JURONG PIONEER JUNIOR COLLEGE
9478 H2 PHYSICS

ENERGY AND FIELDS

Content

- **Energy stores and transfers**
- **Work done by a force**
- **Kinetic energy**
- **Concept of a field**
- **Potential energy**
- **Power and efficiency**

Learning Outcomes

Candidates should be able to:

(a) show an understanding that physical systems can store energy, and that energy can be transferred from one store to another

(b) give examples of different energy stores and energy transfers, and apply the principle of conservation of energy to solve problems

(c) show an understanding that work is a mechanical transfer of energy, and define and use work done by a force as the product of the force and displacement in the direction of the force

(d) derive, from the definition of work done by a force and the equations for uniformly accelerated motion in a straight line, the equation $E_k = \frac{1}{2}mv^2$

(e) recall and use the equation $E_k = \frac{1}{2}mv^2$ to solve problems

(f) show an understanding of the concept of a field as a region of space in which bodies may experience a force associated with the field

(g) define gravitational field strength at a point as the gravitational force per unit mass on a mass placed at that point, and define electric field strength at a point as the electric force per unit charge on a positive charge placed at that point

(h) represent gravitational fields and electric fields by means of field lines (e.g. for uniform and radial field patterns), and show an understanding of the relationship between equipotential surfaces and field lines

(i) show an understanding that the force on a mass in a gravitational field (or force on a charge in an electric field) acts along the field lines, and the work done by the field in moving the mass (or charge) is equal to the negative of the change in potential energy

(j) distinguish between gravitational potential energy, electric potential energy and elastic potential energy

(k) recall that the elastic potential energy stored in a deformed material is given by the area under its force-extension graph and use this to solve problems

(l) derive, from the definition of work done by a force, the equation $\Delta E_p = mg\Delta h$ for gravitational potential energy changes in a uniform gravitational field (e.g. near the Earth's surface)*

(m) recall and use the equation $\Delta E_p = mg\Delta h$ to solve problems**

(n) define power as the rate of energy transfer

(o) show an understanding that mechanical power is the product of a force and velocity in the direction of the force

(p) show an appreciation for the implications of energy losses in practical devices and solve problems using the concept of efficiency of an energy transfer as the ratio of useful energy output to total energy input.

* Learning outcome (c) of topic *Projectile Motion*
** Learning outcome (d) of topic *Projectile Motion*

Introduction

This topic explores the fundamental principles of **Conservation Laws** and **Systems and Interactions**, with a focus on energy stores, transfers, and the role of fields. Energy is stored in various ways, including **kinetic**, **potential** (gravitational, chemical, elastic), **nuclear**, and **internal** energy stores. Energy can be transferred from one store to another through different mechanisms such as **mechanical work, electrical work, heating, and radiation**.

The concept of **work** is central to energy transfers, describing how energy moves between stores through the application of a force. In particular, fields—such as **gravitational, electric, and magnetic fields**—play a crucial role in energy transfers. Objects within a field experience forces that can cause energy to be stored or transferred. For example, an object in a gravitational field gains energy in its gravitational potential store when lifted, while charges in an electric field transfer energy as electrical work.

Analysing energy stores and transfers allows us to describe physical systems without relying solely on Newton's laws. This approach extends beyond mechanics to electromagnetism, thermal physics, and nuclear processes. Because energy is a **scalar** quantity, calculations are often more straightforward than those involving forces, which are vectors. Additionally, considering only the **initial and final** energy stores simplifies analysis by removing the need to track intermediate processes.

By focusing on energy stores, transfers, and the role of fields, we gain a deeper understanding of a wide range of physical phenomena, making energy a unifying concept in science.

1 Work

(c) show an understanding that work is a mechanical transfer of energy, and define and use work done by a force as the product of the force and displacement in the direction of the force

1.1 Definition of Work

The work done by a force on an object is defined as the **product of the force and the displacement** of the object **in the direction of the force**.

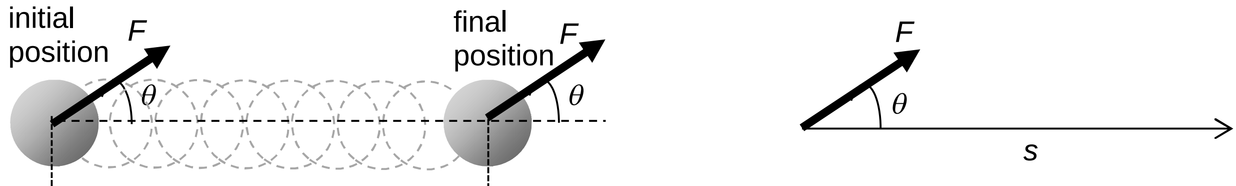
1.2 Work done by a constant force

If the force acting on the object is constant in magnitude and direction throughout the motion of the object, then the work done W by a force F in moving an object through a displacement s is

$$\boxed{W = Fs\cos\theta}$$

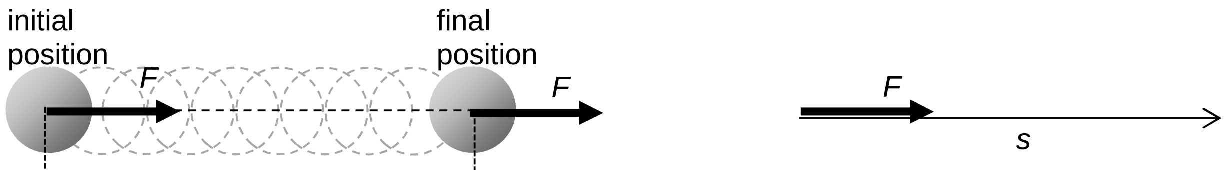
where W is the work done on the object by the constant force F (unit: J)
F is the magnitude of the constant force acting on the object (unit: N)
s is the displacement of the object (unit: m)
θ is the angle between F and s

Fig.1 to Fig. 4 show different scenarios which the equation above can be applied.



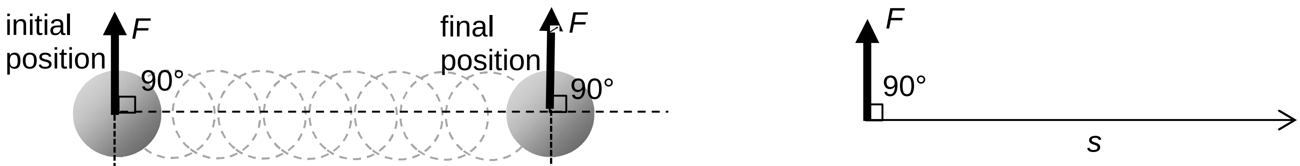
$W = Fs\cos\theta$

Fig. 1: Constant force at an angle θ to the displacement



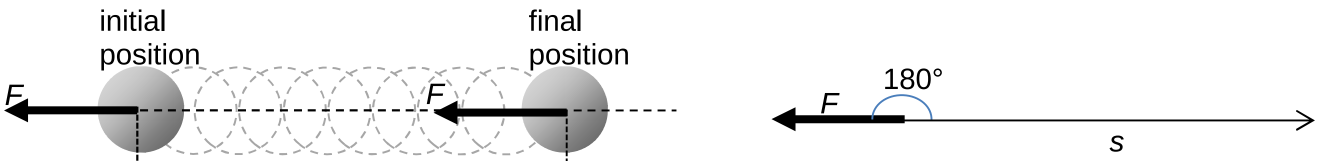
$W = Fs\cos\theta$
$= Fs\cos(0°)$
$= Fs$

Fig. 2: Constant force in the same direction as the displacement



$W = Fs\cos(90°)$
$= 0$

Fig. 3: Constant force 90° to the displacement



$W = Fs\cos\theta$
$= Fs\cos(180°)$
$= -Fs$

Fig. 4: Constant force opposite to the displacement

NOTE:

- The SI unit of work done is the joule (J).

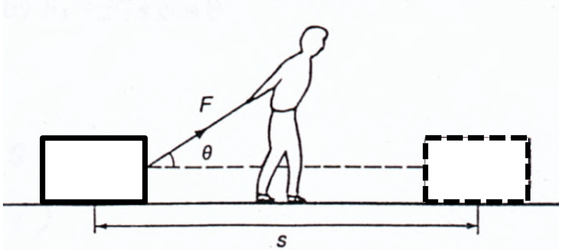
One joule (1 J) is defined as the work done by a force of one newton (1 N) when its point of application moves through a displacement of one metre (1 m) in the direction of the force.

Work done = (1 N) (1 m) (cos 0°) = 1 J

- Work done is a **scalar** quantity.

Example 1

The figure below shows a man pulling a box of mass 20 kg with a constant force of 50 N at an angle θ of 35° to the ground. The box moves through a horizontal distance of 10 m and the frictional force between the box and the ground is 11 N.



Determine the work done by the following forces on the box:

(a) The force applied by the man.

(b) The frictional force on the box.

(c) The normal contact force on the box by the ground.

(d) The gravitational pull by Earth.

(e) The resultant force on the box.

<u>**Solution**</u>

NOTE:

- If a force F is in the **same direction** as the object's displacement, or has a component in the same direction as the object's displacement, the work done by force F on the object is **positive**.

- If a force F is in the **opposite** direction to the object's displacement, or has a component in the opposite direction to the object's displacement, the work done by force F on the object is **negative.**

- Work is a process of transferring energy through the application of a force. If the work done on an object is **positive**, it means energy is **transferred to** the object. If the work done is **negative**, it means energy is **transferred out** from the object.

  In Example 1, the 50 N force exerted by the man on the box results in the man transferring energy to the box. This could be in the form of kinetic energy if the man is pulling a box which is initially at rest, until the box reaches a certain final velocity. On the other hand, the 11 N frictional force exerted on the box results in energy being removed from the box. This is in the form of heat loss to the surroundings. Hence the box cannot reach a higher speed than it could have if the frictional force was absent.

- **Work done on an object <u>by</u> a <u>force</u>**

  A statement like 'Determine the work done on the box' is not clear as there are many forces acting on the box.

  A statement like 'Determine the work done on the box by the man when pulling the box' should be interpreted as 'Determine the work done on the box <u>by</u> the <u>force exerted by the man</u> when pulling the box'.

  A statement like 'Determine the work done on the box by Earth should be interpreted as 'Determine the work done on the box <u>by</u> the <u>gravitational force</u> that is acting on the box'.

- **Work done <u>against</u> a <u>force</u>**

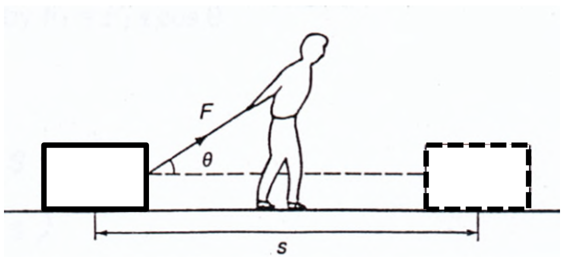
  A statement like 'work done <u>against</u> force F' should be interpreted as:

Work done against force F = − Work done by force F on object

  In Example 1, the work done by the frictional force on the box = −110 J. Therefore, the work done against the frictional force = 110 J.

Example 2

Relook at Example 1.



Determine

(a) the work done by the man.
(b) the work done on the box by the ground.
(c) the work done against friction.

Solution

1.3 Work done by a variable force

- If the force F acting on a body is not constant, the work done by a force can be given by the **area under the *F-x* graph** where ***x* is the displacement in the direction of the force**. Refer to Fig. 5.

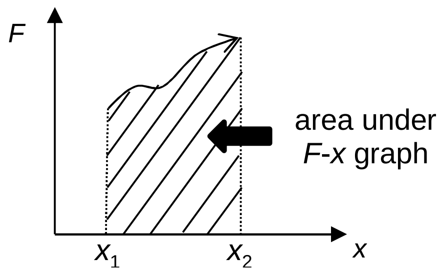


Fig. 5: Work done is given by area under F-x graph

- In general, the work done over a displacement x_1 *to* x_2 is calculated by applying:

$$W = \int_{x_1}^{x_2} F \, dx$$

- We will examine one case of work done by variable force, namely, work done to stretch a spring.

1.4 Work done to stretch a spring

(k) recall that the elastic potential energy stored in a deformed material is given by the area under its force-extension graph and use this to solve problems

- Fig. 6 shows an elastic spring with one end attached to the wall and the other end stretched to a horizontal displacement x by a variable force F. The spring obeys Hooke's Law and has a spring constant k (recall Hooke's Law from Forces and Moments).

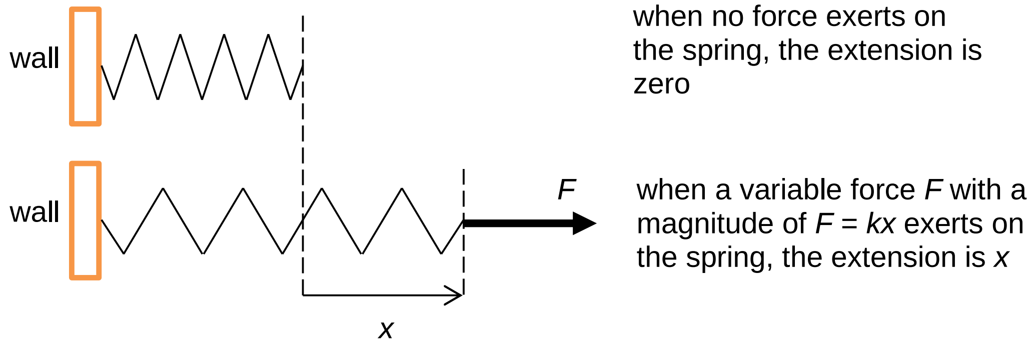
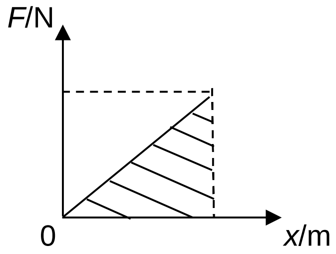


Fig. 6: Extension of a spring

- Work done by the variable force is determined by plotting a F-x graph, where F has a magnitude of $F = kx$.



Work done, W in stretching spring

= **Area under *F-x* graph (shaded)**

$= \frac{1}{2}Fx$

$= \frac{1}{2}(kx)x$

$= \frac{1}{2}kx^2$

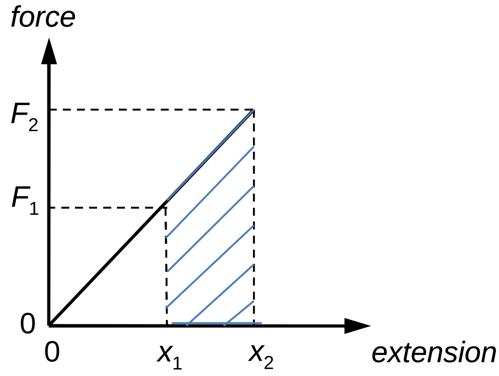
- The work done by the variable force is transferred to the spring and stored as **elastic potential energy.**

Elastic potential energy stored in the spring = $\frac{1}{2}kx^2$

Example 3

Calculate the work done in stretching a spring of force constant 25 N m^{-1} from an extension of 1.0 cm to an extension of 2.0 cm.

Solution



2 Energy Conversion and Conservation

(a) show an understanding that physical systems can store energy, and that energy can be transferred from one store to another

(b) give examples of different energy stores and energy transfers, and apply the principle of conservation of energy to solve problems

(j) distinguish between gravitational potential energy, electric potential energy and elastic potential energy

2.1 Different forms of energy store

- Energy is defined as the **capacity to do work**.
- Energy is a scalar quantity. The SI unit of energy is the joule (J).

The following are some examples of different stores of energy:

energy store	source
mechanical	total mechanical energy = sum of kinetic energies and potential energies
kinetic energy	energy possessed by all objects in motion. e.g. moving car
potential energy	energy possessed by a system by virtue of its relative position or its state of deformation
types of potential energy:	
gravitational potential energy	energy possessed by an object by virtue of its own mass and its position in a gravitational field. e.g. object on top of a building
electric potential energy	energy possessed by an object by virtue of its own charge and its position in an electric field
elastic potential energy	energy possessed by an object by virtue of its state of deformation. e.g. compressed or stretched springs, a bent diving board
electrical	due to flow of charge or current. e.g. current in a closed circuit
chemical	energy possessed by a fuel by virtue of its chemical composition. e.g. oils, wood, food, chemicals in electric cells
nuclear	energy in nucleus of atoms due to nuclear composition. e.g. energy released in atomic bombs, produced by nuclear reactors
radiant (light)	energy in the form of electromagnetic waves. e.g. visible light, radio waves, ultraviolet, x-rays, microwaves
internal energy	energy possessed by particles of matter in the form of kinetic energy due to motion of the particles in the matter and potential energy which depends on the separation between these particles

Renewable and Non-renewable energy

Renewable sources of energy are those that can be **replaced or replenished** each day by the Earth's natural processes. e.g. wind, geothermal, solar and tidal energy.

Wind Park off Copenhagen, Denmark.

Wind generates 20% of Denmark's electricity.

Rise and fall of a buoy generates 20 kW.

Aerial view of Sembcorp Tengeh Floating Solar Farm, Singapore.

Consisting of 122,000 solar panels spread across 10 solar-panel islands, the Tengeh solar farm covers an area of 45 ha, about the size of 45 football fields. It has a capacity of 60 megawatt-peak (MWp). The power generated at Tengeh is sufficient to power the operations of all five local water treatment plants. (Source: The Straits Times)

Non-renewable sources of energy are those that are **finite or exhaustible** because it takes several million years to replace them, e.g. fossil fuels like coal, oil and natural gas, nuclear energy from the fission of uranium nuclei.

2.2 Examples of energy conversion

examples	energy conversions
bouncing ball	• As the ball falls, its gravitational potential energy is converted into kinetic energy. • When the ball hits the ground, the ball is deformed during the collision. Its kinetic energy is converted into elastic potential energy. Some kinetic energy may be lost as thermal energy or sound energy. • The elastic potential energy is converted back into kinetic energy as the ball regains its original shape. • The kinetic energy is converted into gravitational potential energy as the ball bounces upwards, until it reaches its highest position.
diver jumping off a diving board	• The diver uses his gravitational potential energy to do work in bending the diving board. • The work done by man is transferred to the board and stored as elastic potential energy in the board, which is then converted into kinetic energy of the diver as the diver is pushed upwards and off the diving board. • At the same time, some of the elastic potential energy in the board is lost as heat and sound due to dissipative forces in the diving board.
hammering a nail into a wooden block	• A person uses the chemical energy in his muscles to do work against the gravitational force acting on the hammer by the Earth in order to lift the hammer. • The work done is converted into the gravitational potential energy of the hammer in its raised position. • As the hammer falls, its gravitational potential energy is converted into kinetic energy. • When the hammer hits the nail, its kinetic energy is used to do work in driving the nail into the wooden block, producing sound energy in the air and thermal energy in the block, nail and hammer.
hydroelectric plant	• In a hydroelectric plant, water falls from a height on to a turbine causing it to turn. • The turbine turns a coil placed in a magnetic field, thereby generating an electric current. • Therefore, potential energy of the water is converted into kinetic energy of the turbine, which is converted into electrical energy.

2.3 Energy conservation

(e) *recall and use the equation* $E_k = \frac{1}{2} mv^2$ to solve problems

(m) *recall and use the equation* $\Delta E_p = mg\Delta h$ *to solve problems*

- ***Principle of conservation of energy*** *states that:*

> Energy can **neither be created nor destroyed** in any process. It can only be **transformed (converted)** from one store to another or **transferred** from one body to another but the total amount in any isolated system must remain constant.

- We can apply this principle to perform detailed calculations of the total mechanical energy of an isolated system.

- For an isolated system, there is no transfer of energy between the system and its surroundings (e.g. no heat transfer).

- When an isolated system undergoes a change (e.g. an object is displaced, or an object's velocity is changed), the system's total energy remains constant.

- For an isolated system undergoing a change, these are a few common ways to calculate the system's changes in energy:

(potential energy + kinetic energy)$_{\text{initial}}$ = (potential energy + kinetic energy)$_{\text{final}}$

$$(E_p + E_k)_{\text{initial}} = (E_p + E_k)_{\text{final}}$$

(change in potential energy) + (change in kinetic energy) = 0

$$\Delta E_p + \Delta E_k = 0$$

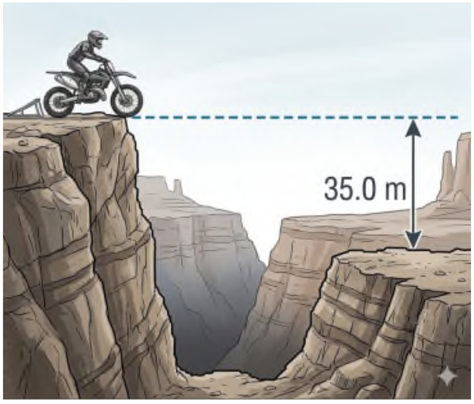
loss in E_p = gain in E_k

gain in E_p = loss in E_k

Example 4

The figure below shows a motorcycle rider leaping between cliffs, across a canyon. At the instant when the motorcycle leaves the higher cliff, it has a horizontal velocity of 38.0 m s^{-1}. The difference in height between the two cliffs is 35.0 m.

Assuming air resistance is negligible, determine the speed with which the motorcycle reaches the lower cliff.



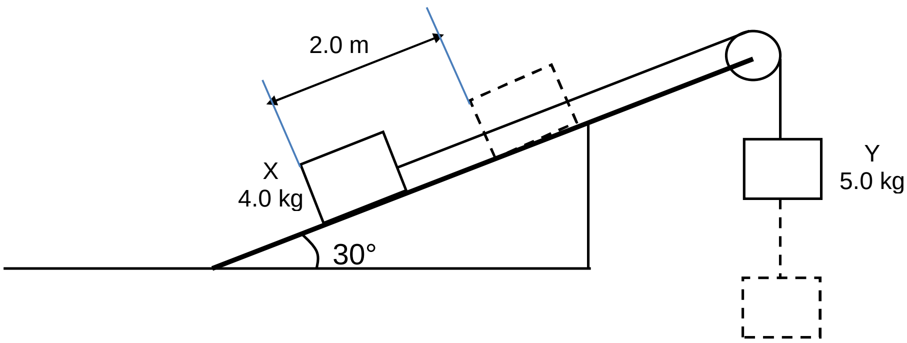
<u>Solution</u>

Example 5

The diagram shows two bodies X and Y connected by a light cord passing over a light, free-running pulley. X starts from rest and moves on a smooth plane inclined at 30° to the horizontal.

(a) Calculate the total final kinetic energy of the system after X has travelled 2.0 m along the plane.

(b) If the plane is rough, and exerts an average frictional force of 5.0 N, calculate the total final kinetic energy of the system after X has travelled 2.0 m along the plane.



Solution

3 Efficiency

(p) show an appreciation for the implications of energy losses in practical devices and solve problems using the concept of efficiency of an energy transfer as the ratio of useful energy output to total energy input

- Practical devices are used to convert one energy store into a store which is useful for mankind. For example, a diesel engine converts chemical energy stored in diesel fuel into mechanical energy for lifting objects.

- When a practical device works, besides converting energy input into useful energy output, some unwanted energy in the form of heat and sound are also produced. These unwanted energies are lost to the surrounding, and should be kept as small as possible.

energy input = useful energy output + wasted energy output

- Efficiency of a practical device is a measure of the useful energy output to the total energy input.

$$\boxed{\eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%}$$

Example 6

A worker pushes a cart of iron ore from a mine to a collection point. He expends 0.10 MJ of energy in the process while the total work done by him is 18 kJ.

Calculate his efficiency.

<u>Solution</u>

Example 7

A car has a mass of 800 kg and the efficiency of its engine is rated at 18 %.

Determine the amount of fuel needed to accelerate the car from rest to 60 km h^{-1}. (Assume that the energy equivalent of 1 litre of fuel is 1.3×10^8 J.)

The engine converts chemical potential energy in the fuel (input energy) to kinetic energy of the car (useful output energy).

4 Derivation of Kinetic Energy and Potential Energy

(d) *derive, from the definition of work done by a force and the equations for uniformly accelerated motion in a straight line, the equation* $E_k = \frac{1}{2}mv^2$

(l) *derive, from the definition of work done by a force, the equation* $\Delta E_p = mg\Delta h$ *for gravitational potential energy changes in a uniform gravitational field (e.g. near the Earth's surface)*

4.1 Derivation of $E_k = \frac{1}{2}mv^2$

- Kinetic energy is a scalar quantity that represents the energy stored with the body due to its motion.

- Consider an object of mass *m* on a horizontal frictionless surface. A constant force *F* acts on the object, causing it to accelerate from an initial velocity of *u* to a final velocity *v*, covering a displacement of *s*.

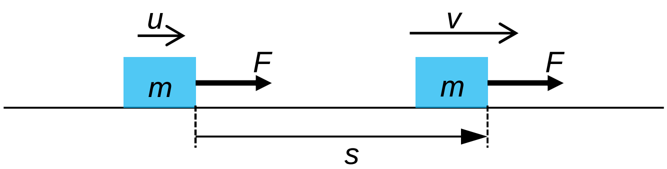


Fig. 7: Object accelerating under constant force

- **Work done by force *F*** on the object is **transferred to** the object as its **kinetic energy store** (law of conservation of energy). An equation for kinetic energy would need to link work done to the velocity of the object. This can be done by applying the kinematics equations.

- Work done by force *F* on the object,

$$W = Fs \quad \text{- - - (1)}$$

- Since the surface is frictionless, *F* is the net force acting on the mass. Hence *F* in equation (1) can be replaced by $F = ma$.

$$W = Fs$$
$$W = (ma)(s) \quad \text{- - - (2)}$$

- Since the net force is constant, **acceleration *a* is constant,** and the kinematics equation of motion applies.

$$v^2 = u^2 + 2as$$
$$a = \frac{1}{2s}(v^2 - u^2) \quad \text{- - - (3)}$$

- Substitute (3) into (2), work done by force *F* is:

$$W = (ma)(s)$$
$$W = \mathrm{m}\left[\frac{1}{2s}(v^2 - u^2)\right](s)$$
$$W = \frac{1}{2}m(v^2 - u^2)$$
$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

- **Work-Energy Theorem** states that:

The ***net work done*** by external forces acting on an object (or a system) is equal to the object's (or system's) ***change in kinetic energy***.

$$\text{work done by net force, } W = (E_\mathrm{k})_\mathrm{final} - (E_\mathrm{k})_\mathrm{initial}$$
$$W = \Delta E_\mathrm{k}$$

- If the object accelerates from rest to a final velocity of *v*, all the work done is transferred into the object's kinetic energy store E_k:

$$\boxed{E_\mathrm{k} = \frac{1}{2}mv^2}$$

Example 8

A car of mass 800 kg moving at 30 m s^{-1} along a horizontal road is brought to rest by a constant frictional force of 5000 N.

(a) Determine the work done by friction.
(b) Hence determine the displacement of the car.

<u>Solution</u>

Example 9

A box of mass 10 kg is accelerated from rest to a speed of 5.0 m s^{-1} along a smooth horizontal road by a horizontal pulling force of 200 N.

Calculate the distance travelled by the box in reaching its final speed.

<u>Solution</u>

4.2 Derivation of $\Delta E_p = mg\Delta h$

- Near the Earth's surface, the gravitational field is **uniform** and the acceleration of free fall g is taken to be constant.

- Consider an object of mass m lifted by a constant force F, vertically upwards at **constant speed** from ground level to a height Δh.

- At constant speed, the applied force must **balance** the weight of object, $F = mg$.

- Since the object's velocity is constant, its kinetic energy is also **constant**.

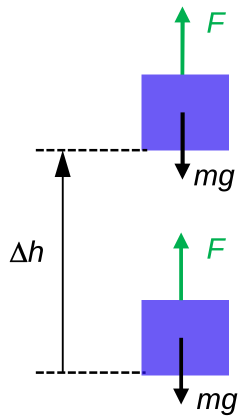


Fig. 8: Object being lifted at constant velocity

- Hence work done by the applied force F on the object is transferred to its gravitational potential energy store, in accordance with the law of conservation of energy.

- Work done by F is

$$W = Fs$$

$$W = (mg)(\Delta h)$$

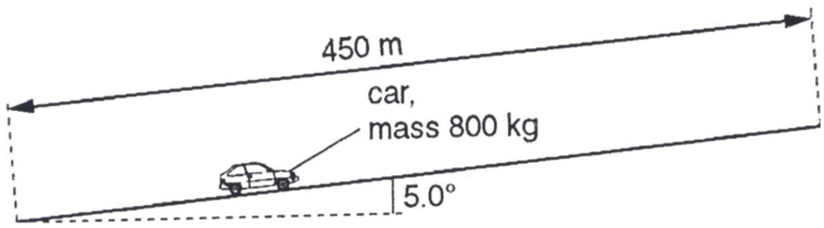
$$\boxed{\therefore\ \Delta E_p = mg\Delta h}$$

Example 10

A car travels upwards along a straight road inclined at an angle of 5.0° to the horizontal, as shown in the figure below.

The length of the road is 450 m and the car has a mass 800 kg.

What is the gain in gravitational potential energy of the car when it reaches the top of the slope?

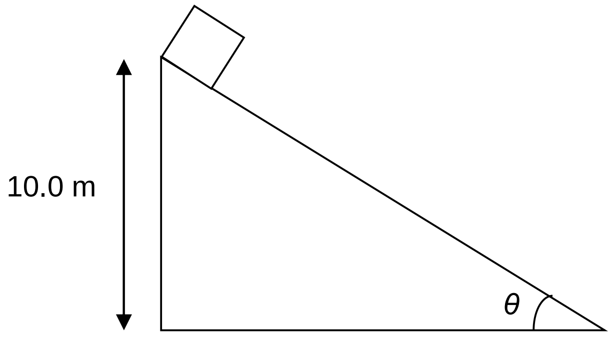


<u>**Solution**</u>

Example 11

A block weighing 800 N moves down a frictionless slope inclined at 20° to the horizontal of height 10.0 m as shown below.

Determine the speed of the block at the bottom of the slope, assuming that the block moves off from the top at an initial speed of 5.0 m s^{-1}.



<u>Solution</u>

5 Concept of a Field and Field Strengths

5.1 Concept of a Field

(f) show an understanding of the concept of a field as a region of space in which bodies may experience a force associated with the field

- A field is a region of a space in which bodies with specific properties, such as mass or charge, may experience a force associated with the field.

- Key characteristics of fields

  - Vector nature: Fields have both magnitude and direction
  - Field lines: Visual representation of the direction and strength of a field (closer lines represent stronger field and vice versa)
  - Action at a distance: Objects can exerts forces through fields without physical contact.

5.2 Gravitational Field Strength and Electric Field Strength

(g) define gravitational field strength at a point as the gravitational force per unit mass on a mass placed at that point, and define electric field strength at a point as the electric force per unit charge on a positive charge placed at that point

5.2.1 Gravitational field strength

- The gravitational field strength is a measure of how strong the gravitational field created by an object is.

Gravitational field strength at a point is defined as the gravitational force per unit mass on a mass at that point.

$$\boxed{g = \frac{F_g}{m} = \frac{GM}{r^2}}$$

where
G is the gravitational constant with a value of 6.67×10^{-11} N m^2 kg^{-2},
M is the mass of object that creates a gravitational field around it,
m is the mass of object that experiences the gravitational force in the field,
r is the distance between the centre of the object and the point in the field, and
F_g is the magnitude of the gravitational force acting between the objects of masses M and m respectively.

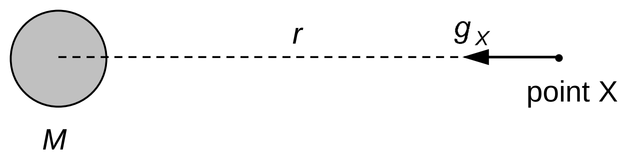


Fig. 9: Gravitational field strength at point X due to an object of mass M is $g_X = \frac{GM}{r^2}$

- It is a vector having both direction and magnitude, therefore addition of gravitational field strengths must be done by vector addition. The direction is the same as the gravitational force. Units for g is N kg^{-1}.

- Note that the gravitational field at point X in Fig. 9 is independent of mass *m.* The value of g only depends on the distance r from the object causing the gravitational field.

Example 12

The mass of the Earth is approximately 80 times the mass of the Moon and the Earth's radius is 3.7 times that of the Moon. The gravitational field strength at the Earth's surface is $10\ \text{N kg}^{-1}$.

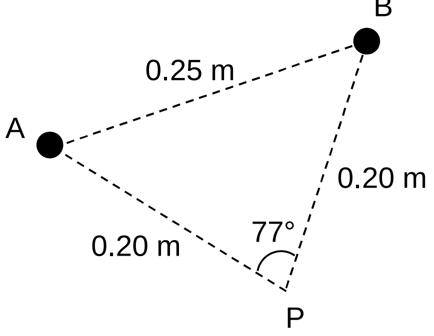
Determine gravitational field strength on the Moon's surface?

<u>Solution</u>

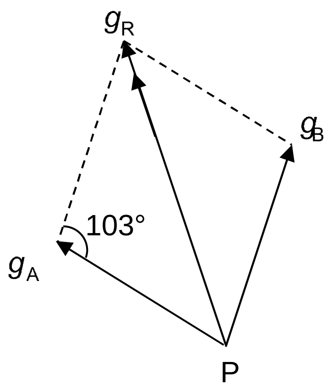
Example 13

The distance between an object A of mass 8000 kg and another object B of mass 8000 kg is 0.25 m.

Calculate the resultant gravitational field strength due to both the objects at a point P 0.20 m away from both A and B.



Solution



5.2.2 Electric field strength

- The electric field strength is a measure of how strong the electric field created by a charge is.

The electric field strength E at a point in an electric field is defined as the electric force per unit charge on a positive charge placed at that point.

- Mathematically,

$$E = \frac{F}{q}$$

where F is the electric force experienced by a charge q. The unit of E is $\mathrm{N\,C^{-1}}$ or $\mathrm{V\,m^{-1}}$.

- Consider a positive point charge $+Q$ placed at point A and a positive test charge q placed at point B, a distance r apart, as shown in Fig. 6. The charge $+Q$ creates an electric field which is radially outwards. The magnitude of the electric field strength at point B is

$$E_B = \frac{F}{q}$$

where F is the electric force of repulsion exerted on the test charge q by $+Q$.

- Substituting F (by Coulomb's law to be discussed in greater detail in the topic of Electric Fields) into the equation,

$$E_{\mathrm{B}} = \frac{\dfrac{Qq}{4\pi\varepsilon_o r^2}}{q}$$

$$E_{\mathrm{B}} = \frac{Q}{4\pi\varepsilon_o r^2}$$

where ε_0 is the permittivity of free space and has a value of 8.85×10^{-12} F m^{-1}.

- The direction of the electric field strength E_{B} is directed to the right, as shown in Fig. 10.

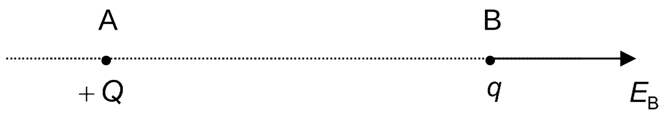


Fig. 10: Electric field strength at point B due to a positive point charge $+Q$

- Similarly, if a negative point charge $-Q$ is placed at point A instead, the magnitude of the electric field strength at point B is

$$E_B' = \frac{F'}{q}$$

where F' is the electric force of attraction exerted on the test charge q by $-Q$.

- Substituting F' into the equation,

$$E_B' = \frac{\dfrac{-Qq}{4\pi\varepsilon_o r^2}}{q}$$

$$E_B' = \frac{-Q}{4\pi\varepsilon_o r^2}$$

- In this case, the direction of the electric field strength E_{B}' is directed to the left, as shown in Fig. 11. This is consistent with what we have learnt so far, that is, the field lines point radially inwards for a negative point charge, $-Q$.

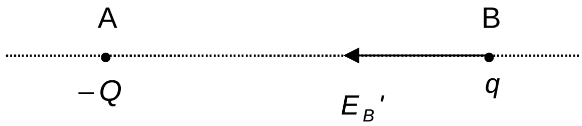


Fig. 11: Electric field strength at point B due to a negative point charge $-Q$

- In general, the magnitude of the electric field strength E at a point, where the point is at a distance r away form a point charge Q is

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

Example 14

Two equal and opposite charges $+q$ and $-q$ are situated 0.90 m apart.

At a point P, 0.30 m from the charge $+q$, the magnitude of the electric field strength due to the charge $+q$ alone is 20.0 N C^{-1}.

Determine the magnitude of the electric field strength at P due to both charges.

Solution

5.3 Field Lines

(h) represent gravitational fields and electric fields by means of field lines (e.g. for uniform and radial field patterns), and show an understanding of the relationship between equipotential surfaces and field lines

5.3.1 Gravitational field lines

- The gravitational field strength g near the surface of the Earth is approximately constant at 9.81 N kg^{-1}, also known as acceleration of free fall, with the value of 9.81 m s^{-2}.
- The approximate constancy should be appreciated by considering the value of g at height h above the surface, where h is small compared to the radius R of the Earth of mass M.

$$g = \frac{GM}{(h+R)^2} \approx \frac{GM}{R^2} \quad \text{since } h << R$$

- Near the surface of the Earth, the gravitational field lines are nearly uniform as shown in Fig. 13.

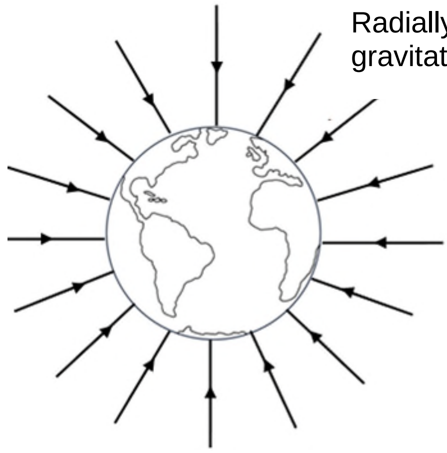


Fig. 12: Radially inwards gravitational field lines around the Earth at a great distance

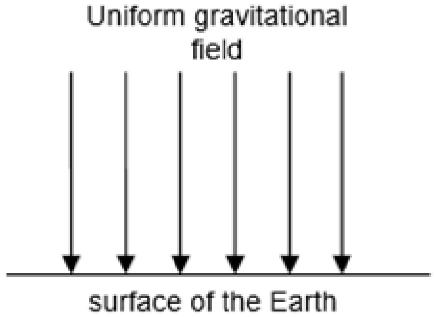
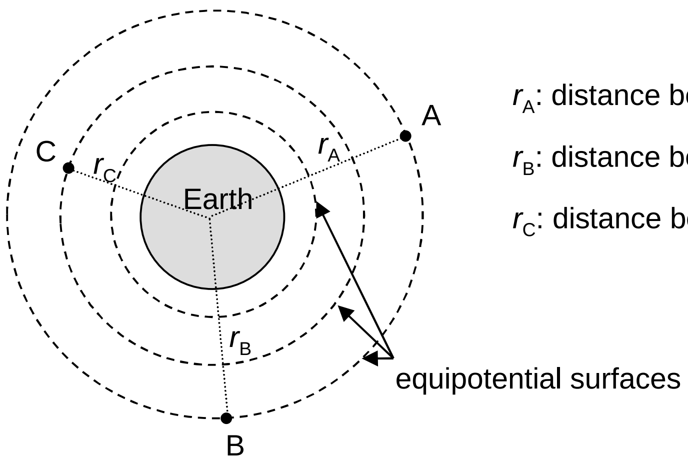


Fig. 13: Uniform gravitational field lines close to the Earth's surface

- All points at the same distance from the centre of the Earth have the same gravitational potential ϕ .



r_A: distance between A and centre of Earth

r_B: distance between B and centre of Earth

r_C: distance between C and centre of Earth

Fig. 14: Equipotential surfaces around the Earth

- The surface where all points on it have the same gravitational potential is known as an equipotential surface. The gravitational field lines are always normal to equipotential surfaces.

- From Fig. 14, since $r_A = r_B > r_C$, $\phi_A = \phi_B > \phi_C$.

5.3.2 Electric field lines

- An electric field can be represented by means of field lines, or lines of force. Field lines are imaginary, and are used to visualise the pattern of the electric field.

- The direction of electric field lines (or lines of force) indicates the direction of force in which a positive test charge would experience at that point.

- In Fig. 15(a), an electric field is set up by a positive point charge, $+Q$. To draw the electric field lines, a positive test charge $+q$ is placed in different regions and the lines of force are drawn. The lines are directed away from $+Q$ as like charges repel. The electric field of a positive point charge is thus as shown in Fig. 15(b).

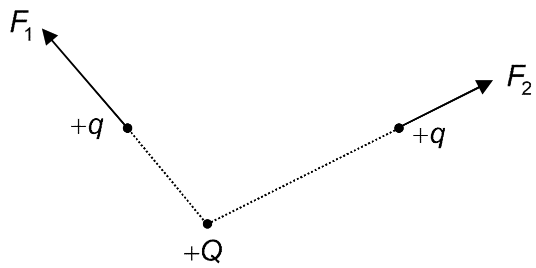


Fig. 15(a): Lines of force on positive test charge $+q$
due to a positive point charge $+Q$

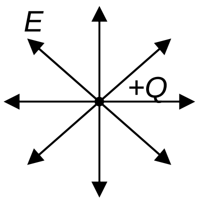


Fig. 15(b): Electric field lines of an isolated
positive point charge $+Q$

- Similarly, for a positive test charge $+q$ placed in different regions near to a negative point charge $-Q$, the test charge experiences forces of attraction. The lines of force are directed towards the negative point charge as shown in Fig. 16(a). The electric field of a negative point charge is thus as shown in Fig. 16(b).

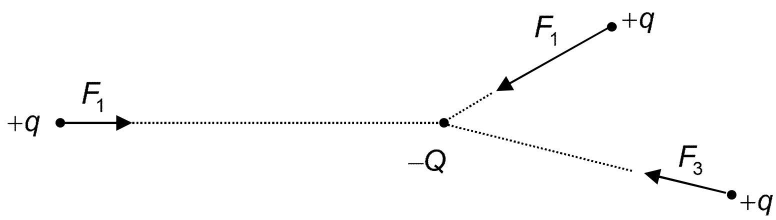


Fig. 16(a): Lines of force on positive test charge
$+q$ due to a negative point charge $-Q$

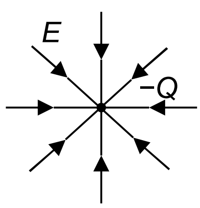


Fig. 16(b): Electric field lines of an isolated
negative point charge $-Q$

5.3.2.1 Characteristics of electric field lines

a. The field lines are always directed away from a positive charge and directed towards a negative charge.

b. The field lines are closer to one another where the field is stronger, and vice versa. In other words, the density of field lines (number of lines per unit area) represents the strength of the electric field.

c. In the direction of an electric field line, the electric potential V is decreasing.

d. The tangent to a field line at a point gives the direction of the electric field E at that point.

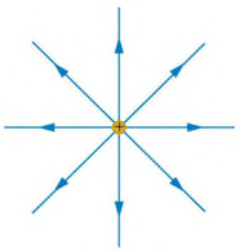
e. The field lines do not cross each other, as the electric field will then have two directions at the point where they cross.

f. For field lines that are parallel and equally spaced, the electric field is said to be uniform. This implies that every point in the field has the same field strength.

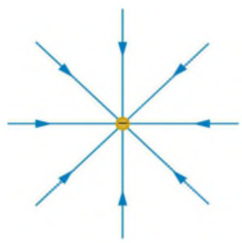
g. The electric field lines are drawn such that they are normal to the surface of conductors or charges.

h. No electric field exists inside a hollow or solid conductor.

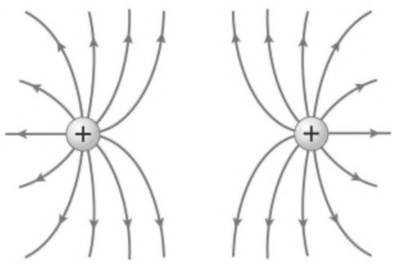
5.3.2.2 Patterns of electric field lines for different arrangements of charges



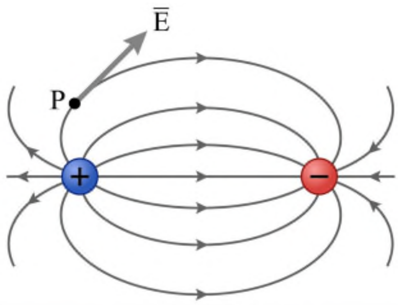
(a) Single positive charge



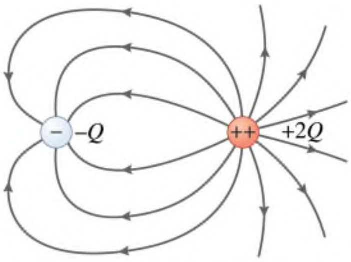
(b) Single negative charge



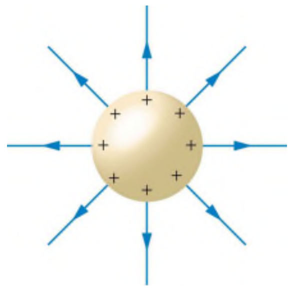
(c) Two like charges



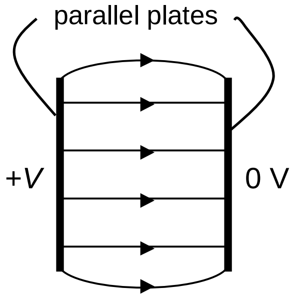
(d) Two unlike charges



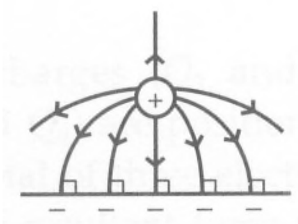
(e) Two unlike charges of different magnitudes



(f) Positively charged conducting sphere



(g) Two parallel conducting plates



(d) Single positive charge and negatively charged plate

- An equipotential line is an imaginary line linking up all the points that have the same potential.
- In the region of an electric field, the equipotential lines are <u>perpendicular</u> to the electric field lines.
- Moving a charge <u>along</u> an equipotential line does <u>not</u> require any work.

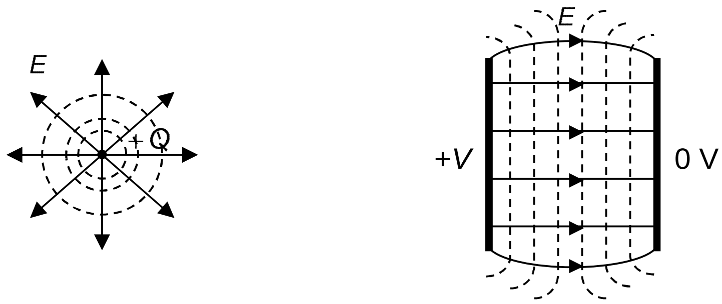


Fig. 17: Equipotential lines (dotted) for point charge and charged parallel plate

5.3.3 Force and Potential Energy in a Field

(i) *show an understanding that the force on a mass in a gravitational field (or force on a charge in an electric field) acts along the field lines, and the work done by the field in moving the mass (or charge) is equal to the negative of the change in potential energy*

- When a body is placed in a uniform field, the body will experience a constant force F acting on it, regardless of which point the body is placed at.

- Examples of a body in a uniform field include:

  - <u>A ball of mass m falling freely near Earth's surface.</u>
    In the region near Earth's surface, Earth's gravitational field is constant (the acceleration of free fall, g, is a constant value of 9.81 m s^{-2}). At any point in this region, the ball always experiences a constant gravitational force of $F_g = mg$.

    As the ball moves from rest under the effect of the gravitational force F_g, the ball will lose gravitational potential energy. That is, work done by the gravitational force causes the ball to lose gravitational potential energy.

  - <u>A particle of charge q placed between 2 oppositely-charged parallel metal plates</u>
    The region between the 2 metal plates has a constant electric field strength of E. At any point in this region, the charge will experience a constant electric force of $F_E = qE$.

    As the charge moves from rest under the effect of the electric force F_E, the charge will lose electric potential energy. That is, work done by the electric force causes the charge to lose electric potential energy.

- Consider a body placed in a uniform field. A constant field force F acts on the body and moves the body along a distance Δx in the direction of the force. The work done by this force F is

$$W = F\Delta x$$

- In a closed system, the principle of conservation of energy dictates that this work done must be compensated by a decrease in potential energy U (i.e. ΔU is negative).

$$\boxed{\begin{aligned} W &= F\Delta x \\ &= -\Delta U \end{aligned}}$$

Re-arranging, $$F = -\frac{\Delta U}{\Delta x}$$

Furthermore (including non-uniform field), $$\boxed{F = -\frac{dU}{dx}}$$

where F is the force acting on the body placed at that point in the field

$\frac{dU}{dx}$ is the **potential energy gradient**.

- Potential energy gradient $\frac{dU}{dx}$ is defined as the change in the potential energy of the body with variation of the distance x from the source of the force field (units: J m^{-1}).
- The negative sign shows that the field force acts in the direction of decreasing potential energy.
- In words, the equation $F = -\frac{dU}{dx}$ means the field force F is equal in magnitude to the potential energy gradient and it acts in the direction of decreasing potential energy.
- Graphically, if a graph of the body's potential energy is plotted against its position in the field, the **negative** of the gradient of the graph gives the value of the force at that position.

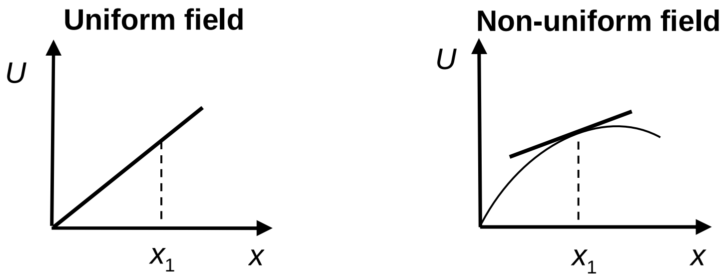


Fig. 18: U-x graphs in Uniform and Non-uniform fields

Example 15

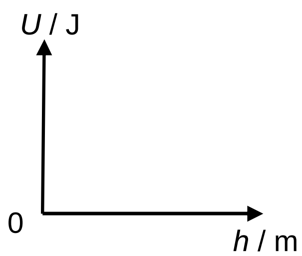
An object of mass 1 kg is raised from Earth's surface to a height of 10 m.

(a) Plot a graph to show how the object's gravitational potential energy varies with its height from Earth's surface.

(b) Use the graph to determine Earth's gravitational pull on the object.

Solution

(a)



(b)

6 Power

(n) *define power as the rate of energy transfer*

(o) *show an understanding that mechanical power is the product of a force and velocity in the direction of the force*

6.1 Definition of Power

- Power is defined as the **rate of energy transfer.**

- Power is a **scalar** quantity.

- The S.I. unit for power is the watt, W.

One watt is defined as the power when one joule of work is done per second, or one joule of energy is transferred per second.

6.2 Average Power

- Average power <*P*> is the total work done *W* divided by the total time interval *t*, OR the transfer of energy ΔE divided by the total time interval *t*.

Mathematically, $\boxed{P = \frac{W}{t}}$ or $\boxed{P = \frac{\Delta E}{t}}$

where P is power (unit: W or J s^{-1})
W is the work done (unit: J)
ΔE is the energy transfer (unit: J)
t is the time taken (unit: s)

6.3 Instantaneous Power

- Consider a constant force *F* being applied on an object such that *F* cancels other forces on the object, resulting in a net force of zero on the object. Under the influence of this force *F*, the object moves at a constant velocity of *v* and cover a displacement *s* over a time interval ***t.*** Refer to Fig. 19.

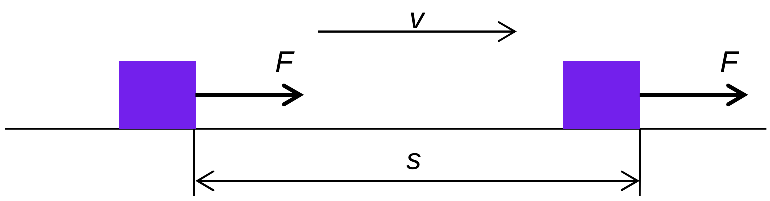


Fig. 19: Work done (or energy transfer) by a constant force moving object at constant velocity

- Work done (or energy transfer) by the force *F*,

$$W = Fs$$

- Power delivered by the force F,

$$P = \frac{W}{t}$$

$$P = \frac{Fs}{t}$$

$$P = F\left(\frac{s}{t}\right)$$

$$\boxed{P = Fv}$$

NOTE:

- Force F is **<u>not the net force</u>** of the object. It is the **applied force** that does work to propel the object forward. If the object is moving at constant velocity, force F will have the same magnitude as the opposing force (e.g. friction, drag). For such a case, the net force on the object is zero.
- The equation is true for both constant and changing force F. When a changing force F acts on the object, the instantaneous power at one instant is the product of the applied force F at that instant and the object's velocity at the same instant.

  Instantaneous Power, $\boxed{P_{\text{instantaneous}} = F_{\text{instantaneous}} \times v_{\text{instantaneous}}}$

Example 16

A cyclist travelling along a horizontal road experiences a constant drag of 20 N. The combined mass of the cyclist and the bicycle is 100 kg and he delivers a constant power.

(a) If his maximum velocity is 5.0 m s^{-1}, what is the power output of the cyclist?
(b) If his velocity at one instant is 4.0 m s^{-1}, what is his acceleration at this instant?

<u>Solution</u>
(a)

(b)

6.4 Electrical Energy Consumption − kilowatt hour (kWh)

- When calculating the cost of electricity consumption, the amount of electrical energy used is denoted in kilowatt-hours (kWh) and not in J (joule).

- 1 kWh is the amount of energy supplied by a power source of 1 kW over a time interval of 1 h.

  i.e. 1 kWh = (1000 W) (3600 s)
  $= 3.6 \times 10^6$ J
  = 3.6 MJ

Example 17

A rice cooker uses 680 W when plugged into a 240 V mains.

It takes 35 min to cook the rice. If the electrical energy costs 28.12 cents per kilowatt-hour, how much does it cost to cook the rice?

Solution

References

1 Hugh D. Young, Roger A. Freedman, *University Physics with Modern Physics (15th edition)*, Pearson

2 Steve Adams and Jonathan Allday, *Advanced Physics*, Oxford University Press

3 Douglas C. Giancoli, *Physics - Principles with Applications (6th edition)*, Prentice Hall

4 Kwok Wai, Loo, *Longman Advanced Level Physics*, Pearson Education

Appendix

Non-renewable/ Exhaustive sources of Energy

1. Oil and gas
2. Coal
3. Nuclear power

Renewable sources

Energy sources that are continually being replenished naturally.

1. <u>Hydroelectric power</u>

   Lots of rainfall in the mountains is the essential requirement for hydroelectricity.

   potential energy store of water in a high-level reservoir → kinetic energy store of moving water → kinetic energy store of rotating turbine → electrical energy store

2. <u>Solar power</u>

   Solar cells are thin wafers made from silicon and they convert solar energy directly into electrical energy. Electrons in the silicon gain energy from light photons to create a voltage.

3. <u>Wind power</u>

   Windmills sited in windy coastal areas convert the kinetic energy of the wind into electrical energy.

4. <u>Tidal power</u>

   The tide rises and falls twice each day. By trapping each high tide and allowing it to flow out through generators, tidal energy can be converted to electrical energy.

5. <u>Geothermal power</u>

   The Earth's interior is very hot and the heat flow can increase the temperature of underground rock basins to 200°C or more. By pumping water down to the hot rocks, steam can be drawn up to generate electricity.

Understanding power efficiency at power stations in Singapore

Brief Description:

The power generation sector accounts for about half of the total fuel consumption in Singapore. This energy is used to generate electricity, which is then used to power homes and industries in Singapore. Given the large amounts of fuel consumed by the power generation sector, improving energy efficiency in power generation would have a significant impact on the overall **energy efficiency** in Singapore.

Power Generation Efficiency:

Since the liberalisation of the electricity market, the power generation companies have moved towards the increased use of natural gas combined-cycle turbine technology. From 2000 to 2006, the electricity generated by natural gas has increased from 19% to 78%. As a result, Singapore's overall power generation efficiency has improved from 38% in 2000 to 44% in 2006. Between 2021 and 2025, natural gas consistently accounted for approximately 95% of Singapore's electricity generation fuel mix.

	2000	2001	2002	2003	2004	2005	2006	2020
electricity generated by natural gas	19%	29%	44%	60%	69%	74%	78%	95%

(Source: Energy Market Authority. www.ema.gov.sg)

Regenerative Braking (Hybrid vehicles, Mass Transit Rail, F1 cars)

Vehicles driven by electric motors use the motor as a generator when using regenerative braking: it is operated as a generator during braking and its output is supplied to an electrical load; the transfer of energy to the load provides the braking effect.

Regenerative braking is used on hybrid/electric cars to recoup some of the energy lost during stopping. This energy is saved in a storage battery and used later to power the motor whenever the car is in electric mode.

Applied to MRT trains: A conventional electric train braking system uses dynamic braking, where the remaining kinetic energy of a moving train is dissipated as waste, mainly in the form of heat. In SMRT trains, this excess energy is not wasted.

Using regenerative braking, the electric motors reverse the current, which slows down the train. At the same time, it generates electricity to be returned to the power distribution system.

This generated electricity is used to power other trains within the network. Any excess electricity is used to offset power demands of other loads such as air conditioners and lighting in stations.

This saves about 15% of the energy costs for running the trains.

http://www.smrt.com.sg/about_us/documents/environment/Regenerative_Braking_in_SMRT_Trains.pdf

~~ THE END ~~