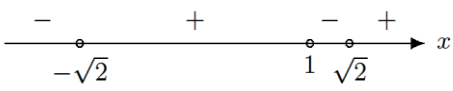
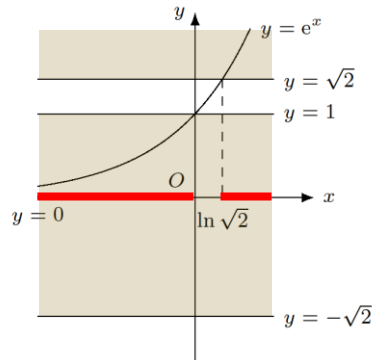


	Solution
1(a)	$x+1 > \frac{1}{x-1}$ $x+1 - \frac{1}{x-1} > 0$ $\frac{(x+1)(x-1)-1}{x-1} > 0$ $\frac{(x+\sqrt{2})(x-\sqrt{2})}{x-1} > 0$  $-\sqrt{2} < x < 1 \text{ or } x > \sqrt{2}$
(b)	 $x < 0 \text{ or } x > \ln \sqrt{2} = \frac{1}{2} \ln 2$

Solution

2 Perimeter of outline = 180

$$3a + 2a + 2b + \frac{1}{2}\pi(a) = 180$$

$$\left(5 + \frac{\pi}{2}\right)a + 2b = 180$$

$$\Rightarrow b = 90 - \left(\frac{5}{2} + \frac{\pi}{4}\right)a$$

Area enclosed by wire,

$$\begin{aligned} A &= 3ab + \frac{1}{2}\pi\left(\frac{a}{2}\right)^2 \\ &= 3a\left[90 - \left(\frac{5}{2} + \frac{\pi}{4}\right)a\right] + \frac{\pi}{8}a^2 \\ &= 270a - \frac{5\pi}{8}a^2 - \frac{15}{2}a^2 \\ &= 270a - \left(\frac{5\pi}{8} + \frac{15}{2}\right)a^2 \end{aligned}$$

$$\frac{dA}{da} = 270 - \left(\frac{5\pi}{4} + 15\right)a$$

When $\frac{dA}{da} = 0$,

$$a = \frac{270}{\frac{5\pi}{4} + 15} \quad \text{or } 14.265$$

$$\frac{d^2A}{da^2} = -\left(\frac{5\pi}{4} + 15\right) < 0$$

Hence the area is maximum when $a = \frac{270}{\frac{5\pi}{4} + 15}$.

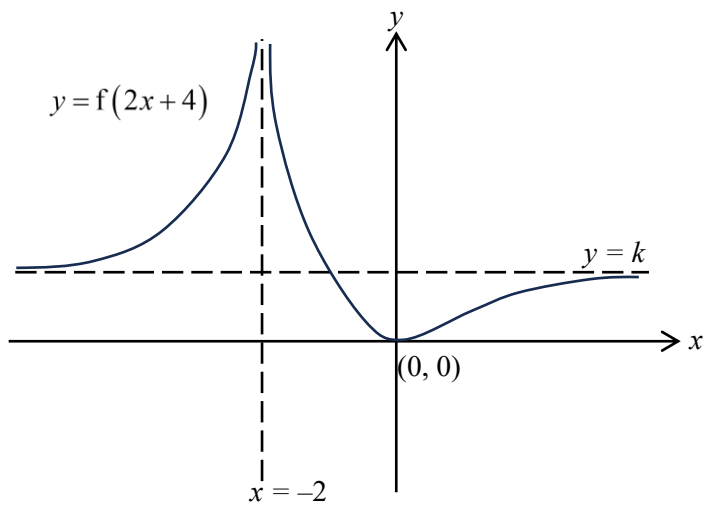
Maximum area

$$\begin{aligned} &= 270\left(\frac{270}{\frac{5\pi}{4} + 15}\right) - \left(\frac{5\pi}{8} + \frac{15}{2}\right)\left(\frac{270}{\frac{5\pi}{4} + 15}\right)^2 \\ &= 1925.82 \text{ cm}^2 \end{aligned}$$

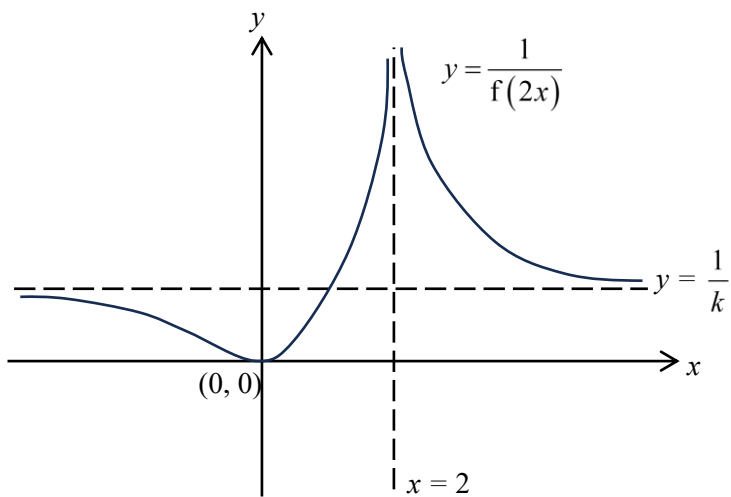
	Solution
3(a)	$AC = \sqrt{13^2 - 5^2} = 12$ $\cos \angle BAC = \frac{AC}{AX} = \frac{12}{13}$
(b)	$AX = \frac{12}{\cos(\angle BAC - \alpha)}$ $= \frac{12}{\cos \angle BAC \cos \alpha + \sin \angle BAC \sin \alpha}$ $= \frac{12}{\frac{12}{13} \cos \alpha + \frac{5}{13} \sin \alpha}$ $= \frac{156}{12 \cos \alpha + 5 \sin \alpha} \quad (\text{shown})$
(c)	Using small angle approximation, $AX \approx \frac{156}{12 \left(1 - \frac{\alpha^2}{2} \right) + 5\alpha}$ $= 156 \left[12 \left(1 + \frac{5}{12} \alpha - \frac{\alpha^2}{2} \right) \right]^{-1}$ $= 156 [12]^{-1} \left[1 + \left(\frac{5}{12} \alpha - \frac{\alpha^2}{2} \right) \right]^{-1}$ $= 13 \left[1 - \left(\frac{5}{12} \alpha - \frac{\alpha^2}{2} \right) + \left(\frac{5}{12} \alpha - \frac{\alpha^2}{2} \right)^2 + \dots \right]$ $= 13 \left[1 - \frac{5}{12} \alpha + \frac{\alpha^2}{2} + \left(\frac{5}{12} \alpha \right)^2 + \dots \right]$ $\approx 13 - \frac{65}{12} \alpha + \frac{1261}{144} \alpha^2$ $p = -\frac{65}{12}, \quad q = \frac{1261}{144} \quad (\text{shown})$

Solution

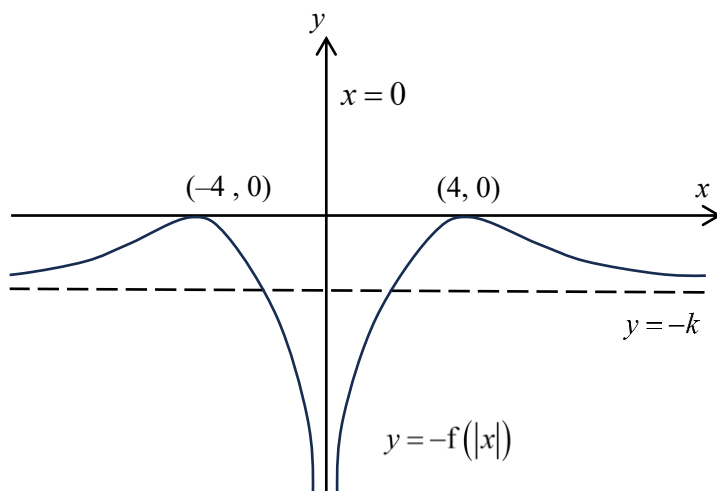
- 4
(a) Translation of 2 units in the negative x -direction.



- (b)



- (c)



Solution**5(a)**

$$S_2 = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{7}{12}$$

Area under curve between $x=1$ and $x=2$ is $\int_1^2 \frac{1}{x} dx = [\ln x]_1^2 = \ln 2$

Since the two rectangles lie under the curve, the area of the two rectangles must be less than the area under curve, so

$$\frac{7}{12} < \ln 2$$

(b)

We can use any integer larger than 2 for n .

Some possible answers are:

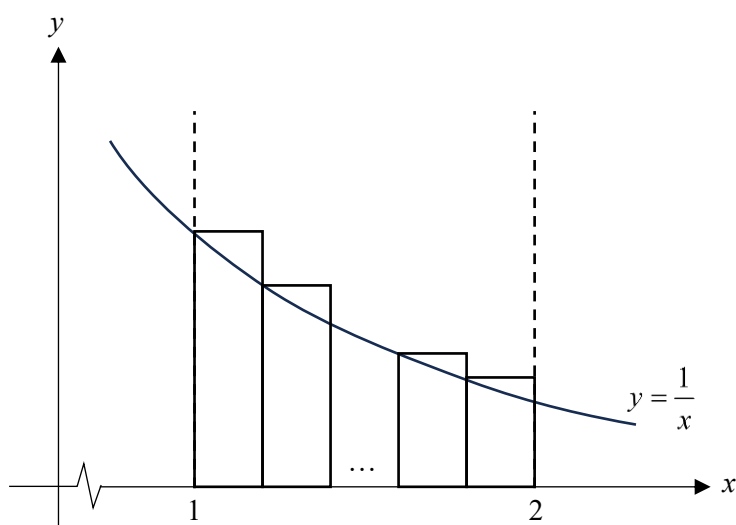
n	S_n
3	$\frac{37}{60}$
4	$\frac{533}{840}$
5	$\frac{1627}{2520}$
6	$\frac{15797}{27720}$

(c)

$$S_n = \left(\frac{1}{n}\right)\left(\frac{1}{\left(\frac{n+1}{n}\right)}\right) + \left(\frac{1}{n}\right)\left(\frac{1}{\left(\frac{n+2}{n}\right)}\right) + \cdots + \left(\frac{1}{n}\right)\left(\frac{1}{\left(\frac{2n}{n}\right)}\right)$$

$$= \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n-1} + \frac{1}{2n}$$

$$= \sum_{r=1}^n \frac{1}{n+r}$$

(d)

Consider n rectangles of equal width in the same interval, drawn above the curve.

Then

$\ln 2 < \text{Area of rectangles}$

$$= \left(\frac{1}{n}\right)\left(\frac{1}{\frac{n}{n}}\right) + \left(\frac{1}{n}\right)\left(\frac{1}{\frac{n+1}{n}}\right) + \dots + \left(\frac{1}{n}\right)\left(\frac{1}{\frac{2n-1}{n}}\right)$$

$$= \frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{2n-1}$$

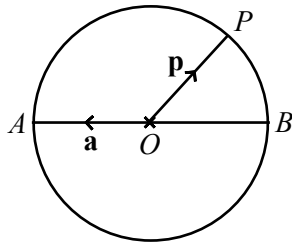
$$= \frac{1}{n} + \left(S_n - \frac{1}{2n}\right)$$

$$= S_n + \frac{1}{n} - \frac{1}{2n}$$

$$= S_n + \frac{1}{2n}$$

Solution

6(a)

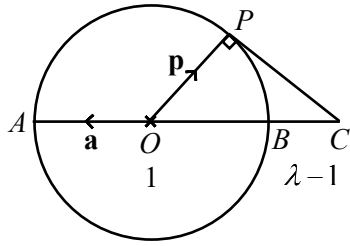


$$\overrightarrow{AP} = \mathbf{p} - \mathbf{a}$$

$$\overrightarrow{BP} = \mathbf{p} + \mathbf{a}$$

$$\begin{aligned}\overrightarrow{AP} \cdot \overrightarrow{BP} &= (\mathbf{p} - \mathbf{a}) \cdot (\mathbf{p} + \mathbf{a}) \\ &= |\mathbf{p}|^2 - |\mathbf{a}|^2 \\ &= |\mathbf{a}|^2 - |\mathbf{a}|^2 \quad (\text{since } OA \text{ and } OP \text{ are radius, } \therefore |\mathbf{p}| = |\mathbf{a}|) \\ &= 0\end{aligned}$$

(b)



By Ratio Theorem,

$$\mathbf{b} = \frac{(\lambda - 1)\mathbf{a} + \mathbf{c}}{\lambda}$$

$$\mathbf{c} = \lambda \mathbf{b} - (\lambda - 1)\mathbf{a}$$

$$\begin{aligned}\mathbf{c} &= \lambda(-\mathbf{a}) - (\lambda - 1)\mathbf{a} \\ &= (1 - 2\lambda)\mathbf{a}\end{aligned}$$

(c)

$$\overrightarrow{PC} = (1 - 2\lambda)\mathbf{a} - \mathbf{p}$$

$$\overrightarrow{OP} \cdot \overrightarrow{PC} = 0$$

$$\mathbf{p} \cdot ((1 - 2\lambda)\mathbf{a} - \mathbf{p}) = 0$$

$$(1 - 2\lambda)\mathbf{p} \cdot \mathbf{a} - \mathbf{p} \cdot \mathbf{p} = 0$$

$$(1 - 2\lambda)|\mathbf{p}||\mathbf{a}|\cos 120^\circ - |\mathbf{p}|^2 = 0$$

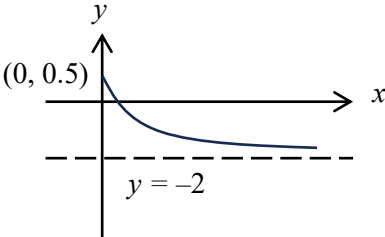
$$-\frac{1}{2}(1 - 2\lambda)|\mathbf{a}|^2 - |\mathbf{a}|^2 = 0$$

$$-\frac{1}{2}(1 - 2\lambda) = 1$$

$$\lambda = \frac{3}{2}$$

	Solution
7(a)	$u_2 = \frac{u_1 + u_3}{2} = \frac{9 + b}{2}$
(b)	$v_2 = \pm\sqrt{9b} = \pm 3\sqrt{b}$
(c)	$u_2 - v_2 = 8$ $v_2 = u_2 - 8$ $v_2^2 = (u_2 - 8)^2$ $9b = \frac{b^2 - 14b + 49}{4}$ $b^2 - 50b + 49 = 0$ $(b - 1)(b - 49) = 0$ $b = 1 \text{ or } b = 49$ If $b = 49$, $r = \sqrt{\frac{49}{9}} = \frac{7}{3}$ but we need $r < 1$ since the sequence is convergent. So $b = 1$. $u_2 = \frac{9 + 1}{2} = 5$ $v_2 = 5 - 8 = -3$

	Solution
8(a)	$(a + bi)^2 = 2i$ $a^2 - b^2 + 2abi = 2i$ $a^2 - b^2 = 0 \quad (1)$ $2ab = 2 \quad (2)$ From (2), $a = \frac{1}{b}$ $\left(\frac{1}{b}\right)^2 - b^2 = 0$ $1 - b^4 = 0$ $(1 + b^2)(1 - b^2) = 0$ $b^2 = 1 \quad (\because b \text{ is real})$ So $b = 1, a = 1$ or $b = -1, a = -1$ (reject since $0 \leq \arg(w) \leq \frac{\pi}{2}$) $w = 1 + i$
(b)	Since $1 + i$ is a root, we can write $z^3 - z^2 + (1 - i)z + s = [z - (1 + i)](z^2 + Bz + C)$ for some (possibly complex) constants B and C. Comparing coefficients, $z^2 \quad : -1 = -1 - i + B \quad \Rightarrow \quad B = i$ $z \quad : 1 - i = -B - Bi + C \quad \Rightarrow \quad C = 0$ $\text{constant: } s = -(1 + i)C \quad \Rightarrow \quad s = 0$ $z^2 + iz = 0$ $z(z + i) = 0$ $z = 0 \text{ or } z = -i$
(c)	Replace z with iv: $(iv)^3 - (iv)^2 + (1 - i)(iv) + s = 0$ $-iv^3 + v^2 + (1 + i)v + s = 0$ Hence, $z = iv \Rightarrow v = -iz$ $v = -i(1 + i) = 1 - i,$ $v = -i(0) = 0 \text{ or}$ $v = -i(-i) = -1$

	Solution
9(ai)	From GC, coordinates are $(-1, 10)$ and $(3, -22)$
(aii)	Largest possible value is $a = -1$ Domain of f^{-1} is equal to range of f , which is $(-\infty, 10]$
(bi)	$g^2(x) = \frac{1 - 2\left(\frac{1-2x}{x+2}\right)}{\left(\frac{1-2x}{x+2}\right) + 2}$ $= \frac{x+2 - 2(1-2x)}{1-2x + 2(x+2)}$ $= \frac{5x}{5}$ $= x$
(bii)	$g^n(x) = \begin{cases} x & \text{if } n \text{ is even} \\ \frac{1-2x}{x+2} & \text{if } n \text{ is odd} \end{cases}$
(ci)	$R_h = [0, \infty)$ $D_g = \mathbb{R} \setminus \{-2\}$ OR $D_g = (-\infty, -2) \cup (-2, \infty)$ Since $R_h \subseteq D_g$, then gh exists.
(cii)	From graph of $y = g(x)$ with domain restricted to $[0, \infty)$  $R_{gh} = \left(-2, \frac{1}{2}\right]$

	Solution
10(a)	$\int_0^4 e^{ x-1 } dx$ $= \int_0^1 e^{1-x} dx + \int_1^4 e^{x-1} dx$ $= [-e^{1-x}]_0^1 + [e^{x-1}]_1^4$ $= -1 + e + e^3 - 1$ $= e^3 + e - 2$
(b)	$\int \sin^2 3x + \tan^2 3x dx$ $= \int \frac{1 - \cos 6x}{2} + \sec^2 3x - 1 dx$ $= \int -\frac{1}{2} \cos 6x + \sec^2 3x - \frac{1}{2} dx$ $= -\frac{1}{12} \sin 6x + \frac{1}{3} \tan 3x - \frac{1}{2} x + C$
(c)	Since $\frac{dx}{du} = 6u^5$, $\int \frac{1}{\sqrt{x} + \sqrt[3]{x}} dx$ $= \int \frac{1}{u^3 + u^2} \cdot 6u^5 du$ $= 6 \int \frac{u^3}{u+1} du$ $= 6 \int u^2 - u + 1 - \frac{1}{u+1} du$ $= 6 \left[\frac{1}{3} u^3 - \frac{1}{2} u^2 + u - \ln(u+1) \right] + C$ $= 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln(\sqrt[6]{x} + 1) + C$

	Solution
11(a)	$\frac{dy}{dt} = -k\sqrt{y}$ $\frac{1}{\sqrt{y}} \frac{dy}{dt} = -k$ $\int \frac{1}{\sqrt{y}} dy = \int -k dt$ $2\sqrt{y} = -kt + C$ When $t = 0$, $y = h$: $C = 2\sqrt{h} + k(0) = 2\sqrt{h}$ $2\sqrt{y} = -kt + 2\sqrt{h}$ $\sqrt{y} = \sqrt{h} - \frac{kt}{2}$
(b)	When $y = 0$, $\sqrt{h} - \frac{kt}{2} = 0$ $\sqrt{h} = \frac{kt}{2}$ $t = \frac{2\sqrt{h}}{k}$
(c)	Since $\cos t$ varies between -1 and 1, $\frac{1 + \cos t}{2} \text{ varies between } 0 \text{ and } 1.$ So the flow is now slower.
(d)	$\frac{dy}{dt} = -k\left(\frac{1 + \cos t}{2}\right)\sqrt{y}$ $\frac{1}{\sqrt{y}} \frac{dy}{dt} = -\frac{k(1 + \cos t)}{2}$ $\int \frac{1}{\sqrt{y}} dy = -\frac{k}{2} \int 1 + \cos t dt$ $2\sqrt{y} = -\frac{k}{2}(t + \sin t) + D$ When $t = 0$, $y = h$: $D = 2\sqrt{h} + \frac{k}{2}(0) = 2\sqrt{h}$ $2\sqrt{y} = -\frac{k}{2}(t + \sin t) + 2\sqrt{h}$ $\sqrt{y} = \sqrt{h} - \frac{kt + k \sin t}{4}$

(e)	When $h = 10$ and $k = 0.1$, time taken in the original model is 63.25 (seconds). For the revised model, from GC, time taken is 126.08 (seconds). $\frac{126.08...}{63.25...} = 1.99$ so the ratio is 1.99.
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