RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Term 2 Week 1 Revision
Topic: Permutations and Combinations and Probability

1 JPJC Prelim 9758/2020/02/Q6

The eleven letters in the word CORONAVIRUS are rearranged to form ‘words’ which may not make sense. Find the probability that

(i) the two ‘R’s are together; [3]

(ii) the vowels (O, A, I, U) are separated; [3]

(iii) either the first letter is a ‘C’ or the last letter is an ‘S’ or both. [3]

Solution	
(i)	No. of ways where the two ‘R’s are together = $\frac{10!}{2!} = 1814400$
	Required probability = $\frac{1814400}{\frac{11!}{2!2!}} = \frac{1814400}{9979200} = \frac{2}{11}$ or 0.182
(ii)	No. of ways to arrange the consonants = $\frac{6!}{2!} = 360$
	No. of ways to slot in the vowels = ${}^7C_5 \times \frac{5!}{2!} = 1260$
	Required probability = $\frac{360 \times 1260}{\frac{11!}{2!2!}} = \frac{453600}{9979200} = \frac{1}{22}$ (or 0.0455)
(iii)	No of ways that the first letter is a C or last letter is an S = $\frac{10!}{2!2!} \times 2 = 1814400$
	No of ways that the first letter is a C and last letter is an S = $\frac{9!}{2!2!} = 90720$
	Required probability $= \frac{1814400 - 90720}{\frac{11!}{2!2!}} = \frac{1723680}{9979200} = \frac{19}{110}$ (or 0.173)

2 CJC Prelim 9758/2020/02/Q6

A bag contains 4 red counters and 6 blue counters. 4 counters are drawn from the bag at random, without replacement.

(a) Calculate the probability that

(i) all the counters drawn are blue, **[1]**

(ii) at least 3 blue counters are drawn, **[2]**

(iii) at least 1 counter of each colour is drawn, **[2]**

(iv) at least 3 blue counters are drawn, given that at least 1 of each colour is drawn. **[2]**

(b) State with a reason whether or not the events "at least 3 blue counters are drawn" and "at least 1 counter of each colour is drawn" are independent. **[1]**

Solution

(a)(i) P (all counters blue) $= \dfrac{{}^{6}C_4}{{}^{10}C_4} = \dfrac{1}{14}$ or $\dfrac{6}{10}\times\dfrac{5}{9}\times\dfrac{4}{8}\times\dfrac{3}{7}$

(a)(ii) <u>Case 1</u> : P(3 blue,1 red) $= \dfrac{{}^{6}C_3 \times {}^{4}C_1}{{}^{10}C_4} = \dfrac{8}{21}$ or $\dfrac{6}{10}\times\dfrac{5}{9}\times\dfrac{4}{8}\times\dfrac{4}{7}\times\dfrac{4!}{3!}$

<u>Case 2</u> : P (all blue) $= \dfrac{1}{14}$

P(at least 3 blue) = P(3 blue, 1 red) + P(all blue) $= \dfrac{8}{21} + \dfrac{1}{14} = \dfrac{19}{42}$

(a)(iii) P (all counters red) $= \dfrac{{}^{4}C_4}{{}^{10}C_4} = \dfrac{1}{210}$ or $\dfrac{4}{10}\times\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}$

P(at least 1 counter of each colour is drawn)
= 1 – P(all blue) – P(all red)
$= 1 - \dfrac{1}{14} - \dfrac{1}{210} = \dfrac{97}{105}$

(a)(iv) P(at least 3 blue | at least 1 of each colour)

$$= \frac{\text{P(at least 3 blue} \cap \text{at least 1 of each colour)}}{\text{P(at least 1 of each colour)}}$$

$$= \frac{\text{P(3 blue,1 red)}}{\text{P(at least 1 of each colour)}}$$

$$= \frac{\frac{8}{21}}{\frac{97}{105}} = \frac{40}{97}$$

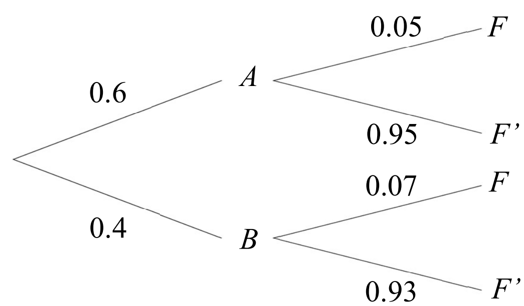
(b) Since P(at least 3 blue | at least 1 of each colour) $= \dfrac{40}{97} \neq \dfrac{19}{42} =$ P (at least 3 blue), the two events are **not** independent.

3 9740/2011/02/Q9

Camera lenses are made by two companies, A and B. 60% of all lenses are made by A and the remaining 40% by B. 5% of the lenses made by A are faulty. 7% of the lenses made by B are faulty.

(i) One lens is selected at random. Find the probability that
(a) it is faulty, **[2]**
(b) it was made by A, given that it is faulty. **[1]**

(ii) Two lenses are selected at random. Find the probability that
(a) exactly one of them is faulty, **[2]**
(b) both were made by A, given that exactly one is faulty. **[3]**

Solution		
(i)	**(a)**	Let A denote the event that "the selected lens is made by A". Let B denote the event that "the selected lens is made by B". Let F denote the event that "the selected lens faulty". Let F' denote the event that "the selected lens is not faulty".  $\mathrm{P}\left(F\right)=0.6\times0.05+0.4\times0.07=0.058$
	(b)	$\mathrm{P}(A\mid F)=\dfrac{\mathrm{P}(A\cap F)}{P(F)}=\dfrac{0.05\times0.6}{0.058}\approx0.517$
(ii)	**(a)**	$\mathrm{P}\left(\text{exactly one is faulty}\right)=0.058\times0.942\times2=0.109272$ Note: This is an exact answer.
	(b)	$\mathrm{P}\left(\text{both are made by } A \mid \text{exactly one is faulty}\right)$ $=\dfrac{\mathrm{P}\left(\text{both are made by } A\cap\text{exactly one is faulty}\right)}{\mathrm{P}\left(\text{exactly one is faulty}\right)}$ $=\dfrac{\mathrm{P}\left(A\cap F\right)\times\mathrm{P}\left(A\cap F'\right)\times2!}{0.109272}$ $=\dfrac{0.6\times0.05\times0.6\times0.95\times2!}{0.109272}$ ≈0.313

4 RI Prelim 9758/2020/02/Q6

For events X and Y, it is given that $\mathrm{P}(X \cap Y') = \frac{1}{2}$, $\mathrm{P}(X \cup Y') = \frac{3}{4}$ and $\mathrm{P}(X | Y') = \frac{50}{63}$.

Find

(i) $\mathrm{P}(Y')$, **[2]**

(ii) $\mathrm{P}(X)$, **[2]**

(iii) $\mathrm{P}(X \cap Y)$ and state with a reason whether X and Y are independent events. **[3]**

Solution	
(i)	$\mathrm{P}(X\|Y') = \frac{\mathrm{P}(X \cap Y')}{\mathrm{P}(Y')}$ $\mathrm{P}(Y') = \frac{\mathrm{P}(X \cap Y')}{\mathrm{P}(X\|Y')}$ $= \frac{\left(\frac{1}{2}\right)}{\left(\frac{50}{63}\right)}$ $= \frac{63}{100}$
(ii)	$\mathrm{P}(X \cup Y') = \mathrm{P}(X) + \mathrm{P}(Y') - \mathrm{P}(X \cap Y')$ $\mathrm{P}(X) = \mathrm{P}(X \cup Y') + \mathrm{P}(X \cap Y') - \mathrm{P}(Y')$ $= \frac{3}{4} + \frac{1}{2} - \frac{63}{100}$ $\mathrm{P}(X) = \frac{31}{50}$
(iii)	$\mathrm{P}(X) = \mathrm{P}(X \cap Y) + \mathrm{P}(X \cap Y')$ $\mathrm{P}(X \cap Y) = \mathrm{P}(X) - \mathrm{P}(X \cap Y')$ $= \frac{31}{50} - \frac{1}{2}$ $= \frac{3}{25}$ Since $\mathrm{P}(X \cap Y) = \frac{3}{25} \neq \frac{31}{50} \times \frac{37}{100} = \mathrm{P}(X) \times \mathrm{P}(Y)$, X and Y are not independent events OR Since $\mathrm{P}(X) = \frac{31}{50} \neq \frac{50}{63} = \mathrm{P}(X \| Y')$, X and Y' are not independent events So X and Y are not independent events