



CANDIDATE
NAME

CG

MATHEMATICS

Paper 2

9758/02

31 August 2026

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your CG, index number and name on the work you hand in.

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

Section A: Pure Mathematics [40 marks]

1 Do not use a calculator in answering this question.

The complex number z is such that $z = 1 - \sqrt{3}i$.

- (a) Find $|z|$ and $\arg(z)$. [2]

The points A , B and C represent the complex numbers z , iz and $iz + 2$ respectively.

- (b) Mark the origin O and the points A , B and C on an Argand diagram, illustrating clearly the geometrical relationship between them. [3]
- (c) Hence, find the exact value of $\arg(iz + 2)$. [2]

- 2 (a) The non-zero vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are such that $\mathbf{a} \times \mathbf{c}$ is parallel to $\mathbf{a} \times \mathbf{b}$. Show that $\mathbf{c} = m\mathbf{a} + n\mathbf{b}$, where m and n are constants. [3]
- (b) Given that $\mathbf{a}$ is a non-zero vector and $\mathbf{b}$ is a unit vector such that $|\mathbf{a} \times \mathbf{b}| = 2$ and $|\mathbf{a} \cdot \mathbf{b}| = 2.5$, show that the acute angle between the vectors $\mathbf{a}$ and $\mathbf{b}$ is $\tan^{-1}\left(\frac{4}{5}\right)$ and find the exact value of $|\mathbf{a}|$. [4]

3 The function f is defined by

$$f: x \mapsto \frac{1}{(x-1)(x-2)}, \quad x \in \mathbb{R}, x \neq 1, x \neq 2.$$

- (a) Sketch the graph of $y = f(x)$, giving the equations of any asymptotes and the coordinates of any turning points. [2]
- (b) Hence state the range of f . [1]
- (c) Explain why the function f^2 does not exist. [1]

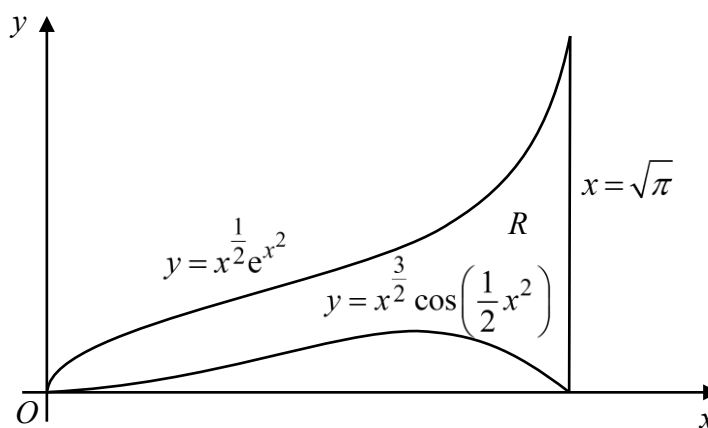
The domain of f is further restricted to $(-\infty, a]$ where a is a constant.

- (d) State the largest value of a such that the function f^{-1} exists. [1]
- (e) Hence find $f^{-1}(x)$ and state its domain. [4]

4 (a) Find $\int x e^{2x^2} dx$. [2]

(b) Use the substitution $w = x^2$ to find $\int x^3 \cos x^2 dx$. Hence, using double angle formulae, show that

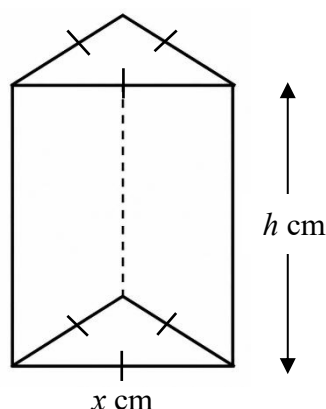
$$\int x^3 \cos^2\left(\frac{1}{2}x^2\right) dx = \frac{1}{8}\left(2x^2 \sin x^2 + 2\cos x^2 + x^4\right) + C$$
 [5]



The diagram shows part of the curves $y = x^{\frac{3}{2}} \cos\left(\frac{1}{2}x^2\right)$ and $y = x^{\frac{1}{2}} e^{x^2}$, where $0 \leq x \leq \sqrt{\pi}$. The region R is bounded by the curves and the line $x = \sqrt{\pi}$.

(c) Find the exact volume of the solid generated when R is rotated through 2π radians about the x -axis. [3]

5



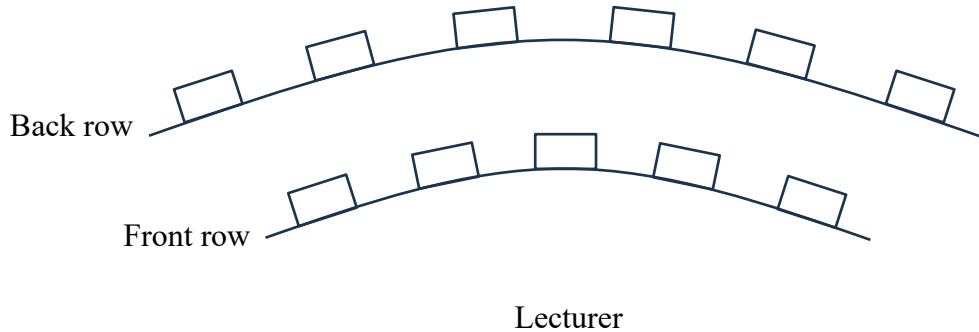
The diagram shows an open container made in the form of an upright prism with an equilateral triangular base of side x cm, and height h cm. The material used for the base costs 2 cents per cm^2 , while the material used for the sides costs 5 cents per cm^2 . The total cost of the container is a fixed value of C cents.

(a) Show that the capacity, $V \text{ cm}^3$ of the container is given by $\frac{\sqrt{3}}{60} x \left(C - \frac{\sqrt{3}}{2} x^2 \right)$. [3]

(b) Use differentiation to show that V is maximum when the cost of the base is $\frac{C}{3}$ cents. [4]

Section B: Probability and Statistics [60 marks]

- 6** A mini-Lecture Theatre has 11 seats in 2 rows as shown in the diagram below.



Find the number of ways that a class of 5 girls and 3 boys can be seated

- (a) without restriction, [1]
 (b) with the girls and the boys occupying different rows. [2]

The lecturer has prepared 8 different questions and will ask each student in the class to answer one question in turn.

- (c) Find the number of ways this can be done if no two consecutive questions are answered by boys. [2]

The class needs to form two groups of 4 each for a group activity.

- (d) Find the number of ways that the groups can be formed if the 3 boys cannot be in the same group. [2]

- 7** A technician has a set of 5 cables, 3 of which are faulty. He tests the cables one by one, without replacement, in a random order until all the faulty cables are tested. The random variable X denotes the number of tests carried out until all the faulty cables are tested.

- (a) Show that $P(X = 3) = \frac{1}{10}$ and find the probability distribution of X . [3]
 (b) The technician noted that the first cable tested was faulty. Find the probability that exactly 4 tests were required. [2]

The testing process incurs a cost, $\$C$, which depends on the number of tests carried out and is given by the formula $C = 5 + 10X$.

- (c) Find $\text{Var}(C)$. [2]
 (d) The testing process is carried out on 80 sets of cables independently. Using a suitable approximation, find the probability that the mean testing cost exceeds $\$51.50$. [3]

- 8 The number of active users of a new mobile application, y thousand users, after x number of months since its launch, is given in the following table.

x	1	2	4	6	8	10
y	85	55	38	31	27	25

- (a) Sketch a scatter diagram of the data. [1]
- (b) In the context of the question, comment whether a linear model would be appropriate. [1]
- (c) Suppose that the relationship between x and y is modelled by an equation of the form $y = \frac{a}{x} + b$, where a and b are constants. Using the scatter diagram, explain whether a is positive or negative. [1]
- (d) Find the product moment correlation coefficient and the equation of the regression line for the model in part (c). [2]
- (e) Use your equation to estimate the number of active users of the new mobile application 9 months after the launch. Give two reasons why you would expect this estimate to be reliable. [3]
- 9 A recycling centre collects plastic bottles from households. It is estimated that, on average, 6% of the plastic bottles collected are non-recyclable. Each day, the recycling centre randomly selects 40 plastic bottles for checking and the number of non-recyclable plastic bottles found is the random variable X .

- (a) State, in the context of the question, two assumptions needed to model X by a binomial distribution. [2]

You are now given that X can be modelled by a binomial distribution.

- (b) Find the probability that, on a randomly chosen day, at least 4 non-recyclable plastic bottles are found. [2]
- (c) Find the probability that, in a randomly chosen week of 5 days, at least 4 non-recyclable plastic bottles are found on no more than 2 days. [2]

The recycling centre also collects cardboard boxes. The number of non-recyclable cardboard boxes also follows a binomial distribution. The probability that a randomly selected cardboard box is non-recyclable is p . The recyclability of plastic bottles is independent of the recyclability of cardboard boxes.

A quality control officer randomly selects a sample of 3 plastic bottles and 2 cardboard boxes for inspection. The probability that the sample contains at most 1 non-recyclable item is 0.9.

- (d) Write down an equation satisfied by p . Hence, find the value of p . [4]

- 10** A company produces three electrical components, Type *A*, Type *B* and Type *C*. The masses, in grams, of Type *A* components follow the distribution $N(250, 3^2)$ and the masses, in grams, of Type *B* components follow the distribution $N(240, 4^2)$. The masses of the electrical components are independent of each other.

- (a) Find the probability that the total mass of a randomly chosen Type *A* component and a randomly chosen Type *B* component exceeds 480 grams. [3]
- (b) Find the probability that the total mass of 2 Type *A* components is less than three times the mass of a Type *B* component. [4]

A worker selects 10 Type *A* components at random.

- (c) Find the probability that at least 25% of these components have masses between 250 grams and 255 grams. [3]

The masses, in grams, of Type *C* components are normally distributed such that 80% of the components have masses within ± 7 grams of the mean and 35% have masses less than 312 grams.

- (d) Find the mean, μ , and the standard deviation, σ , of the masses of Type *C* components. [3]

- 11** A food manufacturer uses an automated machine to fill sachets with chilli sauce. The machine is designed to dispense a mean mass of 50 grams of chilli sauce into each sachet. The food manufacturer wishes to test whether the mean mass of chilli sauce dispensed into each sachet is more than 50 grams.

The masses, x grams, of a random sample of 40 sachets of chilli sauce are summarised as follows.

$$\sum(x - 50) = 80.5 \quad \sum(x - 50)^2 = 2415$$

- (a) Calculate unbiased estimates of the population mean and variance of the masses of chilli sauce in the sachets. [2]
- (b) Test, at the 5% level of significance, whether the mean mass of chilli sauce dispensed into each sachet is more than 50 grams. You should state your hypotheses and define any symbols that you use. [5]
- (c) Explain why there is no need for the food manufacturer to know anything about the population distribution of the masses of chilli sauce in the sachets. [1]

The machine is subsequently adjusted, and the mass of chilli sauce in each sachet now follows a normal distribution with population standard deviation of 6 grams. The food manufacturer wishes to test whether the mean mass of chilli sauce in each sachet differs from 50 grams. A second random sample of n sachets of chilli sauce is taken, and its mean is found to be 51.7 grams.

- (d) Find the set of values that n can take if there is sufficient evidence at the 10% level of significance that the mean mass of chilli sauce in each packet differs from 50 grams. [4]