1. The remainder when $x^3 + kx^2 + 8$ is divided by $(x-3)$ is 12 more than the remainder when it is divided by $(x+1)$. Find the value of k. [3]

2. Solve the equation $2x^3 + 2x^2 - 11x + 3 = 0$, giving your answers in exact form. [5]

3. Given that $x^4 - 2x^2 + x - 1 \equiv (x^2 + A)(x+1)(x-1) + Bx + C$ for all values of x, find the value of A, of B and of C. [5]

4. If $\log_8 x = a$ and $\log_2 y = b$, express xy and $\dfrac{x}{y}$ in terms of a and b. Given that $xy = 128$ and $\dfrac{x}{y} = 32$, find the value of a and of b. [5]

5. Solve the following equations:

   (a) $3^{2-x} = 8^{2x+3}$ [4]

   (b) $\log_5 x - 6\log_x 5 = 1$ [5]

6. The mass, m grams, of a radioactive substance present at a time t weeks after first being observed is given by the formula $m = ae^{bt}$ where a and b are constants. The initial mass of the substance is 120 g.

   (i) If $t = \ln 32$ when the mass is half of its initial value, find the value of a and of b. [4]

   (ii) Hence, determine the amount of radioactive substance after 20 weeks. [2]

7. The variables x and y are related in such a way that when xy is plotted against $\dfrac{x}{y}$, a straight line graph is obtained. The line passes through the points $A(-2, 5)$ and $B(4, -1)$. Form an equation in x and y and show that it reduces to $xy^2 = 3y - x$. [3]

8. Express the equation $y = ab^x + 2$ in the form $\mathrm{Y} = m\mathrm{X} + C$ for drawing a straight line graph. State the variables to be plotted on each axis and explain how the constants a and b can be obtained in terms of m and C. [4]

End of Paper

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1. $f(x) = x^3 + kx^2 + 8$

$x-3 \quad f(3) = 3^3 + 9k + 8$

$= 9k + 35$

$x+1 \quad f(-1) = (-1)^3 + k + 8$

$= k + 7$

$9k + 35 = k + 7 + 12$

$9k + 35 = k + 19$

$8k = -16$

$k = -2$

3

$\frac{37.1}{40} = 93\%$

90%

2. $2x^3 + 2x^2 - 11x + 3 = 0$

try $x+3$

$x+3 \quad f(-3) = 2(-3)^3 + 2(-3)^2 - 11(-3) + 3$

$= 0$

$x+3$ is a factor

$2x^3 + 2x^2 - 11x + 3 \equiv (x+3)(2x^2 - 4x + 1)$

$x = -3$ or $x = \frac{4 \pm \sqrt{16-8}}{4}$

$= \frac{4 \pm \sqrt{8}}{4}$

$= \frac{4 \pm 2\sqrt{2}}{4}$

$= \frac{2 \pm \sqrt{2}}{2}$

3

3. $x^4 - 2x^2 + x - 1 = (x^2 + A)(x+1)(x-1) + Bx + C$

$= (x^2 + A)(x^2 - 1) + Bx + C$

$= x^4 + (A-1)x^2 + Bx + C - A$

$A = -1$, $B = 1$, $C = -2$

3

4. $\log_8 x = a$

$\frac{\log_2 x}{\log_2 8} = a$

$\log_2 x = 3a$

$x = 2^{3a}$

$\log_2 y = b$

$y = 2^b$

$\therefore xy = 2^{3a} \times 2^b$

$= 2^{3a+b}$

$\therefore \frac{x}{y} = \frac{2^{3a}}{2^b}$

$= 2^{3a-b}$

$2^{3a+b} = 128$

$2^{3a+b} = 2^7$

$3a+b = 7$

$2^{3a-b} = 32$

$2^{3a-b} = 2^5$

$3a-b = 5$

$2b = 2$

$\therefore b = 1$

$\therefore a = 2$

5

5. (a) $3^{2-x} = 8^{2x+3}$

$2\lg 3 - x\lg 3 = 2x \lg 8 + 3\lg 8$

$2x \lg 8 + x \lg 3 = 2\lg 3 - 3\lg 8$

$x = \frac{2\lg 3 - 3\lg 8}{2\lg 8 + \lg 3}$

$= -0.769$ (3 s.f.)

(b) $\log_5 x - 6\log_x 5 = 1$

$\log_5 x - \frac{6\log_5 5}{\log_5 x} = 1$

Let $\log_5 x$ be y

$y - \frac{6}{y} = 1$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$\log_5 x = 3$ or $\log_5 x = -2$

$x = 125$ $x = \frac{1}{25}$

9

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6. (i) $m = ae^{bt}$

$120 = ae^{b(0)}$

$= a$

$60 = 120e^{b(\ln 32)}$

$\frac{1}{2} = e^{b(\ln 32)}$

$\frac{1}{2} = 32^b$

$2^{-1} = 2^{5b}$

$5b = -1$

$b = -\frac{1}{5}$

$\therefore a = 120,\ b = -\frac{1}{5}$

(ii) $m = 120e^{-\frac{1}{5}(20)}$

$= 120e^{-4}$

$= 2.20g$ (3s.f.)

6

7. $m = \frac{5-(-1)}{-2-4}$

$= -1$

$-1 = -1(4) + c$

$c = 3$

$xy = -1\left(\frac{x}{y}\right) + 3$

$xy = -\frac{x}{y} + 3$

$xy^2 = 3y - x$

3

8. $y = ab^x + 2$

$\lg y = \lg a + x \lg b + \lg 100$

$\lg y = \lg b(x) + \lg 100a$

$\therefore$ Plot $\lg y$ on the vertical or Y-axis, plot x on the horizontal or X-axis

$\lg b = m$

$\therefore b = 10^m$

$\lg 100a = c$

0

$\lg 100 + \lg a = c$

$\lg a = c-2$

$\therefore a = 10^{c-2}$

$y = ab^x + 2$

Take lg on both sides

$\lg(y-2) = \lg(ab^x)$

$\lg(y-2) = \lg a + \lg b^x$

$\lg(y-2) = \lg a + x \lg b$

Plot $\lg(y-2)$ against x, to obtain a straight-line graph.

then

$\lg b = m$

$b = 10^m$

$\lg a = c$

$a = 10^c$