Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2016
Secondary Three

INTEGRATED MATHEMATICS PAPER 1

2 hours

Thursday **6 October 2016** **0845 - 1045**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Answer questions 1 – 12 before attempting the Bonus Question.
2. Answer the whole of question 12 on the INSERT provided.
3. Write your answers and working on the separate answer paper provided.
4. Write your name, register number and class on all the work you hand in.
   At the end of the examination, fasten all your work securely together.
5. Omission of essential working will result in loss of marks.
6. Write in dark blue or black pen on both sides of the paper.
7. You may use an HB pencil for any diagrams or graphs.
8. Do not use staples, paper clips, glue or correction fluid.
9. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 80.
3. The use of an approved scientific calculator is expected, where appropriate.
4. You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

TRIGONOMETRY

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

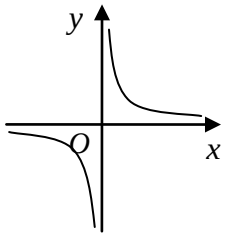
1. It is given that y is directly proportional to x^3, and $y = 10$ when $x = 2$.

(i) Find an equation connecting x and y. [2]

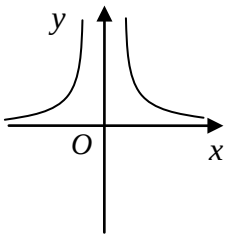
(ii) Calculate the value of x when $y = 80$. [1]

2. **(a)** The sketch of the graphs $y = x^n$ are shown below. Write down a possible value of n for each of the graphs. [2]

(i)

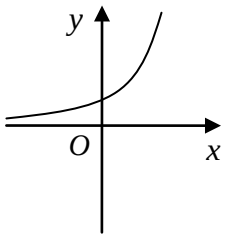


(ii)

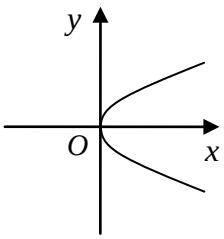


(b) Write down a possible equation for each of the graphs. [2]

(i)



(ii)



3. The following equations

(i) $y = (x + a)(1 - x^2)$,

(ii) $x^2y - ay - bx = 0$,

may be represented by a straight line in the form $Y = mX + c$, where a, b, m and c are constants while X and Y are functions of x and/or y. **Copy** the following table and complete it with an expression each for Y, X, m and c. [4]

	Y	X	m	c
$y = (x + a)(1 - x^2)$		x		
$x^2y - ay - bx = 0$				b

4. In a school fun fair, a booth sold three types of items: keychains, notebooks and pens. The table illustrates the number of items sold during different sessions of the day at the booth.

	Keychains	Notebooks	Pens
Session 1	20	35	48
Session 2	30	50	54
Session 3	54	72	60

The information in the table above can be represented by a matrix $\mathbf{P} = \begin{pmatrix} 20 & 35 & 48 \\ 30 & 50 & 54 \\ 54 & 72 & 60 \end{pmatrix}$.

The price for each item was as follows:

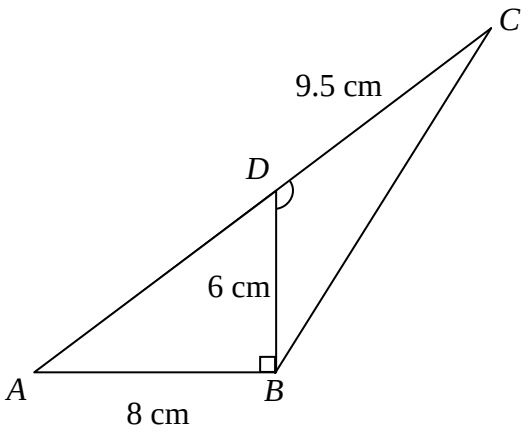
Keychain: \$1 Notebook: \$3 Pen: \$2

(a) **(i)** Write down a matrix **Q** such that the elements of the matrix product **PQ** give the total takings for each session. Find the product **PQ**. [2]

(ii) 10% of the takings will be donated to the school's Pocket Money Fund. Write down a row matrix **T** such that the element of the matrix product **TPQ** gives the total amount of donation from the booth. [1]

(b) It was later discovered that the number of each type of items sold in Session 3 was wrongly recorded to be 20% more than the actual number. Write down a row matrix **R** such that the elements of the matrix product **RP** give the correct number of each type of items sold in Session 3. Find the product **RP**. [2]

5. In the diagram, ADC is a straight line. $\angle ABD = 90°$, $AB = 8$ cm, $BD = 6$ cm, and $DC = 9.5$ cm.



(a) Without the use of calculators, find the value of

(i) $\cos \angle BDC$, [2]

(ii) $\sin \angle BDC$. [1]

Hence, calculate the area of triangle BDC. [1]

(b) Find the shortest length from B to AC. [2]

(c) Calculate the length of BC. [2]

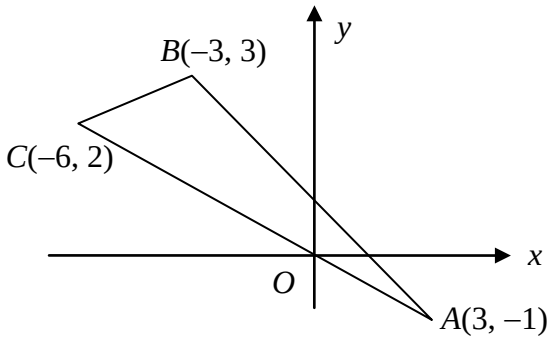
6. Given that $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Find the matrix $\mathbf{C}$ such that $\mathbf{A} + \mathbf{C} = \mathbf{B} + 3\mathbf{I}$. [2]

(ii) Find $\mathbf{A}^{-1}$. Hence, find the matrix $\mathbf{D}$ such that $\mathbf{AD} = \mathbf{B}^2$. [4]

7. **The solution of this question by accurate drawing will not be accepted.**

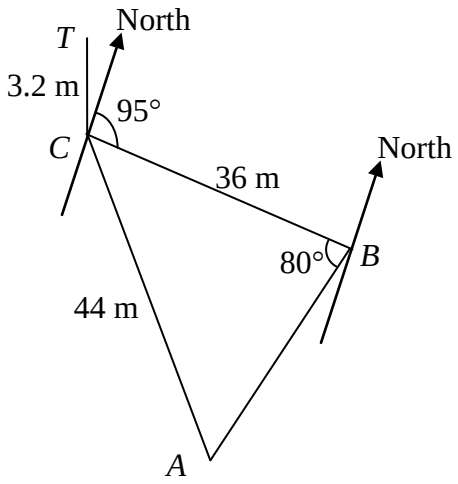
The diagram shows a triangle ABC, where A, B and C are (3, –1), (–3, 3) and (–6, 2) respectively.



(i) Find the area of triangle ABC. [2]

(ii) Point X lies on the line segment AB such that $BX:XA = 2:1$. Find the coordinates of point X. [2]

8. The diagram shows a triangular field ABC, where B is 36 m from C on a bearing of 095°, $\angle ABC = 80°$ and $AC = 44$ m.

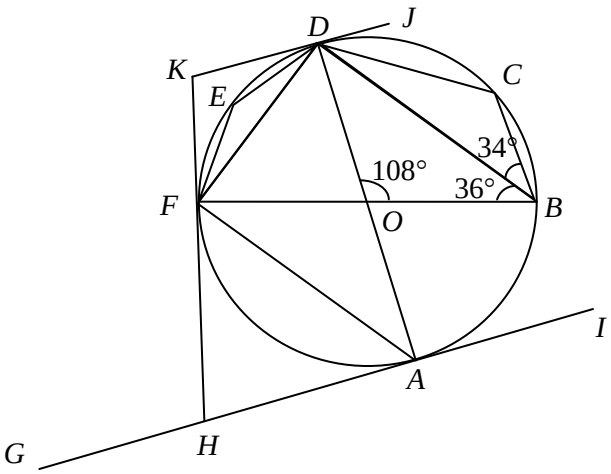


(i) Calculate how far C is west of B. [2]

(ii) Find the bearing of A from B. [2]

(iii) A flagpole CT of height 3.2 m stands at C. An air drone is flying from A, towards the flagpole CT, at an angle of elevation of $x°$ from A. Find the minimum whole number x such that the air drone will not crash onto the flagpole. [2]

9.



The points A, B, C, D, E and F lie on the circumference of a circle with centre O. AD and FB are diameters of the circle. The lines $GHAI$, HFK and KDJ are tangents to the circle. It is given that $\angle BOD = 108°$, $\angle CBD = 34°$ and $\angle DBO = 36°$.

(a) Find

(i) $\angle DAF$, [1]

(ii) $\angle CDJ$, [1]

(iii) $\angle FED$. [1]

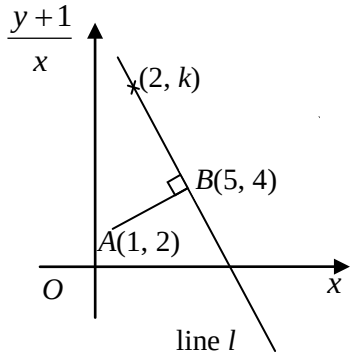
(b) Giving your reasons, find $\angle DCB$. [2]

(c) **(i)** Prove that $AOFH$ is a cyclic quadrilateral. [2]

(ii) Hence, find $\angle GHF$. State your reasons clearly. [2]

(d) Given that the length of chord DB = 6 cm, calculate the length of the diameter BF. [2]

10. The diagram shows part of a straight line l obtained by plotting $\frac{y+1}{x}$ against x.



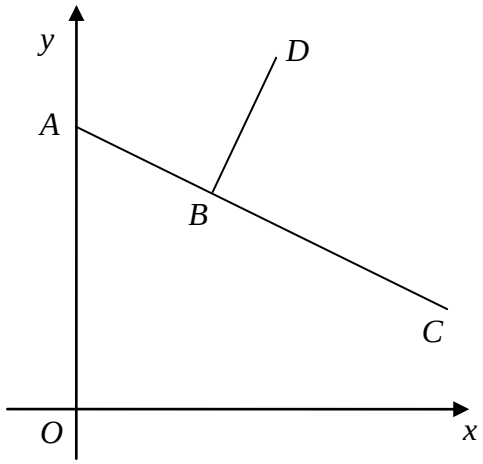
(i) Line AB is perpendicular to line l. A and B are (1, 2) and (5, 4) respectively. Show that the equation of line l is $y = -2x^2 + 14x - 1$, when y is expressed in terms of x. [4]

(ii) If (2, k) lies on line l, find the value of k. [2]

(iii) Find the vertical-intercept of the new line formed when line l is plotted as a graph of $\frac{y-17x}{x+1}$ against x. [2]

11. The solution of this question by accurate drawing will not be accepted.

In the diagram, the line segments AC and BD intersect at point B (4, 6). It is given that A, C and D are (0, 8), (10, 3) and (6, 10) respectively.



(a) Find the equation of the line parallel to AD and passing through B. [2]

(b) Show that AB is perpendicular to BD. [3]

(c) If $AXCD$ is a parallelogram, state the coordinates of X. [2]

(d) **(i)** Find the centre-radius form of the equation of the circle which passes through A, B and D. [3]

(ii) Write down the coordinates of Y such that BY is a diameter of the circle in **(d)(i)**. [1]

12. Answer the whole of this question on the INSERT provided.

The graph of $y = \frac{1}{2}\left(\frac{1}{x^2} - x\right)$ is partially shown on the INSERT.

(a) By drawing a tangent, find the gradient of the graph at the point where $x = -0.8$. [3]

(b) Find the x-coordinate of a point on the curve at which the gradient is -1. [1]

(c) By drawing suitable straight lines on the graph, find

(i) the solution to the equation $\frac{1}{x^2} - x = 4$, [3]

(ii) the range of values of x for which $x^3 + 6x^2 - 1 < 0$ and $x \neq 0$. [3]

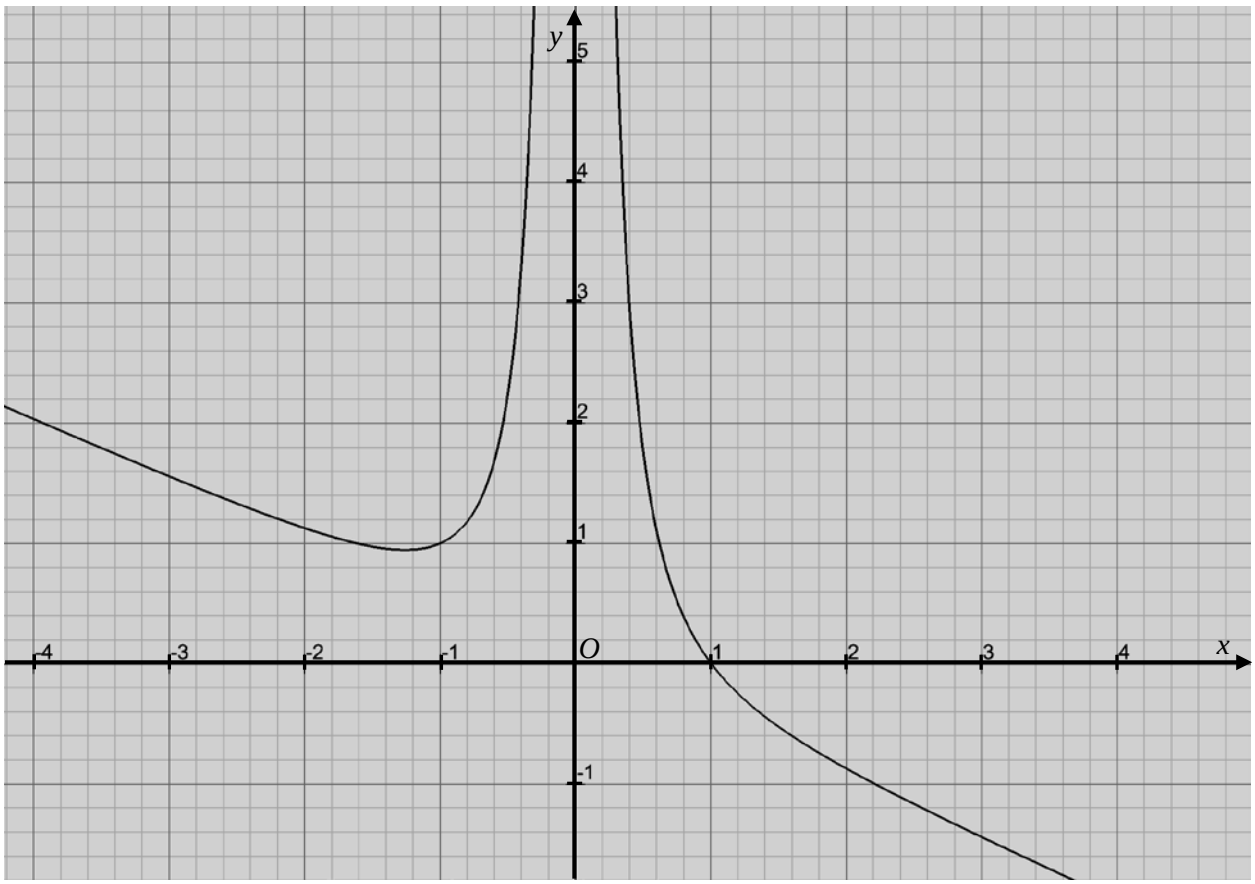
Bonus

Let ABC be an equilateral triangle and M be a point inside the triangle. Let D, E, F be points on BC, CA and AB respectively such that MD, ME and MF are perpendicular to BC, CA and AB respectively. Given that M lies on the arc of a circle with BC as its diameter, find the value of $\angle FDE$. State your reasons clearly. [2]

End of Paper

INSERT

Answer the whole of Question 12 on this piece of INSERT.
Tie this INSERT together with the rest of your answer scripts.



2016 Sec 3 EOY IM1 Answer Key

<table>
<tr><td>1i</td><td>$y = \frac{5}{4}x^3$</td></tr>
<tr><td>1ii</td><td>$x = 4$</td></tr>
<tr><td>2ai</td><td>Negative odd integers e.g. –1</td></tr>
<tr><td>2aii</td><td>Negative even integers e.g. –2</td></tr>
<tr><td>2bi</td><td>Given answer in the form:
$y = ka^{mx}$, where k, m>0, a>1. E.g. $y = 2^x$
OR
$y = ka^{mx}$, where k>0, 0<a<1, m<0.
E.g. $y = (0.5)^{-x}$</td></tr>
<tr><td>2bii</td><td>Given answer in the form:
$y^2 = kx$, where k >0. E.g. $y^2 = x$</td></tr>
<tr><td>3</td><td>
<table>
<tr><td></td><td>Y</td><td>X</td><td>m</td><td>c</td></tr>
<tr><td>$y = (x+a)(1-x^2)$</td><td>$\frac{y}{1-x^2}$</td><td>x</td><td>1</td><td>a</td></tr>
<tr><td>$x^2y - ay - bx = 0$</td><td>xy</td><td>$\frac{y}{x}$</td><td>a</td><td>b</td></tr>
</table>
</td></tr>
<tr><td>4ai</td><td>$\mathbf{Q} = \begin{pmatrix}1\\3\\2\end{pmatrix}$; $\mathbf{PQ} = \begin{pmatrix}221\\288\\390\end{pmatrix}$</td></tr>
<tr><td>4aii</td><td>$\mathbf{T} = \begin{pmatrix}0.1 & 0.1 & 0.1\end{pmatrix}$</td></tr>
<tr><td>4b</td><td>$\mathbf{R} = \begin{pmatrix}0 & 0 & \frac{5}{6}\end{pmatrix}$; $\mathbf{RP} = \begin{pmatrix}45 & 60 & 50\end{pmatrix}$</td></tr>
<tr><td>5ai</td><td>$-\frac{6}{10} = -\frac{3}{5}$</td></tr>
<tr><td>5aii</td><td>$\sin\angle BDC = \frac{4}{5}$; Area of triangle BDC =22.8 cm^2</td></tr>
<tr><td>5b</td><td>Shortest length = 4.8 cm</td></tr>
<tr><td>5c</td><td>BC = 14.0 cm (3s.f.)</td></tr>
<tr><td>6i</td><td>$\mathrm{C} = \begin{pmatrix}2 & -4\\0 & 0\end{pmatrix}$</td></tr>
<tr><td>6ii</td><td>$\mathrm{A} = \frac{1}{14}\begin{pmatrix}4 & -2\\1 & 3\end{pmatrix}$
$\mathrm{D} = \frac{1}{14}\begin{pmatrix}30 & -30\\-3 & 3\end{pmatrix}$ or $\begin{pmatrix}\frac{15}{7} & -\frac{15}{7}\\-\frac{3}{14} & \frac{3}{14}\end{pmatrix}$</td></tr>
</table>

7i	Area of triangle ABC = 9 sq units
7ii	$X = (1, \frac{1}{3})$
8i	35.9 m (3s.f.)
8ii	Bearing = 195°
8iii	$\angle CAT$ = 4.2° (1d.p.) Minimum whole number x = 5
9ai	$\angle DAF = 36°$
9aii	$\angle CDJ = 34°$
9aiii	$\angle FED = 144°$
9b	$\angle DCB = 126°$
9cii	$\angle GHF = 108°$
9d	Diameter = 7.42 cm
10i	$y = -2x^2 + 14x - 1$
10ii	k = 10
10iii	Vertical-intercept = –1
11a	Eqn: $y = \frac{1}{3}x + \frac{14}{3}$
11c	X = (4, 1)
11di	$(x-3)^2 + (y-9)^2 = 10$
11dii	Y(2, 12)
12a	Gradient ≈ 1.45 (Accept 1.13 to 1.87)
12b	1.26 (Accept 1.1 to 1.4)
12ci	Draw $y = 2$ $x \approx -3.94, -0.54$ or 0.47 (Accept $-4 \le x < -3.8$, $-0.6 < x < -0.4$ and $0.4 < x < 0.6$)
12cii	Draw $y = 3$ $-0.42 < x < 0.4$, and $x \neq 0$ (Accept $-0.5 < x < -0.4$ instead of –0.42)
Bonus	30°