Hwa Chong Institution 华侨中学
Mathematics (SMTP)

Name: Wan An Hao (27) Class: 321 Date: 29th July

MA303.2.1 – Linear Conversion

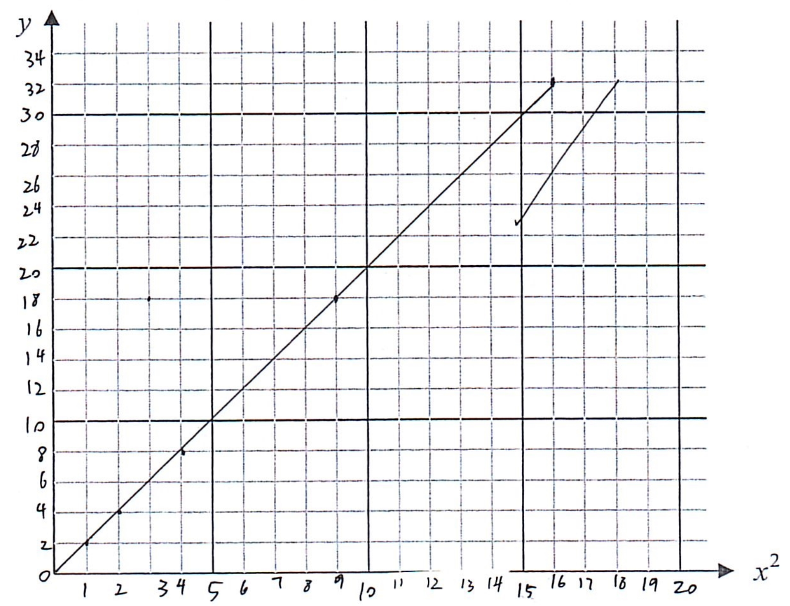
At the end of this unit, you should be able to

- **convert non-linear equations into linear form $Y = mX + c$**
- **solve unknown parameters using linear law**

E1. Fill in the table below given that $y = 2x^2$.

x	0	1	2	3	4
x^2	0	1	4	9	16
y	0	2	8	18	32

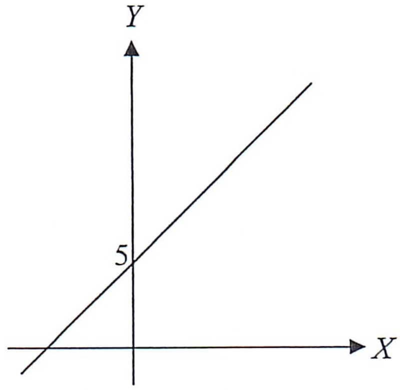
Draw a graph of y against x^2, plotting the points accurately. Describe the graph.



Let x and y be two variables related such that $F(x, y) = m \cdot G(x, y) + c$, where m and c are constants and $F(x, y)$, $G(x, y)$ being functions involving x and y. If the graph of $F(x, y)$ is drawn against $G(x, y)$, it will be a straight line of gradient m and a vertical intercept of value c.

E2. The equation $xy^2 = \frac{2}{x} + 5$ involving x and y, if drawn with xy^2 against $\frac{1}{x}$, is a straight line with gradient 2 and vertical intercept 5.

To see how, let $Y = xy^2$ and let $X = \frac{1}{x}$, the equation then becomes $xy^2 = 2\left(\frac{1}{x}\right) + 5 \Rightarrow Y = 2X + 5$. The latter equation is a straight line if drawn with Y against X, and the value of m is 2 and the value of c is 5.

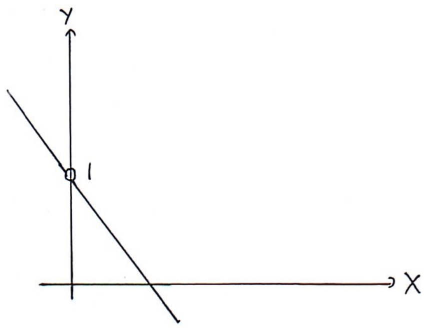


P1. For each of the following, express in the form $Y = mX + c$, where both X and Y are in terms of x and/or y, ensuring that m and c are constants.

(a) $y = x^2 - 2x^3$

$\frac{y}{x^2} = 1 - 2x$

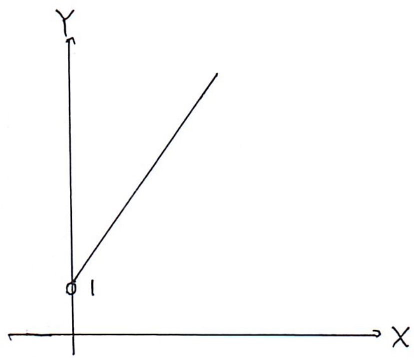
$Y = -2X + 1$, $Y = \frac{y}{x^2}$, $X = x$



(b) $y = 2x + \frac{1}{x}$

$xy = 2x^2 + 1$

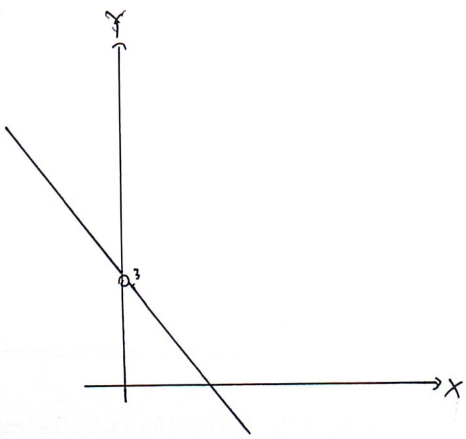
$Y = 2X + 1$, $Y = xy$, $X = x^2$



(c) $y^2 = 3\sqrt{x} - \frac{y}{x}$

$\frac{y^2}{\sqrt{x}} = -\frac{y}{x\sqrt{x}} + 3$

$Y = -X + 3$, $Y = \frac{y^2}{\sqrt{x}}$, $X = \frac{y}{x\sqrt{x}}$

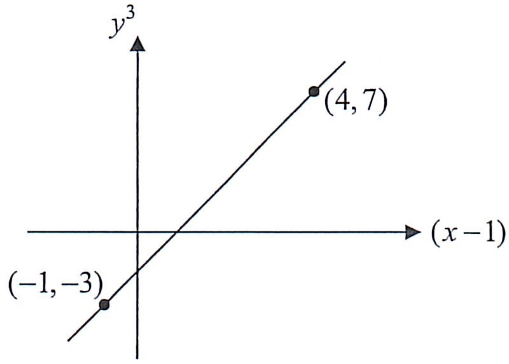


E3. A straight line graph of y^3 drawn against $(x-1)$ passes through the points $(-1,-3)$ and $(4,7)$.

(a) Find the gradient of the line.

$$\text{Gradient} = \frac{7-(-3)}{4-(-1)}$$

$$= 2$$



(b) By letting $Y = y^3$ and $X = x-1$, find the equation of straight line in terms of X and Y.

$$(Y-7) = 2(X-4)$$

$$Y = 2X - 1$$

(c) Hence, determine the equation relating x and y. Find the value of y when $x = 2$.

$$y^3 = 2(x-1) - 1$$

$$y = \sqrt[3]{2x-3}$$

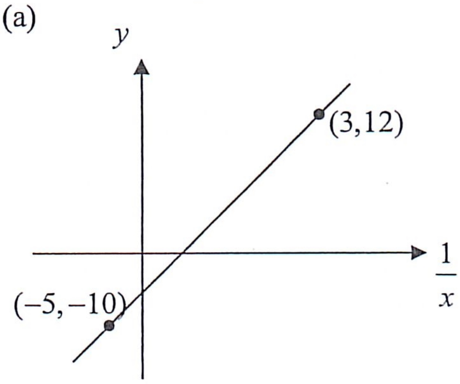
Sub $x=2$

$$y = \sqrt[3]{2(2)-3}$$

$$y = 1$$

P2. In the following straight-line graphs, find y in terms of x.

(a)



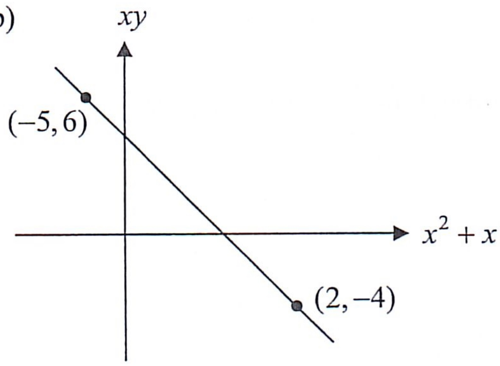
Gradient $= \frac{12-(-10)}{3-(-5)}$

$= \frac{11}{4}$

$y-12 = \frac{11}{4}\left(\frac{1}{x}-3\right)$

$y = \frac{11}{4x} + \frac{15}{4}$

(b)



Gradient $= \frac{6-(-4)}{-5-2}$

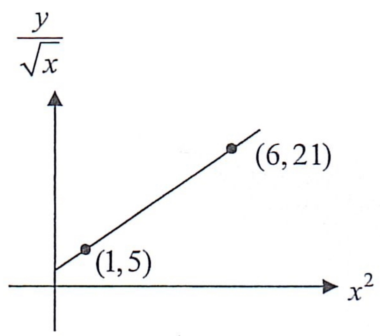
$= -\frac{10}{7}$

$xy-(-4) = -\frac{10}{7}(x^2+x-2)$

$xy = -\frac{10}{7}x^2 - \frac{10}{7}x - \frac{8}{7}$

$y = -\frac{10}{7}x - \frac{10}{7} - \frac{8}{7x}$

(c)



$$\text{Gradient} = \frac{21-5}{6-1}$$
$$= \frac{16}{5}$$
$$\frac{y}{\sqrt{x}} - 5 = \frac{16}{5}(x^2 - 1)$$
$$\frac{y}{\sqrt{x}} = \frac{16}{5}x^2 - \frac{16}{5} + 5$$
$$y = \frac{16}{5}x^2\sqrt{x} + \frac{9}{5}\sqrt{x}$$

P3. The variables x and y are related by the equation $y = \dfrac{a}{bx-1}$. When the graph of $\dfrac{1}{y}$ is drawn against x, it shows a straight line passing through $(5,3)$ and $(-1,-1)$. Find the values of a and b.

$$\frac{1}{y} = \left(\frac{b}{a}\right)x - \frac{1}{a}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-(-1)}{5-(-1)}$$
$$= \frac{2}{3}$$
$$Y = mX + c$$
$$Y = \frac{2}{3}X + c$$
$$3 = \frac{2}{3}(5) + c$$
$$3 = \frac{10}{3} + c$$
$$c = -\frac{1}{3}$$
$$Y = \frac{2}{3}x - \frac{1}{3}$$
$$\frac{1}{y} = \frac{2x-1}{3}$$
$$y = \frac{3}{2x-1}$$
$$\therefore a = 3$$
$$\therefore b = 2$$