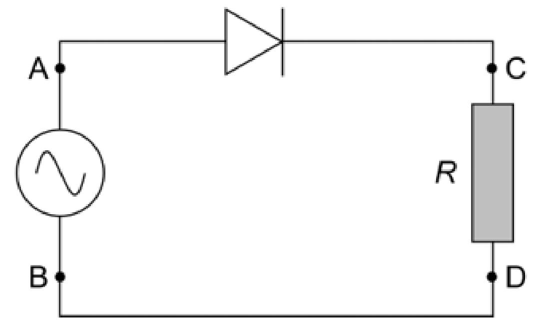


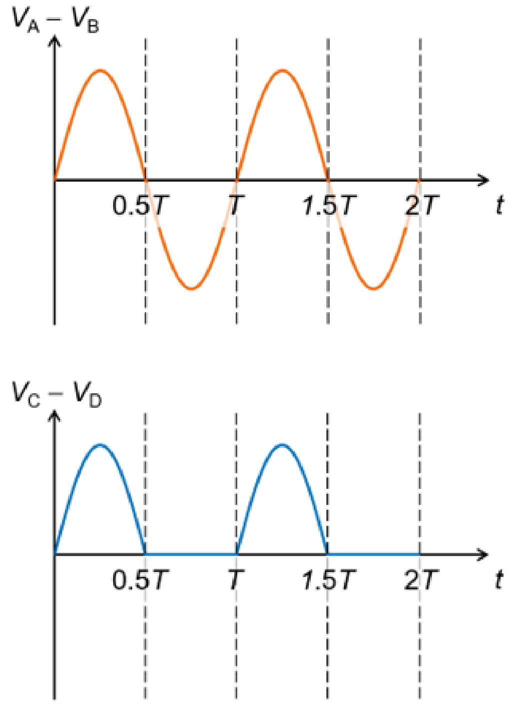
6. For an ideal transformer, $\dfrac{V_S}{V_P} = \dfrac{I_P}{I_S} = \dfrac{N_S}{N_P}$

7. For a constant power supplied by the generator P and transmission voltage V, the current in the cables is $I = \dfrac{P}{V}$.

If total resistance of cables is R, the power lost across cables is $P_{loss} = I^2 R$

8. Rectification is the conversion of alternating current to direct current. Diodes are used for rectification. A diode is a device that conducts current predominantly in one direction.





First ½ cycle:	The diode is in **forward bias**.
$t = 0$ to $t = 0.5T$ $V_A > V_B$	The current flows round the circuit as the diode acts like a closed switch. The a.c. source applies a potential difference across the load and drives a current through the load. The p.d. across R is the same as the voltage of the a.c. supply given that the diode is ideal.
Second½ cycle:	The diode is in **reverse bias**.
$t = 0.5T$ to $t = T$ $V_A < V_B$	No current flows as the diode acts like an open switch. The p.d. across R is zero.
	This is repeated for each cycle of a.c. input. The current through R is unidirectional and so the p.d. across R is direct. Although it fluctuates, the p.d. across R does not change direction.

Chapter 19: Quantum Physics

1. A **photon** is a quantum of electromagnetic energy.
Energy of a photon $E = hf$, where h is Planck constant and f is frequency

2.

Behaviour	Evidence
Light behaves as a wave	Interference / Diffraction of light
Light behaves as particles	Photoelectric effect
Electrons behave as particles	Electrons have mass and charge, and undergo collision
Electrons behave as a wave	Electron diffraction

3. **Photoelectric effect**

The **photoelectric effect** is a phenomenon in which electrons are emitted from a metal surface when light of sufficiently high frequency is incident on the surface.

It provides **evidence for the particulate nature** because:

1) **There is a minimum frequency, threshold frequency, of the incident electromagnetic (EM) radiation, above which only then will photoemission take place,** unlike wave theory which predicts that if light of sufficient intensity is used, the electrons would absorb enough energy to escape.

2) There is an **almost immediate emission of photoelectrons from the material surface even with low intensity EM radiation that was above the threshold frequency**, whereas classical wave theory states that electrons should absorb energy over a period of time before it gains enough energy to escape the metal.

3) There is a **maximum kinetic energy of the photoelectrons which is dependent on the frequency of the incident EM radiation rather than the intensity of the incident radiation**, unlike classical wave theory which believes that by using light with higher intensity (i.e. larger amount of energy striking the metal surface), the kinetic energy of an ejected electron can be increased.

The **threshold frequency** (f_0) for a metal is the minimum frequency of electromagnetic radiation below which no emission of photoelectrons occurs.

Work function (Φ) : the minimum amount of energy necessary for an electron to escape from the *surface* of a material.

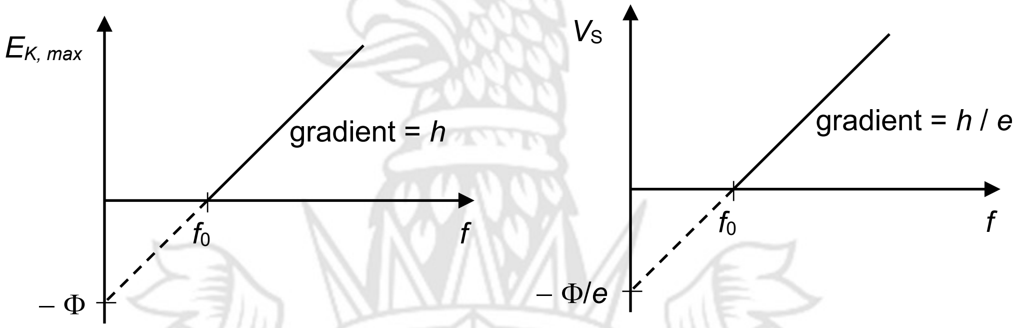
$$\Phi = hf_0$$

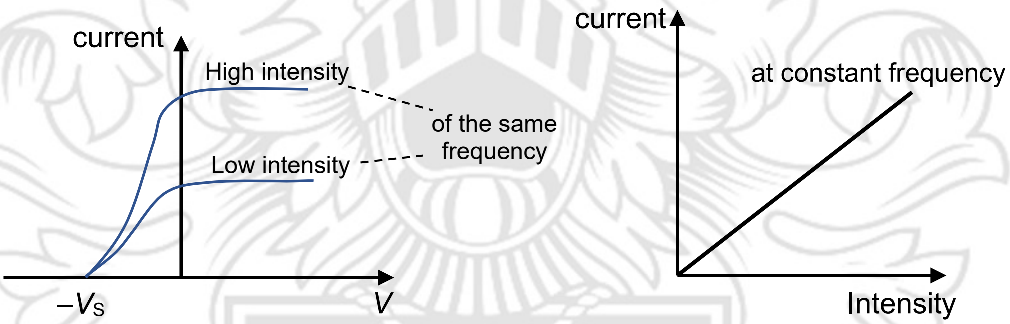
Photoelectric equation : $hf = \Phi + \frac{1}{2} m_e v_{max}^{\;2}$

Stopping potential: the negative potential of collector with respect to the emitter which prevents the most energetic photoelectrons from reaching the collector and hence results in zero photocurrent.

$$\frac{1}{2}m_e v_{\max}{}^2 = eV_S$$

where $\frac{1}{2}m_e v_{\max}{}^2$ is the $E_{K,\max}$ of the photoelectron and V_s is the stopping potential





The intensity I of a beam of photons is the energy E, transmitted per unit area, A per unit time t,

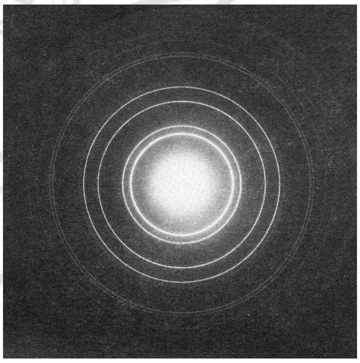
$$\text{Intensity } I = \frac{\text{power}}{\text{area}} = \frac{E}{At} = \frac{Nhf}{At} = nhf$$

where n is the number of photons passing a unit area per unit time.

An **<u>increase in intensity</u>** (at constant frequency) of incident radiation means a greater number of photons striking the metal surface per second and therefore a greater number of photoelectrons can be emitted per unit time, hence a **<u>greater photoelectric current</u>**.

4. <u>Electron Diffraction</u>

When a beam of electron falls on a thin layer of graphite atoms, the electrons disperse and their pattern is captured on a coated fluorescent screen. The pattern is identical to that obtained due to interference of electromagnetic radiation. This is known as electron diffraction which provides evidence for the wave nature of particles.



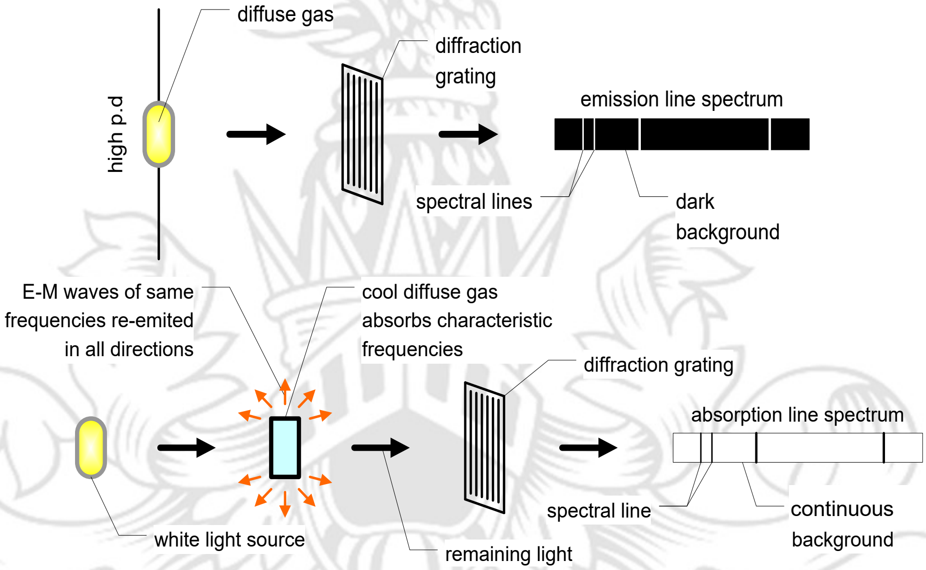
5. de Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$

6. Line spectrum

Discrete electron energy levels exists in isolated atoms (e.g. atomic hydrogen) and electron transitions between these levels leads to spectral lines.

The *emission line spectrum* consists of discrete bright lines of definite wavelengths on a dark background.

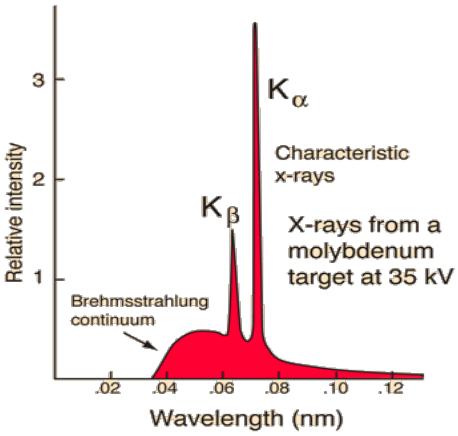
The *absorption line spectrum* consists of dark lines on a continuous bright spectrum.



Energy of photon emitted when an electron transits from energy level E_2 to E_1

$$h\,f = E_2 - E_1$$

7. **X-ray spectrum**



- **Continuous spectrum**
  Electrons emit electromagnetic radiation when they are slowed down upon hitting the target metal. X-rays are emitted if the incident electron had been accelerated by a high voltage. Not all electrons are stopped in a single collision. Thus, X-rays cover a wide range of wavelengths, giving rise to the continuous spectrum.

- **Minimum wavelength**
  When an electron loses all of its kinetic energy in a single collision with target metal, the energy of the electron (eV) is transformed completely into the energy of the X-ray photon (hf_{max}).

$$eV = \frac{1}{2}m_e v_e^2 = hf_{max} = h\frac{c}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV}$$

- **Characteristic peaks**
  Incident electron knock an electron out of the K-shell (n = 1) of the target metal. Characteristic x-rays are emitted when electrons from other levels (e.g. n = 2 and n = 3) transit to fill the vacancies in the K-shell. Characteristic peaks depend on the nature of the target material.

8. **Heisenberg's uncertainty principle** states if a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its x-component of momentum is made with uncertainty Δp, the product of the two uncertainties can never be smaller than h (Planck constant).

$$\Delta p \, \Delta x \gtrsim h$$

The uncertainties do not arise from imperfections in the measuring instruments but rather is an inherent property of matter.

Chapter 20: Nuclear Physics

Rutherford's Alpha Particle Scattering

1. In Rutherford's alpha particle scattering experiment, most alpha particles pass through the gold foil with little or no deflection. About 1 in 10^4 alpha particles are deflected by angles bigger than 90^0, including 180^0.

- The fact that some are deflected by 180^0 shows that the nucleus must have the same type of charge as the alpha particles (positive) so that the alpha particles experience a coulombic repulsion.
- The nucleus must be very small as very few alpha particles come close enough to the charged nucleus to experience the large coulombic force which causes the large deflections.
- The atom is largely empty space as most of the alpha particles through gold foil with little or no deflection.
- The nucleus must be very massive so that when the alpha particle approaches the nucleus, the nucleus will not recoil too much/ get knocked out.

The Nucleus

1. The atomic nucleus consists of

	nucleons	charge	rest mass
(a)	protons	1.6×10^{-19} C	1.672622×10^{-27} kg
(b)	neutrons	0	1.674927×10^{-27} kg

2. The nucleons (i.e., protons and neutrons) attract each other with the strong nuclear force. It is 'strong' because it is able to overcome the colossal coulomb repulsion between the protons that are separated by a mere 10^{-15} m and hold them together.

3. The strong force is very short range and its effect is negligible at distances greater than 10^{-15} m. This explains why atoms with very large atomic numbers are unstable – the short-range strong force is overwhelmed by the long-range coulomb's repulsion when there are too many protons in the nucleus.

Mass Defect and Binding Energy

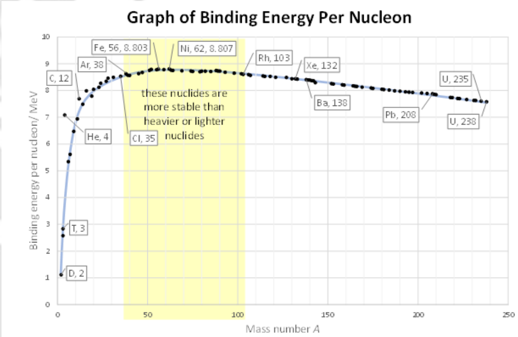
1. When Z protons and $(A-Z)$ neutrons (A is the mass number) form a nucleus, the mass of the nucleus is less than the total mass of the individual protons and neutrons. This mass difference is known as the mass defect (Δm).

2. The mass defect is consistent with Einstein's famous mass-energy equation: $E = mc^2$. When nucleons form a nucleus, they are bound to each other. This means that they have less energy as compared to the state in which they are separated far from each other. Less energy implies less mass, and hence the mass defect.

3. **Nuclear binding energy** is the energy equivalent of the mass defect of a nucleus. It is the energy required to separate to infinity all the nucleons of a nucleus.
$$B.E. = \Delta m c^2$$

4. The larger the **binding energy per nucleon** of an atom, the more stable the atom will be.

5. Since the graph of BE per nucleon against mass number peaks at about $A \approx 60$, both fusion of light nuclides and fission of heavy nuclides are energy-favourable.



6. A nuclide (not to be confused with nuclei) is a species of nucleus or atom characterized by a specific number of protons and neutrons. E.g. $^{235}_{92}U$, $^{238}_{92}U$ and $^{235}_{90}Th$ are different nuclides.

7. Isotopes are nuclides that have the same number of protons but different number of neutrons.

8. During a nuclear reaction, a loss of mass implies that energy is released (in the form of KE or gamma-ray photons). Mass-energy (conversion using $E = mc^2$) is conserved in nuclear processes.

9. The mass of nuclides is often expressed in terms of the unified atomic mass unit (abbrev: u).
$1\,u = \frac{1}{12} \times$ mass of a carbon-12 atom $= 1.660539 \times 10^{-27}$ kg
Mass of proton and neutron are less than 1% more than 1 u. Hence, **for rough calculations**, mass of one nucleon can be taken to be equal to 1 u.

10. In a nuclear reaction, both atomic number and mass number must be conserved:
e.g. $^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$

11. **Nuclear fission** is the splitting of a heavy nucleus into two lighter nuclei of approximately the same mass.
Nuclear fusion occurs when two light nuclei combine to form a nucleus of greater mass.

Radioactivity

1. **Radioactivity** or **radioactive decay** is the process whereby a radionuclide is transformed into a more stable nuclide by emitting some forms of ionizing radiation.
Radioactive decay is **spontaneous** because the probability of decay of a nucleus is unaffected by any external factors such as temperature, pressure or chemical composition.
Radioactive decay is **random** as it is impossible to predict which nucleus will decay next even though the probability of decay per unit time is identical for all nuclei. This results in fluctuations in the count rate measured by a detector.

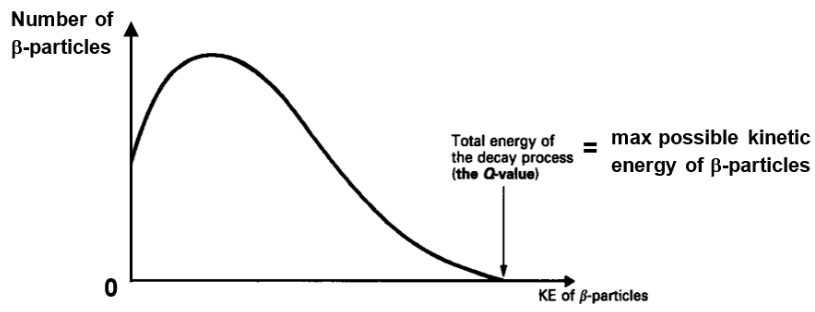
2. The 3 types of radioactive decays in the H2 syllabus are:

(a) **Alpha Decay** – the spontaneous emission of an α-particle (helium-4 nucleus) by a radioactive nucleus.
(b) **Beta Decay** – the spontaneous emission of a β-particle (electron) by a radioactive nucleus.
(c) **Gamma Decay** – the emission of a γ-ray photon when a nucleus at a higher energy state (after nuclear reactions) de-excites to lower energy states. This process is analogous to the de-excitation of electrons in atoms.
(d) **Comparison of Properties**

Properties	Alpha	Beta	Gamma
Nature	He-4 nucleus	electron	γ-ray photon
Charge	$+2e$	$-e$	0
Energy	well-defined 4 – 9 MeV	range from 0.01 to 5 MeV	discrete energies (keV – MeV)
Ionizing power	strong	moderate	very weak
Range in air	a few centimetre	a few metres	very long
Penetrating power	low	medium	high

Existence of neutrino

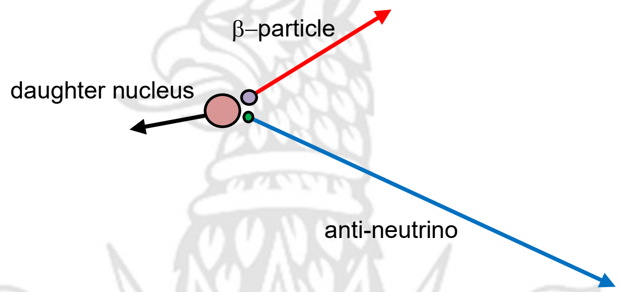
1. Conservation of Energy:
(1) The sum of the kinetic energy of the daughter nucleus and the β-particle is less than the total energy released in a given decay process $\Rightarrow$ this violates the Principle of Conservation of Energy, if these are the only two particles produced in a β-decay. Therefore, another particle, the anti-neutrino, must have been emitted and it carries away the rest of the energy produced.
(2) The kinetic energies of the β-particles emitted in a particular decay reaction are not fixed but range from zero to a maximum value $\Rightarrow$ the energy is shared between the β-particle and the anti-neutrino.



2. Conservation of Momentum:

(1) The tracks (formed in a Cloud Chamber) of the daughter nucleus and the β-particle alone do not form a straight line $\Rightarrow$ momentum is not conserved, if these are the only two particles formed after the decay. Hence this violates the Principle of Conservation of Momentum.

(2) Only by including the momentum of the invisible anti-neutrino can the Principle of Conservation of Momentum be satisfied.



Radioactive Decay Law

1. **Decay constant** λ is defined as the **probability per unit time** of the decay of a nucleus.
The **activity** A of a radioactive source is the **number of radioactive decays per unit time** or **rate of radioactive decay** in the source.
For a sample of N radioactive atoms, the rate of decay is proportional to N.

$$A = \lambda N \qquad (1)$$

$$\frac{dN}{dt} = -\lambda N \qquad (2)$$

$$N = N_o e^{-\lambda t} \qquad (3)$$

where N_o is the number of radioactive nuclei at time t = 0.

$$A = A_o e^{-\lambda t} \qquad (4)$$

Half-Life

1. The **half-life** $t_{1/2}$ of a radioactive nuclide is the time taken for the **number of undecayed nuclei** to be **reduced to half its original number.** $t_{1/2} = \dfrac{\ln 2}{\lambda}$

2. Since $\lambda = \dfrac{\ln 2}{T_{1/2}}$, we can show that $e^{-\lambda t} = \left(\dfrac{1}{2}\right)^{t/T_{1/2}}$. Equations (3) and (4) can be written as

$$N = N_o \left(\frac{1}{2}\right)^{t/t_{1/2}} \quad \text{and} \quad A = A_o \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

These formulae offer very quick ways to compute N and A compared to equations (3) and (4) because usually exam questions give $t_{1/2}$ rather than λ.

Effects of ionising radiation on living tissues and cells

1. Ionizing radiation can cause cancer and other health effects.
Ionizing radiation can break chemical bonds in atoms and molecules. Damage occurring at the cellular or molecular level can disrupt the processes that control the rate at which cells grow and replace themselves and permit the uncontrolled growth of cancer cells.
Acute health effects include burns and radiation sickness. Radiation sickness can cause premature aging or even death.