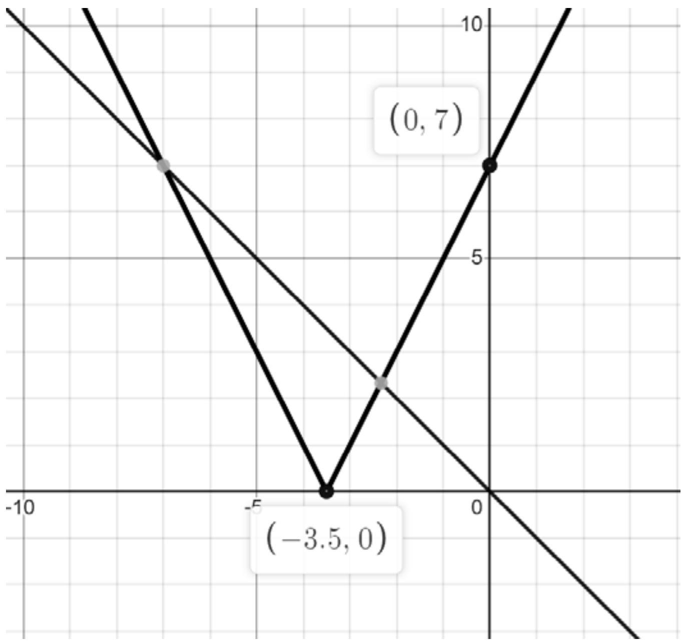
PAPER i Sec 3 Solutions for students

Qn	Solutions
1	$8^{x+3y}=4$ $2^{3(x+3y)}=2^2$ $3x+9y=2--------(1)$ $2x-3y=1--------(2)\times 3$ $\text{Fr}(2):$ $6x-9y=3-----(3)$ $(1)+(3):$ $9x=5$ $x=\frac{5}{9}$ $\frac{15}{9}+9y=2$ $y=\frac{1}{27}$
2(a)	$3\sqrt{216}-\sqrt{24}+\frac{4}{\sqrt{6}}$ $=3\sqrt{36(6)}-\sqrt{6(4)}+\frac{4\sqrt{6}}{6}$ $=3(6)\sqrt{6}-2\sqrt{6}+\frac{2}{3}\sqrt{6}$ $=\sqrt{6}\left(18-2+\frac{2}{3}\right)$ $=16\frac{2}{3}\sqrt{6}$ $=\frac{50\sqrt{6}}{3}$

2(b)	$8\left(5^{2x-1}\right)-5^{x}=2\left(5^{x+2}\right).$ $8\left(5^{2x-1}\right)-5^{x}=50\left(5^{x}\right)$ $\frac{8}{5}\left(5^{2x}\right)-51\left(5^{x}\right)=0$ $8\left(5^{2x}\right)-255\left(5^{x}\right)=0$ let $a=5^{x}$ $8a^{2}-255a=0$ $a\left(8a-255\right)=0$ $a=0,\ a=\frac{255}{8}$ $5^{x}=0(rej.),\quad 5^{x}=\frac{255}{8}$ $x=\frac{\lg\frac{255}{8}}{\lg 5}$ $=2.15\ (3s.f)$
3(i)	$2x^{3}-128=2\left(x^{3}-64\right)$ $=2\left(x-4\right)\left(x^{2}+4x+16\right)$
3(ii)	$2x^{3}-128=4-x$ $2x^{3}-128-\left(4-x\right)=0$ $2\left(x-4\right)\left(x^{2}+4x+16\right)-\left(4-x\right)=0$ $2\left(x-4\right)\left(x^{2}+4x+16\right)+\left(x-4\right)=0$ $\left(x-4\right)\left[2\left(x^{2}+4x+16\right)+1\right]=0$ $\left(x-4\right)\left(2x^{2}+8x+33\right)=0$ $x=4,\quad 2x^{2}+8x+33=0$ $x=\frac{-8\pm\sqrt{64-4(2)(33)}}{4}(reject)$

4(a)	$3x^2+16x-12-3x+2>0$ $3x^2+13x-10>0$ $(3x-2)(x+5)>0$ $x<-5 \;\; or \;\; x>\frac{2}{3}$
4(b)	$2y-b=ax \Rightarrow y=\frac{ax+b}{2}$ $4x^2+5y^2-20=0,$ $4x^2+5\left(\frac{ax+b}{2}\right)^2-20=0,$ $4x^2+5\left(\frac{a^2x^2+2axb+b^2}{4}\right)-20=0$ $16x^2+5\left(a^2x^2+2axb+b^2\right)-80=0$ $\left(16+5a^2\right)x^2+10abx+5b^2-80=0$ discriminant, $b^2-4ac=0$ $100a^2b^2-4\left(16+5a^2\right)\left(5b^2-80\right)=0$ $100a^2b^2-4\left(80b^2-1280+25a^2b^2-400a^2\right)=0$ $-320b^2+5120+1600a^2=0$ $320b^2=5120+1600a^2$ $\therefore b^2=16+5a^2$ (shown)
5(a)	$x-5=4\sqrt{x}$ $(x-5)^2=16x$ $x^2-10x+25-16x=0$ $x^2-26x+25=0$ $(x-25)(x-1)=0$ $x=25, \quad x=1(reject)$

5(b)	$\log_3 x^2 + 2\log_x 3 = 5$ $2\log_3 x + 2\left(\dfrac{\log_3 3}{\log_3 x}\right) = 5$ let $\log_3 x = a$, $\therefore 2a^2 + 2 = 5a$ $2a^2 - 5a + 2 = 0$ $(2a-1)(a-2) = 0$ $a = 0.5, \quad a = 2$ $\log_3 x = 0.5, \quad \log_3 x = 2$ $x = \sqrt{3}, \qquad x = 3^2 = 9$
6(i)	$\lvert -2x-7 \rvert \le -x$ 
6(ii)	$y = 2x + 7$ ------(1) $y = -x$ --------(2) $2x + 7 = -x$ $3x = -7$ $x = \dfrac{-7}{3}$ $y = -2x - 7$ ----(1) $y = -x$ ------(2) $-2x - 7 = -x$ $\therefore x = -7$

	$\therefore\ -7 \le x \le -2\frac{1}{3}$ Method 2 $(-2x-7)^2 = (x)^2$ $4x^2 + 28x + 49 = x^2$ $3x^2 + 28x + 49 = 0$ $x = \frac{-7}{3}$ or $x = -7$ $-7 \le x \le -2\frac{1}{3}$ Method 3 $-2x-7 = x$ $-3x = 7$ $x = \frac{-7}{3}$ $-2x-7 = -x$ $x = -7$ $-7 \le x \le -2\frac{1}{3}$
7(a)	$y = a(x-9)^2 + 18$ At $(3,0): 0 = a(36) + 18$ $a = -\frac{1}{2}$ $\therefore y = -\frac{1}{2}(x-9)^2 + 18$ $y = -\frac{1}{2}(x^2 - 18x + 81) + 18$ $= -\frac{1}{2}x^2 + 9x - \frac{45}{2}$

	Method 2 $y = ax^2 + bx + c$ $9a + 3b + c = 0 ------(1)$ $81a + 9b + c = 18 -----(2)$ $169a + 13b + c = 10 ----(3)$ solve simultaneous equations $a = \frac{-1}{2}, b = 9, c = \frac{-45}{2}$ $y = \frac{-1}{2}x^2 + 9x - \frac{45}{2}$
7(b) (i)	$3x + 2\pi r = 20$ $r = \frac{20 - 3x}{2\pi}$
7(b) (ii)	Area of triangle $= \frac{1}{2}x(x)\sin 60°$ $= \frac{\sqrt{3}x^2}{4}$ Area of circle $= \pi\left(\frac{20 - 3x}{2\pi}\right)^2$ $= \frac{(20 - 3x)^2}{4\pi}$ $\therefore A = \frac{\sqrt{3}\pi x^2 + (20 - 3x)^2}{4\pi}$

7(b) (iii)	$\therefore A=\dfrac{\sqrt{3}\pi x^2+\left(20-3x\right)^2}{4\pi}$ $=\dfrac{\sqrt{3}\pi x^2+\left(400-120x+9x^2\right)}{4\pi}$ $=\dfrac{1}{4\pi}\left[\left(9+\sqrt{3}\pi\right)x^2-120x+400\right]$ $=\dfrac{1}{4\pi}\left(9+\sqrt{3}\pi\right)\left[x^2-\dfrac{120x}{9+\sqrt{3}\pi}+\dfrac{400}{9+\sqrt{3}\pi}\right]$ $=\dfrac{\left(9+\sqrt{3}\pi\right)}{4\pi}\left[\left(x-\dfrac{60}{9+\sqrt{3}\pi}\right)^2-\left(\dfrac{60}{9+\sqrt{3}\pi}\right)^2+\dfrac{400}{9+\sqrt{3}\pi}\right]$ $=\left(\dfrac{9+\sqrt{3}\pi}{4\pi}\right)\left(x-\dfrac{60}{9+\sqrt{3}\pi}\right)^2$ $+\left(\dfrac{9+\sqrt{3}\pi}{4\pi}\right)\left[\dfrac{-3600}{(9+\sqrt{3}\pi)^2}+\dfrac{400}{9+\sqrt{3}\pi}\right]$ $=\left(\dfrac{9+\sqrt{3}\pi}{4\pi}\right)\left(x-\dfrac{60}{9+\sqrt{3}\pi}\right)^2+11.994$ Hence, minimum total area $=12.0m^2$ (3s.f.) Method 2: $1.1492x^2-9.5493x+31.831$ $=1.1492(x^2-8.3095x+27.698)$ $=1.1492[(x-4.1548)^2-(4.1548)^2+27.698]$ $=1.1492(x-4.1548)^2+11.99$ $\therefore$ min. area = 12.0 m^2
8(i)	coordinates of $S=(x, y)$ $\left(\dfrac{-3+x}{2},\dfrac{3+y}{2}\right)=\left(-1,4\right)$ $\dfrac{-3+x}{2}=-1,\quad \dfrac{3+y}{2}=4$ $x=1,\quad y=5$ $\therefore S:\left(1,5\right)$

8(ii)	radius $= \sqrt{(4-3)^2 + (-1+3)^2}$ $= \sqrt{5}$ units equation of circle is $(x+1)^2 + (y-4)^2 = 5$ $x^2 + 2x + 1 + y^2 - 8y + 16 = 5$ $x^2 + y^2 + 2x - 8y + 12 = 0$
8(iii)	grad $RS = \dfrac{5-3}{1-2}$ $= -2$ $grad\ \ QS = \dfrac{5-4}{1+1}$ $= \dfrac{1}{2}$ $gradRS \times gradQS = -2 \times \dfrac{1}{2} = -1$ $Hence,\ RS \perp QS,\ \ \angle RSQ = 90°.$ RS *is a tangent to the circle.* (shown)
8(iv)	Area of quadrilateralPTRS $= \dfrac{1}{2}\begin{vmatrix} -3 & 0 & 2 & 1 & -3 \\ 3 & 2 & 3 & 5 & 3 \end{vmatrix}$ $= \dfrac{1}{2}\left[(-6+0+10+3)-(0+4+3-15)\right]$ $= \dfrac{1}{2}(7-(-8))$ $= 7.5$ units^2 Method 2: Area of quadrilateralPTRS $= \dfrac{1}{2}(5)(1) + \dfrac{1}{2}(5)(2)$ $= 7.5$ units^2
9(i)	$\mathrm{f}(x) = 2e^x$ Step 1: Reflect the graph of $\mathrm{f}(x) = 2e^x$ along x-axis to get $h(x) = -2e^x$ Step 2: Translate 5 units in the positive y-direction, the graph of $h(x) = -2e^x$ to get the graph of $g(x) = 5 - 2e^x$.

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<tr>
<td>9(ii)</td>
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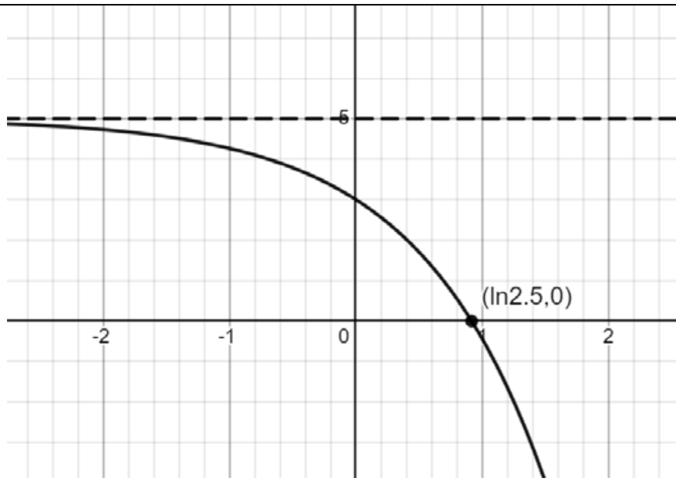

$when\ y = 0,$
$5 - 2e^x = 0$
$2e^x = 5$
$e^x = \frac{5}{2}$
$x = \ln\frac{5}{2}$
$x = \ln 2.5$
$when\ x = 0,$
$y = 5 - 2e^0 = 3$
y-intercept: (0, 3)
x-intercept: (ln2.5, 0)
asymptote: $y = 5$
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<tr>
<td>9(iii)</td>
<td>
$\ln\left(\frac{x+6}{4}\right) = x$
$e^x = \frac{x+6}{4}$
$2e^x = \frac{x+6}{2}$
$-2e^x = -\frac{x}{2} - 3$
$5 - 2e^x = -\frac{x}{2} + 2$
$\therefore y = -\frac{x}{2} + 2$
$y = 2 - \frac{x}{2}$
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10a	$A=\begin{pmatrix}-2 & -3\\ 4 & 3\end{pmatrix}$ $B=\begin{pmatrix}-2 & 1\\ -3 & 5\end{pmatrix}$ $3A-B=3\begin{pmatrix}-2 & -3\\ 4 & 3\end{pmatrix}-\begin{pmatrix}-2 & 1\\ -3 & 5\end{pmatrix}$ $=\begin{pmatrix}-6 & -9\\ 12 & 9\end{pmatrix}-\begin{pmatrix}-2 & 1\\ -3 & 5\end{pmatrix}$ $=\begin{pmatrix}-4 & -10\\ 15 & 4\end{pmatrix}$
10b (i)	$C=\begin{pmatrix}290 & 240 & 200\\ 235 & 362 & 117\end{pmatrix}$
10b (ii)	$D=\begin{pmatrix}240\\ 95\\ 110\end{pmatrix}$
10b (iii)	$E=CD$ $=\begin{pmatrix}290 & 240 & 200\\ 235 & 362 & 117\end{pmatrix}\begin{pmatrix}240\\ 95\\ 110\end{pmatrix}$ $=\begin{pmatrix}69600+22800+22000\\ 56400+34390+12870\end{pmatrix}$ $=\begin{pmatrix}114400\\ 103660\end{pmatrix}$
10b (iv)	The elements in matrix E represents the total profit made for selling 290 air con units, 240 units of water heater and 200 units of humidifier in 2021 and 235 air con units, 362 units of water heater and 117 units of humidifier in 2020 respectively.
11(i)	$\mathrm{f}(-2)=0$ $\mathrm{f}(3)=185$ $-8a+28+2b-4=0$ $-8a+2b=-24---(1)$ or $-4a+b=-12---(1)$ $27a+63-3b-4=185$ $27a-3b=126---(2)$ or $9a-b=43---(2)$ $(1)+(2):5a=30$ $a=6$ $b=12$

11(ii)	$f(x)=6x^3+7x^2-12x-4$ $\begin{array}{rrrrr} & 3x & +\frac{7}{2} & & \\ \cline{2-5} 2x^2+3 & \big)\, 6x^3 & +7x^2 & -12x & -4 \\ -) & 6x^3 & & +9x & \\ \hline & & 7x^2 & -21x & -4 \\ & -) & 7x^2 & & +\frac{21}{2} \\ \hline & & & -21x & -\frac{29}{2} \end{array}$ Remainder $=-21x-\dfrac{29}{2}$
12(i)	$\tan\theta=\dfrac{6}{3}$ $\theta=\tan^{-1}(2)=1.1071$ $\angle PQB=\pi-1.1071=2.0344$ $\therefore \angle PQB=2.03$
12(ii)	$radius BQ=\sqrt{3^2+6^2}$ $=\sqrt{45}$ $=3\sqrt{5}$ $CP=3\sqrt{5}+3$ $arc\ BRP=r\theta=3\sqrt{5}(2.0344)$ $=13.6$ Perimeter $CPRBC=6+CP+arc\ BRP$ $=6+3+\sqrt{45}+13.65$ $=29.4$ cm
12(iii)	area of shaded region $=\dfrac{1}{2}r^2\theta-\dfrac{1}{2}r^2\sin\theta$ $=\dfrac{1}{2}\left(\sqrt{45}\right)^2(2.0344)-\dfrac{1}{2}\left(\sqrt{45}\right)^2\sin 2.0344$ $=25.6\text{ cm}^2$

13(i)	$2x^2 - ax - 3y = 0,$ $3y = 2x^2 - ax$ $y = \frac{2}{3}x^2 - \frac{a}{3}x$ $\frac{y}{x} = \frac{2}{3}x - \frac{a}{3}$
13(ii)	$\frac{y}{x} - 1 = \frac{2}{3}x - \frac{a}{3} - 1$ $\frac{y-x}{x} = \frac{2}{3}x - \frac{a}{3} - 1$ gradient, $\frac{h-2}{3} = \frac{2}{3}$ $\therefore h - 2 = 2$ $h = 4$ $\frac{y-x}{x} = \frac{2}{3}x - \frac{a}{3} - 1$ At $(2,2)$, $2 = \frac{2}{3}(2) - \frac{a}{3} - 1$ $= \frac{4}{3} - \frac{a}{3} - 1$ $= \frac{1}{3} - \frac{a}{3}$ $6 = 1 - a$ $a = -5$
14 (i)	$(BC)^2 = 60^2 + 72^2 - 2(60)(72)\cos 75^\circ$ $BC = 80.92$ $BC = 80.9$ m
14(ii)	$\frac{BC}{\sin 75^\circ} = \frac{72}{\sin \angle ABC}$ $\sin \angle ABC = \frac{72 \sin 75^\circ}{80.918}$ $= 0.8595$ $\angle ABC = 59.3^\circ$

14 (iii)	$\angle NAB = 360^\circ - 217^\circ - 75^\circ$ $= 68^\circ$ $\angle PBA = 180 - 68$ $= 112^\circ$ bearing of C from B $= 59.3^\circ + 112^\circ$ $= 171.3^\circ$