

Topic: Relations and functions

Question 1 [2008 EOY, Q13]	marks
A function f is defined by $f : x \mapsto \frac{1}{x} - 2$, where $-2 \leq x \leq 2$, $x \neq 0$. (i) Sketch the graph of $y = f(x)$, showing clearly any asymptote(s) and intersection(s) with the axes.	[3]
(ii) Find an expression for f^{-1}, the inverse function of f, and state its domain.	[3]

Question 2 [2008 EOY, Q14]	marks
Sketch the graphs of $y = \ln x$ and $y = e^x$ on the same axes.	[2]
Describe using geometrical transformations how you would obtain the graphs of (a) $y = \ln x^2$, and	[2]
(b) $y = 2 + e^{-x}$ from the graphs you have drawn above. You are not required to draw them.	[2]

Question 3 [2009 EOY, Q4]	marks
Sketch the graphs of $y = x - 2$ and $y = x^2 - 3x - 4$ for the domain $-2 \leq x \leq 5$ on the same axes and hence state the number of solutions of the equation $x - 2 = x^2 - 3x - 4$.	[5]
Question 4 [2009 EOY, Q8]	marks
Function f is defined by $f : x \mapsto 3x + 1$. (i) Write down the inverse function of f in similar form.	[2]

(ii) Describe using geometrical transformation(s) how you would obtain the graph of $y = -f(x)$ from the graph of $y = f(x)$. You are not required to draw them.	[1]
Question 5 [2010 EOY, Q15]	marks
A function f is defined by $f : x \mapsto \frac{2x+k}{3+x}$, where $x \neq -3$ and k is a real constant. (i) If $f(-1) = -1\frac{1}{2}$, find the value of k. .	[2]
(ii) Find an expression for $f^{-1}(x)$, where f^{-1} is the inverse function of f, and state its domain.	[3]
(iii) Find the x-coordinates of the points of intersection, if they exist, when $f(x) = f^{-1}(x)$.	[2]

Question 6 [2011 EOY, Q8]	marks
(i) Solve the inequality $2x - 3 < 5$. Hence state the solution of $-4 < 2x - 3 < 5$.	[3]
(ii) By sketching appropriate graphs or otherwise determine the number of root(s) for the equation $\frac{2}{x} = 2^x$.	[3]
Question 7 [2012 EOY, Q4]	marks
(a) Sketch the graph $y = x - 3 - a$ where $0 < a < 3$, showing clearly the intercepts with the axes.	[3]

(b) Find the range of values of k, in terms of a, for which the equation $ x-3 - a = k$ (i) has no solution, (ii) has two solutions of opposite signs.	[2]
Question 8 [2012 EOY, Q14]	marks
Function f is defined by $f : x \mapsto \frac{2x+13}{x+3}$. (a) (i) State the largest possible domain for f.	[1]
(ii) A and B are such that $\frac{2x+13}{x+3} = A + \frac{B}{x+3}$ for all values of x except $x = -3$. By long division or otherwise, find the value of A and of B.	[2]

(a)(iii)

State precisely a sequence of transformations by which the graph of

$y = \frac{2x+13}{x+3}$ may be obtained from the graph of $y = \frac{1}{x}$.

[3]

(b) The function f has an inverse if its domain is restricted to $x > k$, state the least value of k and find an expression for $f^{-1}(x)$ corresponding to this domain for f .

[4]

Question 9 [2014 EOY, Q11]	marks
A function f is defined, for $x \neq 0$, by $f : x \mapsto \frac{a}{x} + 1$, where a is a real constant. Given that $2f(-1) + f^{-1}(2) = 4$, calculate the value of a.	[4]

Question 10 [2014 EOY, Q13]	marks
(i) Sketch the graphs of $y = 10^x$ and $y = \left \frac{1}{2} (2x - 1)(x - 2) \right$ on the same axes, showing clearly the x-intercepts, y-intercepts and turning points, if any. Hence, state the range of values of x such that $10^x < \left \frac{1}{2} (2x - 1)(x - 2) \right$.	[5]
(ii) Describe the sequence of transformations to obtain a graph of $y = 2 \ln(x - 1)$ from the graph of $y = e^x + 1$.	[2]

Question 11 [2015 EOY, Q5]	marks
It is given that f is a function such that $f : x \mapsto 2 - \sqrt{4 - x}$, $x \leq k$. (a) Write down the largest possible value of k.	[1]
(b) If $k = 3$, (i) find $f^{-1}(x)$ and state the domain of f^{-1},	[3]
(b) (ii) solve for x when $f(x) = f^{-1}(x)$.	[3]

Question 12 [2015 EOY, Q12]	marks
Sketch the graph of $y = \frac{4}{3} 3 - 2x$, showing clearly any intercepts with the axes. By sketching the graph of $y = 4 + 3x - x^2$, solve for the range of x when $12 + 9x - 3x^2 > 4 3 - 2x$.	[5]