# Topical Revision Paper 9

## Term 3 Unit 9: Points, Lines and Shapes

**Question 1:**
The vertices of a triangle *ABC* are *A* (5, 10), *B* (0, 0) and *C* (–6, 8).

**(i)** Show that the triangle *ABC* is an isosceles triangle. [2]

**(ii)** Calculate the area of triangle *ABC*. [2]

**(iii)** Find the coordinates of the foot of perpendicular from *A* to *BC*. [1]

**(iv)** Given that four points *A*, *B*, *C* and *D* form a rhombus, find the coordinates of the point *D*. [3]

**(v)** Find the equation of *AC*. Hence or otherwise, find the coordinates of the foot of perpendicular from *B* to *AC*. [5]

**Solution:**
(i)

$$AB = \sqrt{(5-0)^2 + (10-0)^2} = \sqrt{125} \text{ units}$$

$$BC = \sqrt{(-6-0)^2 + (8-0)^2} = 10 \text{ units}$$

$$AC = \sqrt{(5+6)^2 + (10-8)^2} = \sqrt{125} \text{ units}$$

*AB* = *AC*
Hence, triangle *ABC* is isosceles.

(ii)
Area of triangle ABC

$$= \frac{1}{2}\begin{vmatrix} 0 & 5 & -6 & 0 \\ 0 & 10 & 8 & 0 \end{vmatrix}$$

$$= 50 \text{ units}^2$$

(iii)
Foot of perpendicular from *A* to *BC* in the isosceles triangle *ABC* is the midpoint of *BC*. That is

$$\left(\frac{-6+0}{2}, \frac{8+0}{2}\right) = (-3, 4)$$

(iv)

Let the coordinate of *D* be (*x*, *y*). *ABDC* is the rhombus.

Since the diagonals of a rhombus bisect each other,

$$\left(\frac{x+5}{2},\frac{y+10}{2}\right)=(-3,4)$$

$$x=-11, y=-2$$

$$D:(-11,-2)$$

(v)

Gradient of *AC* = $\frac{8-10}{-6-5}=\frac{2}{11}$

Equation of the line *AC*:

$$y-10=\frac{2}{11}(x-5)$$

$$y=\frac{2}{11}x+\frac{100}{11}$$

Gradient of the line perpendicular to *AC* = $-\frac{11}{2}$

Equation of the line perpendicular to *AC* through *B*:

$$y=-\frac{11}{2}x$$

Solving the simultaneous equations, we have

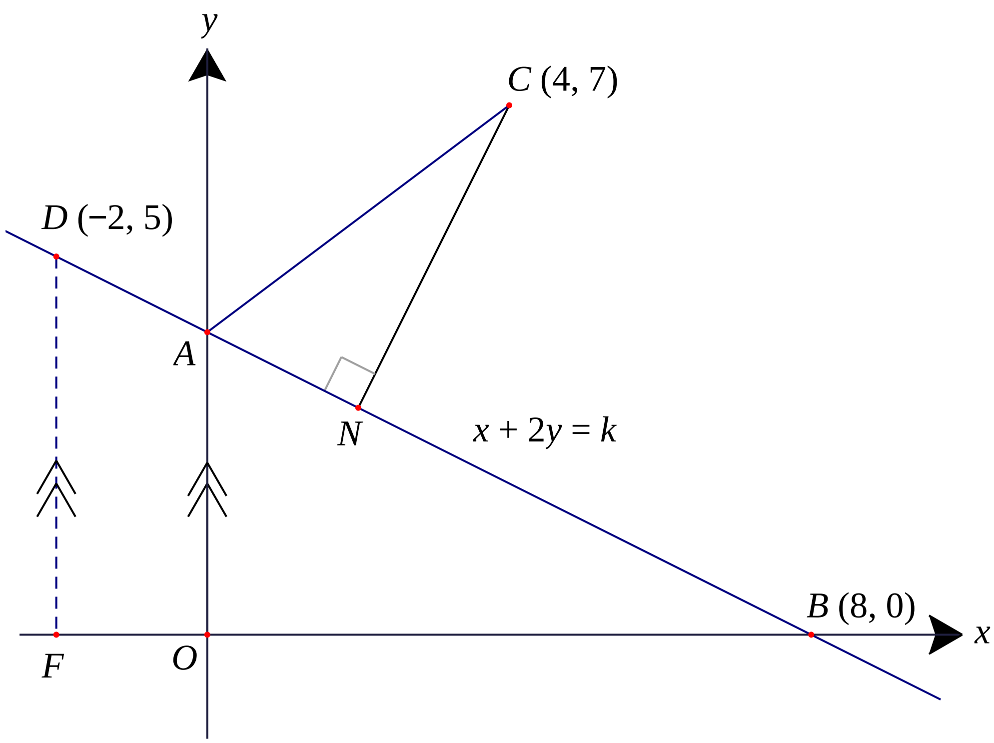
$$x=-\frac{8}{5}$$

$$y=\frac{44}{5}$$

The coordinate of the foot of perpendicular are $\left(-\frac{8}{5},\frac{44}{5}\right)$.

**<u>Question 2 :</u>**

The diagram below shows triangle *ANC*. Point *B* has coordinates $(8,0)$, Point *C* has coordinates $(4,7)$ and Point *D* has coordinates $(-2,5)$. The equation of the straight line *DB* is $x+2y=k$ and this line intersects the *y*-axis at *A*. Point *N* lies on the line *DB* such that *CN* is perpendicular to line *DB*. Point *F* is on the *x*-axis such that *DF* is parallel to the *y*-axis.




**(i)** Find the value of $k$, in the equation of straight line $DB$. [2]

**(ii)** Find

**(a)** the gradient of $CN$, [2]

**(b)** the equation of $CN$. [1]

**(iii)** Calculate the coordinates of $N$. [2]

**(iv)** Is $CN$ a perpendicular bisector of $BD$? Explain your answer. [1]

**(v)** Calculate the area of triangle $DFB$. [3]

**(vi)** Show clearly that the distance between $D$ and $B$ is 11.18 units, correct to 2 decimal places. [2]

**(vii)** Find the perpendicular distance of $F$ to $BD$. [2]

**Solution:**

(i)

$x+2y=k$

Substitute $(-2,5)$ into the equation to find value of *k*,

$(-2)+2(5)=k$

$k=8$

(ii)

Rewriting the equation of *DB*, we have $y=-\frac{1}{2}x+4$.

**(a)** Gradient of *DB* = $-\frac{1}{2}$

Gradient of *CN* = 2

**(b)** $y-7=2(x-4)$

Equation of *CN* is $y=2x-1$

(iii)

$2x-1=-\frac{1}{2}x+4$

$x=2, y=3$

$N:(2,3)$

(iv)

Midpoint of *BD* = $\left(\frac{-2+8}{2},\frac{5+0}{2}\right)=\left(3,2\frac{1}{2}\right)$

Therefore *CN* is not a perpendicular bisector of *BD* since *N* is not the midpoint of *BD*.

(v)

Coordinate of $F=(-2,0)$

Area of triangle *DFB* = $\frac{1}{2}\times10\times5=25$ units$^2$

(vi)

Distance of *DB* = $\sqrt{(-2-8)^2+(5-0)^2}=\sqrt{100+25}=11.18$ units

(vii)

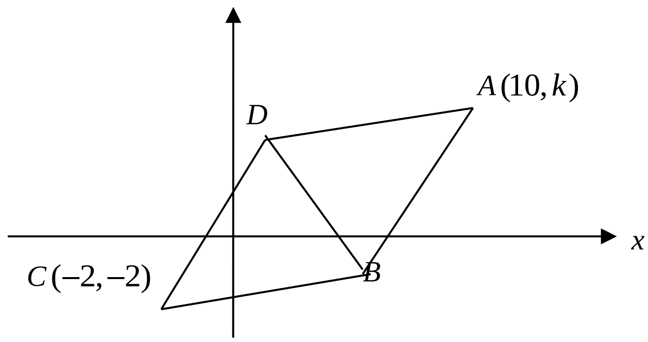
$\frac{1}{2}\times\sqrt{125}\times d=25$ or $\frac{1}{2}\times11.18\times d=25$

$d=4.47$ or $2\sqrt{5}$ $\quad d=4.47$

**Question 3:**

**Solutions to this question by accurate drawing will not be accepted.**

*y*




The diagram shows a rhombus *ABCD* with vertices *A*(10,*k*), *B*, *C*(–2,–2) and *D* in which the equation of line *BC* is given to be $8y-x+14=0$ and the equation of line *BD* is $2y+3x-16=0$.

**(i)** Show that $k = 6$. [3]

**(ii)** Find the coordinates of *B* and of *D*. [6]

**(iii)** Find the area of the rhombus *ABCD*. [2]

**<u>Solution:</u>**

(i)

Diagonals of rhombus bisect each other. Midpoint.

$BD$ is $2y+3x-16=0$

when $x=\dfrac{-2+10}{2}=4,$

$2y+3(4)-16=0$

$y=\dfrac{4}{2}=2$

$\dfrac{k-2}{2}=2$

$k=6$ (shown)

(ii)

$BC$ and $BD$ intersect at $B$

$x=8y+14$

$3x=42+24y=16-2y$

$26y=-26$

$y=-1$

$x=-8+14=6$

$B:(6,-1)$

Let $D = (x, y)$

$$\frac{x+6}{2} = 4$$

$x = 2$

$$\frac{y-1}{2} = 2$$

$y = 5$

$D:(2,5)$

(iii)

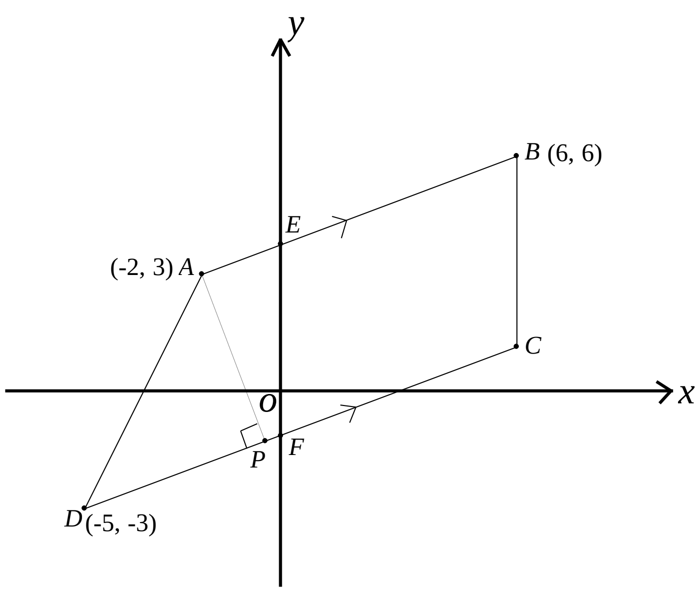
Area of *ABCD*

$$= \frac{1}{2}\begin{vmatrix} -2 & 6 & 10 & 2 & -2 \\ -2 & -1 & 6 & 5 & -2 \end{vmatrix}$$

$= 52 \text{ unit}^2$

**<u>Question 4:</u>**

**Solutions to this question by accurate drawing will not be accepted.**




The diagram shows a trapezium *ABCD* in which *AB* is parallel to *DC* and *BC* is a vertical line. The vertices of the trapezium are *A*(–2, 3), *B*(6, 6), *C* and *D*(–5, –3). The perpendicular from *A* to *DC* meets *DC* at the point *P*.

**(i)** Find the coordinates of *P* and *C*. [6]

The lines *AB* and *DC* cut the *y*-axis at *E* and *F* respectively.

| | |
|---|---|
| **(ii)** Find the area of the parallelogram *BCFE*. [2] | |

Solution:

(i)

$$m_{AB} = m_{DC} = \frac{6-3}{6+2} = \frac{3}{8}$$

Equation of *AP*:

$$y-3 = -\frac{8}{3}(x+2) \text{ -------------------- (1)}$$

Equation of *DC*:

$$y+3 = \frac{3}{8}(x+5) \text{ ------------------------ (2)}$$

(2) – (1): $\frac{3}{8}(x+5)+\frac{8}{3}(x+2)=6$

$$\left(\frac{3}{8}+\frac{8}{3}\right)x = 6-\frac{15}{8}-\frac{16}{3}$$

$$x = -\frac{29}{73} \quad [\text{or} -0.39726 \approx -0.397]$$

$$y = -\frac{8}{3}(-\frac{29}{73}+2)+3 = -\frac{93}{73} = -1\frac{20}{73} \quad [\text{or} -1.2739 \approx -1.27]$$

$$\therefore \quad P:\left(-\frac{29}{73}, -1\frac{20}{73}\right)$$

Sub $x = 6$ into (2): $y = \frac{3}{8}(6+5)-3 = \frac{7}{8} = 1\frac{1}{8}$ [or 1.125]

$\therefore$ the coordinates of *C* are $\left(6, 1\frac{1}{8}\right)$

(ii)

Area of *BCFE* = $6 \times \left(6-1\frac{1}{8}\right) = \frac{117}{4} = 29\frac{1}{4}$ units$^2$