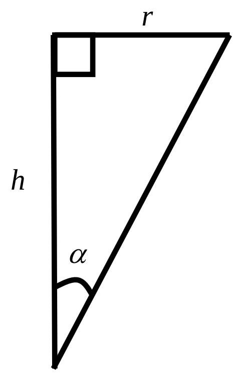
<table>
<tr><th colspan="2">2016 H2 Maths Paper 2</th></tr>
<tr><th colspan="2">SECTION A</th></tr>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>1</td><td>

Let h m and V m^3 be the depth and volume of water respectively at time t minutes.

Given: $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.1 \quad ---(1)$ and $\tan\alpha = 0.5$

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t} \quad ---(2)$, so we find V in terms of h first.



$\tan\alpha = 0.5 \Rightarrow r = 0.5h$

$V = \frac{1}{3}\pi(0.5h)^2 h = \frac{1}{12}\pi h^3$

$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{4}\pi h^2 = \frac{\pi h^2}{4} \quad ---(3)$

Sub (1) & (3) into (2):

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{4}{\pi h^2} \times 0.1 = \frac{0.4}{\pi h^2}$

When $V = 3$, $\frac{1}{12}\pi h^3 = 3 \Rightarrow h = \left(\frac{36}{\pi}\right)^{\frac{1}{3}}$,

so $\frac{\mathrm{d}h}{\mathrm{d}t} = 0.0251$ (3 s.f.)

The rate of increase of the depth of water is 0.0251 m per minute.

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<tr><td>2(a)(i)</td><td>

$\int x^2 \cos nx \,\mathrm{d}x = \frac{x^2 \sin nx}{n} - \frac{2}{n}\int x \sin nx \,\mathrm{d}x$

Integration by parts

$u = x^2 \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \cos nx$

$\frac{\mathrm{d}u}{\mathrm{d}x} = 2x \qquad v = \frac{\sin nx}{n}$

$\int x \sin nx \,\mathrm{d}x = -\frac{x\cos nx}{n} - \int\left(-\frac{\cos nx}{n}\right)(1)\,\mathrm{d}x$

$= -\frac{x\cos nx}{n} + \frac{1}{n}\int \cos nx \,\mathrm{d}x$

$= -\frac{x\cos nx}{n} + \frac{1}{n^2}\sin nx$

$u = x \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \sin nx$

$\frac{\mathrm{d}u}{\mathrm{d}x} = 1 \qquad v = -\frac{\cos nx}{n}$

$\therefore \int x^2 \cos nx \,\mathrm{d}x = \frac{x^2 \sin nx}{n} - \frac{2}{n}\left(-\frac{x\cos nx}{n} + \frac{1}{n^2}\sin nx\right)$

$= \frac{x^2 \sin nx}{n} + \frac{2x\cos nx}{n^2} - \frac{2\sin nx}{n^3} + c$ where c is an arbitrary constant

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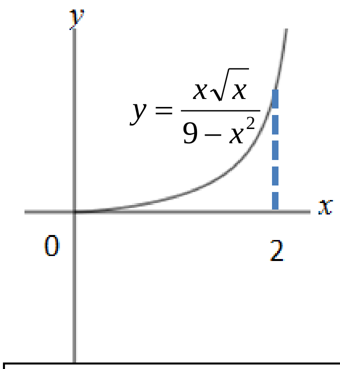
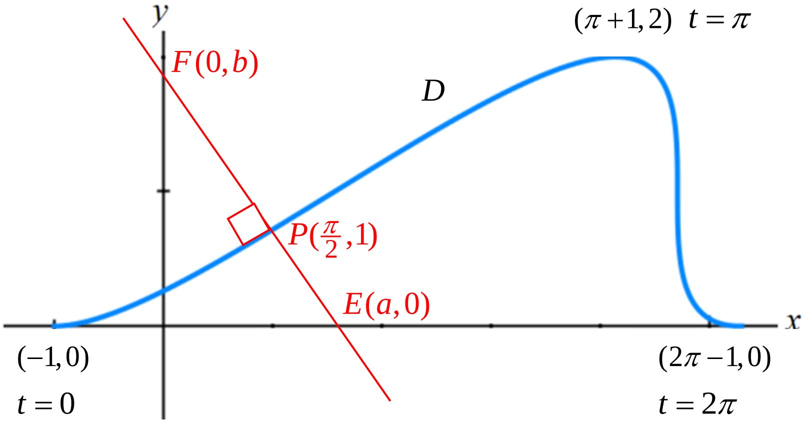
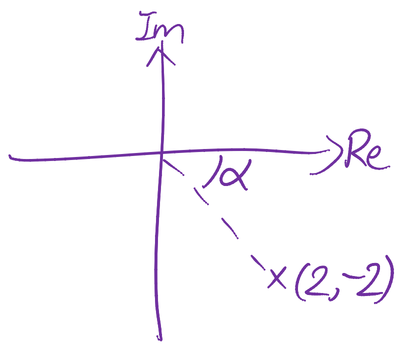
$\sin(n\pi) = 0$
$\cos(2n\pi) = 1,\ \cos\left[(2n-1)\pi\right] = -1$

(a)(ii)	$\int_{\pi}^{2\pi} x^2 \cos nx \, \mathrm{d}x$ $= \left[\frac{x^2 \sin nx}{n} + \frac{2x\cos nx}{n^2} - \frac{2\sin nx}{n^3}\right]_{\pi}^{2\pi}$ $= \left[\frac{4\pi^2 \sin 2n\pi}{n} + \frac{4\pi\cos 2n\pi}{n^2} - \frac{2\sin 2n\pi}{n^3}\right] - \left[\frac{\pi^2 \sin n\pi}{n} + \frac{2\pi\cos n\pi}{n^2} - \frac{2\sin n\pi}{n^3}\right]$ $= \frac{4\pi\cos 2n\pi}{n^2} - \frac{2\pi\cos n\pi}{n^2}$ ($\because$ the other terms are 0) $= \frac{4\pi(1)}{n^2} - \frac{2\pi(1)}{n^2}$ (when n is even) or $\frac{4\pi(1)}{n^2} - \frac{2\pi(-1)}{n^2}$ (when n is odd) $= \frac{2\pi}{n^2}$ or $\frac{6\pi}{n^2}$
(b)	$y = \frac{x\sqrt{x}}{9-x^2} \Rightarrow y^2 = \frac{x^2(x)}{(9-x^2)^2}$  Volume of solid $= \pi\int_0^2 y^2 \mathrm{d}x$ $= \pi\int_0^2 \frac{x^3}{\left(9-x^2\right)^2} \mathrm{d}x$ $= \pi\int_9^5 \frac{(9-u)\sqrt{9-u}}{u^2} \cdot \frac{-1}{2\sqrt{9-u}} \mathrm{d}u$ $= \pi\int_9^5 \frac{u-9}{2u^2} \mathrm{d}u$ $= \frac{\pi}{2}\int_9^5 u^{-1} - 9u^{-2} \, \mathrm{d}u$ $= \frac{\pi}{2}\left[\ln\lvert u\rvert + 9u^{-1}\right]_9^5$ $= \frac{\pi}{2}\left[\left(\ln 5 + \frac{9}{5}\right) - \left(\ln 9 + 1\right)\right]$ $= \frac{\pi}{2}\left(\ln\frac{5}{9} + \frac{4}{5}\right)$ Let $u = 9 - x^2 \Rightarrow x^2 = 9 - u$ $\frac{\mathrm{d}u}{\mathrm{d}x} = -2x = -2\sqrt{9-u}$ $\int \mathrm{d}x = \int -\frac{1}{2\sqrt{9-u}} \mathrm{d}u$ When $x = 2,\ u = 5;$ $x = 0,\ u = 9.$
3 (i)	$x = t - \cos t,\ y = 1 - \cos t,$ for $0 \le t \le 2\pi$ When D meets the x-axis, $y = 0 \Rightarrow \cos t = 1 \Rightarrow t = 0$ or $t = 2\pi$ $x = -1$ or $x = 2\pi - 1$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\sin t}{1 + \sin t}$ At the maximum point, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \sin t = 0$ $t = 0, \pi, 2\pi$ When $t = 0$ or 2π, we get the points on the x-axis. When $t = \pi$, we get the maximum point. $x = \pi - \cos\pi = \pi + 1,\ y = 1 - \cos\pi = 2$

	 Note: The extra line is for part (iii).
(ii)	Area required $=\int_{-1}^{x_1} y\,\mathrm{d}x=\int_0^a y\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\mathrm{d}t$ $=\int_0^a (1-\cos t)(1+\sin t)\,\mathrm{d}t$ $=\int_0^a \left(1-\cos t+\sin t-\frac{1}{2}\sin 2t\right)\mathrm{d}t$ $=\left[t-\sin t-\cos t+\frac{1}{4}\cos 2t\right]_0^a$ $=\left[a-\sin a-\cos a+\frac{1}{4}\cos 2a\right]-\left[-1+\frac{1}{4}\right]$ $=\frac{3}{4}+a-\sin a-\cos a+\frac{1}{4}\cos 2a$
(iii)	At P, where $t=\frac{\pi}{2}$, $x=\frac{\pi}{2}$, $y=1$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sin\frac{\pi}{2}}{1+\sin\frac{\pi}{2}}=\frac{1}{2}$. Equation of normal at P, $y-1=-2\left(x-\frac{\pi}{2}\right)$ $y=-2x+\pi+1$ At E, $y=0$, $x=a=\frac{1+\pi}{2}$ At F, $x=0$, $y=b=1+\pi$ $\therefore$ Area of triangle OEF is $=\frac{1}{2}ab=\frac{1}{2}\left(\frac{1+\pi}{2}\right)(1+\pi)=\left(\frac{1+\pi}{2}\right)^2$ units^2
4(b)(ii)	$w=2-2\mathrm{i}$ $\lvert w\rvert=\sqrt{2^2+(-2)^2}=\sqrt{8}$, $\alpha=\tan^{-1}\frac{2}{2}=\frac{\pi}{4}$ $w=\sqrt{8}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{4}\right)}$ $w^*w^n=\sqrt{8}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{4}\right)}\times\left(\sqrt{8}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{4}\right)}\right)^n$ $=\left(\sqrt{8}\right)^{n+1}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{4}-\frac{n\pi}{4}\right)}$ 

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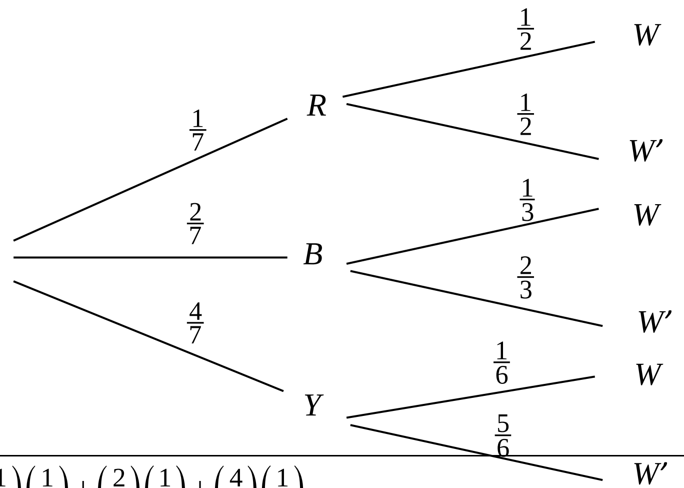
<tr><td></td><td>$\arg(w^* w^n) = \ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$
$\frac{\pi}{4} - \frac{n\pi}{4} = \frac{\pi(4m-3)}{2}, m \in \mathbb{Z}$
$1 - n = 2(4m-3)$
$n = 1 - 8m + 6$
$= 7 - 8m$
The smallest positive whole number value of n is 7 when $m = 0$.

Do the following to get the generalisation of $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$:
1. Observe that $\frac{\pi}{2}(1), \frac{\pi}{2}(5), \frac{\pi}{2}(9), \ldots$
2. Factorise $\frac{\pi}{2}$, you would have 1, 5, 9, …
3. The sequence follows an AP with first term 1, common difference 4.
4. Find the mth term, $T_m = 4m - 3$
Note: Do not use n for the generalisation</td></tr>
<tr><td></td><td><u>Section B</u></td></tr>
<tr><td>5</td><td>Let W be the event that a player wins a game.
Let R, B and Y be the events that the spinner comes to rest over the red, blue and yellow sections respectively.

</td></tr>
<tr><td>(i)</td><td>$\mathrm{P}(W) = \left(\frac{1}{2}\right)\left(\frac{1}{7}\right) + \left(\frac{2}{7}\right)\left(\frac{1}{3}\right) + \left(\frac{4}{7}\right)\left(\frac{1}{6}\right)$
$= \frac{11}{42}$</td></tr>
<tr><td>(ii)</td><td>$\mathrm{P}(B \mid W) = \frac{\mathrm{P}(B \cap W)}{\mathrm{P}(W)}$
$= \frac{\left(\frac{2}{7}\right)\left(\frac{1}{3}\right)}{\frac{11}{42}} = \frac{4}{11}$</td></tr>
<tr><td>(iii)</td><td>Required probability $= \mathrm{P}(W \cap R)\mathrm{P}(W \cap B)\mathrm{P}(W \cap Y)(3!)$
$= \left(\frac{1}{7}\right)\left(\frac{1}{2}\right)\left(\frac{2}{7}\right)\left(\frac{1}{3}\right)\left(\frac{4}{7}\right)\left(\frac{1}{6}\right)(3!)$
$= \frac{4}{1029}$</td></tr>
<tr><td>6(iii)</td><td>Let X be the age of employees in the company and μ be the population mean age.
Test $\mathrm{H}_0 : \mu = 37$ against $\mathrm{H}_1 : \mu < 37$ at 5% level of significance
Under H_0, $\overline{X} \sim \mathrm{N}(37, \frac{140}{80})$ approx. by Central Limit Theorem since $n = 80$ is large.
Using a one-tail z-test,</td></tr>
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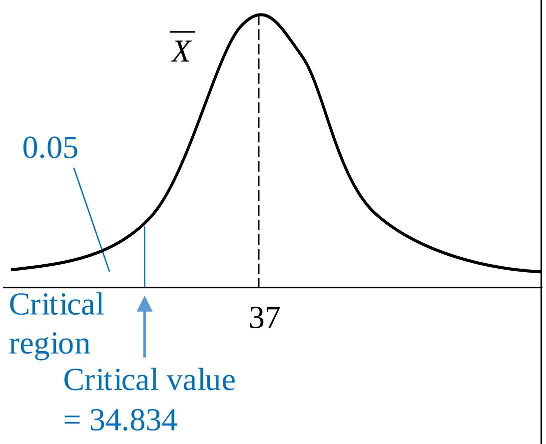
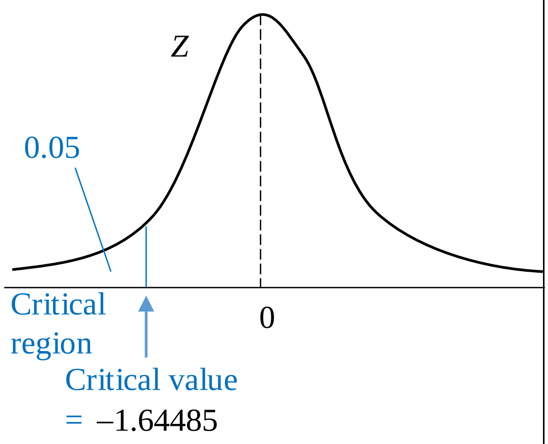
<tr><td></td><td>
</td><td>
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<tr><td></td><td>Critical value = 34.8
Critical region: $\overline{x} \le 34.8$

For H_0 to be rejected, $\overline{x}$ must lie in the critical region.
$\therefore \overline{x} \le 34.8$</td><td>Critical value = −1.64485
Critical region: $z \le -1.64485$
$z_{cal} = \dfrac{\overline{x}-37}{\sqrt{\dfrac{140}{80}}}$
For H_0 to be rejected, z_{cal} must lie in the critical region.
$\therefore z_{cal} \le -1.64485$
$\dfrac{\overline{x}-37}{\sqrt{\dfrac{140}{80}}} \le -1.64485$
$\overline{x} \le 34.8$</td></tr>
<tr><td></td><td colspan="2">The set of values of $\overline{x}$ is $\{\overline{x} \in \mathbb{R} \mid \overline{x} \le 34.8\}$</td></tr>
<tr><td>(iv)</td><td colspan="2">Now do a one-tail z-test at the α% level of significance.
For H_0 not to be rejected,
$p\text{-value} = \mathrm{P}(\overline{X} < 35.2) > \dfrac{\alpha}{100}$
$0.086809 > \dfrac{\alpha}{100}$
$\alpha < 8.68$ (3 s.f.)
This info suggests we use a p-value method.
$\mu_0 : 37$
$\sigma : \sqrt{140}$
$\overline{x} : 35.2$
$n : 80$
The set of values of α is $\{\alpha \in \mathbb{R} \mid 0 < \alpha < 8.68\}$</td></tr>
<tr><td>7(i)</td><td colspan="2">No. of ways $= 4 \times 3 \times 2$ or ${}^4\mathrm{C}_3 \times 3!$
$= 24$</td></tr>
<tr><td>(ii)</td><td colspan="2">No. of ways they are all men $= 6 \times 5 \times 4$ or ${}^6\mathrm{C}_3 \times 3!$
$= 120$
Required no. of ways = Total without restriction – 24 – 120
$= 10 \times 9 \times 8 - 24 - 120$
$= 576$</td></tr>
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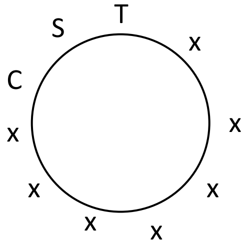
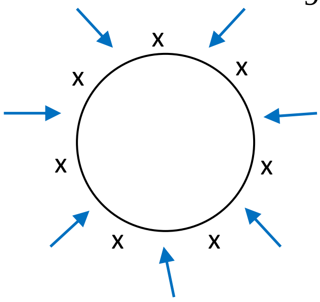
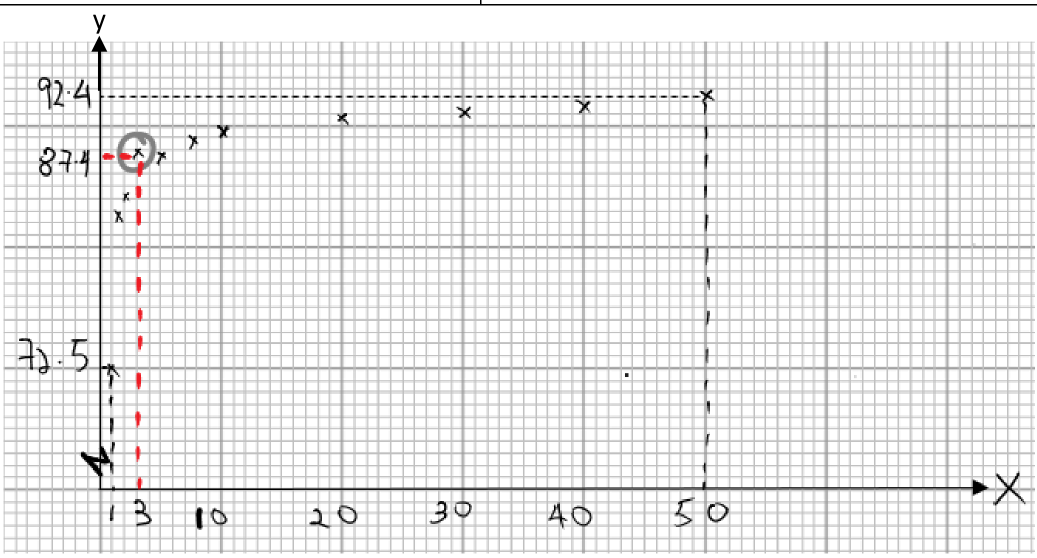
(iii)	Required **probability** $=\dfrac{7!\times 3!}{9!}=\dfrac{1}{12}$  (iv) Required probability $=\dfrac{6!\times\left({}^7C_3\times 3!\right)}{9!}=\dfrac{5}{12}$ 
8(i)	 **[Do note that it was instructed by the question that the scatter diagram is to be drawn on a graph paper]**
(ii)	The scatter diagram shows a non-linear relationship between *x* and *y*. Thus a linear model in the form of $y = ax + b$ is not appropriate.
(iii)	From the scatter diagram, as *x* increases, *y* increases. Hence *c* is negative. As *x* approaches infinity, *y* approaches *d*, which is a positive limit (representing the maximum efficiency as power increases).
(iv)	Product moment correlation coeefficient, $r = -0.980$ (3 s.f.) $c = -17.484 \approx -17.5, \quad d = 91.750 \approx 91.8$
(v)	Using the model $y = -\dfrac{17.484}{x} + 91.750$, $x = 3 \Rightarrow y = 85.9$ (3.s.f.) The value that Xian copied wrongly is estimated to be 85.9. This estimate is reliable as *x* = 3 is within the data range for *x* and $\lvert r\rvert \approx 1$ indicates strong linear correlation between $\frac{1}{x}$ and *y*.
9(a)	Given: $X \sim \mathrm{N}(15, a^2)$ $\mathrm{P}(X < 10) = 0.25 \Rightarrow \mathrm{P}\left(Z < \frac{10-15}{a}\right) = 0.25$ $\frac{-5}{a} = -0.67449$ $a = 7.41$ (3 s.f.)

(b)	Given: $Y \sim \mathrm{B}(4, p)$ and $\mathrm{P}(Y=1)+\mathrm{P}(Y=2)=0.5$ $\binom{4}{1}p(1-p)^3+\binom{4}{2}p^2(1-p)^2=0.5$ $4p(1-3p+3p^2-p^3)+6p^2(1-2p+p^2)=0.5$ $4p-12p^2+12p^3-4p^4+6p^2-12p^3+6p^4=0.5$ $2p^4-6p^2+4p=0.5$ $4p^4-12p^2+8p=1$ (shown!) From GC, $p=-2.01,\ 1.25,\ 0.166,\ 0.599$ (3 s.f.) As $0 \le p \le 1$, $p=0.166$ or 0.599