



H2 math last-minute reminders

Disclaimer: These are general guidelines I got from my notes, teachers and friends. Reminders and styles of writing may differ from school to school, so please follow your own teachers'/school's guidelines/facts in internal exams. For the actual As...it's kinda up to you lol

⚠ ALWAYS check denominator and check if need to add extra conditions or reject stuff eg reject $x=1$ cause denominator is $x-1$ etc

⚠ all graphs need endpoints!!

⚠ If qn asks a set of values, give SET NOTATION!!!

⚠ generally don't do similarity for non triangle shapes like trapezium. Try to split up and find the triangles within

⚠ RMB similarity rules. Eg similar sides x/y mean volumes are of the ratio x^3/y^3 . And area is ratio x^2/y^2

⚠ need to simplify surds that can be simplified — eg root 4

“Find range of this function algebraically” —> can try using discriminant

- usually it's an equation where y can be the coefficient of x terms
- Find discriminant in terms of y and do discriminant $>$ or equal 0 for real roots (for function to be defined)

!!There are more things and equations you can solve using GC than you think... if qn doesn't want exact and you have a horrendous combination of variables like $x + \sin x$ and whatever, just use GC draw graph

!!“Express x in terms of y ” unconventional qns have stuff like need to use quadratic formula (esp when theres like x and x^2 in the same equation...) discriminant, identities etc...

If $y = f(x)$, then $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$

E.g. to prove $\frac{d}{dx}(x^2) = 2x$ from 1st principles

Let $y = f(x) = x^2$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

$$= \lim_{\delta x \rightarrow 0} \frac{(x + \delta x)^2 - x^2}{\delta x}$$

$$= \lim_{\delta x \rightarrow 0} \frac{x^2 + 2x\delta x + (\delta x)^2 - x^2}{\delta x}$$

$$= \lim_{\delta x \rightarrow 0} (2x + \delta x)$$

$$= 2x$$

jic...im not sure if this will be tested

Functions

- if checking inverse one to one exists, be mindful of mod signs eg if the expression is $y = e^{\text{mod}(x+z)}$ note that $y=e$ both when $x=1-z$ and $x=-1-z$
- graph of F!!! not graph of $f(x)$ or graph of $y=f(x)$ or whatever, keep it simple bro
- If showing inequality, try to use discriminant??

Vectors 🤖🤖!?



Non-zero vectors $\underline{a}, \underline{b}, \underline{c}$ are such that $\underline{a} \times \underline{b} = \underline{c} \times \underline{a}$.

Given $\underline{b} \neq -\underline{c}$, find a linear v/s between $\underline{a}, \underline{b}, \underline{c}$.

$$\underline{a} \times \underline{b} = \underline{c} \times \underline{a}$$

$$\underline{a} \times \underline{b} - \underline{c} \times \underline{a} = \underline{0}$$

$$\underline{a} \times \underline{b} + \underline{a} \times \underline{c} = \underline{0}$$

$$\underline{a} \times (\underline{b} + \underline{c}) = \underline{0} \Rightarrow \text{Note: } \underline{a} = \underline{0} \text{ or } \underline{b} = \underline{0} \text{ or } \underline{a} \parallel \underline{b}$$

Given $\underline{a} \neq \underline{0}$, $\underline{b} \neq -\underline{c}$ implying $\underline{b} \neq \underline{c} \neq \underline{0}$, $\underline{a}$ is parallel to $(\underline{b} + \underline{c})$. $\therefore \underline{a} = \lambda(\underline{b} + \underline{c})$, $\lambda \neq 0$

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- for stuff like (a), try moving everything to LHS and apply vectors dot or multprop properties

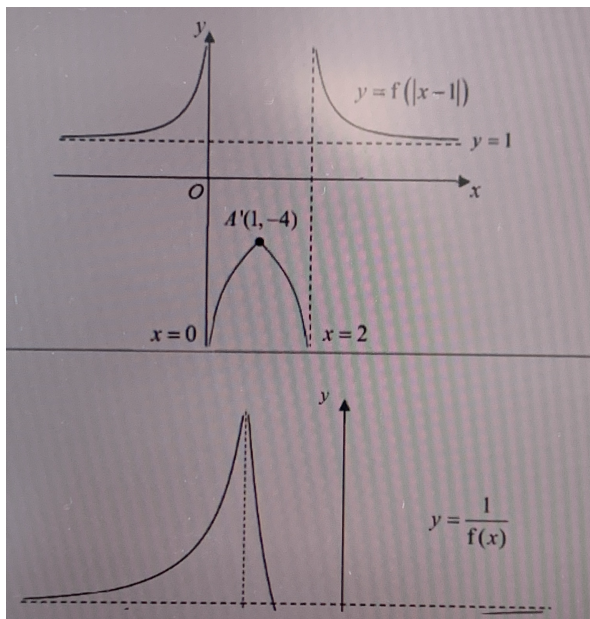
Differentiation

- parametric: CHANGE WINDOW SETTINGS ALWAYS to check the parametric graph. Dude. You gotta change it you really gotta. Like x -100/100, y -100/100. Zoom all the way out. Seriously they love testing this kind because the gc will NEVER give you the full range. NEVER. 100% of the time it's got hidden asymptotes or it's not zoomed out enough.

Graphs

- Parametric Cartesian Conversion
 - For fractions: try to make t the subject then sub into y expression
 - For powers: change to indices or whatever (divide)
 - ESP for trigo: change to indices and use trigo identities somehow it will work out??
 - can have +/-
 - May need to combine x and y eg $x+y$ to get t , whatever

Graphs – Transformation



If the translation down is within the mod (the graph on top) the axes gets changed so eg you flip the graph around $x=1$ instead of $x=0$

Sequences and Series 🙄🙄🙄🙄🙄🔥🔥🔥🔥🔥

- every time they ask you prove it's a whatever sequence/find U_n , that kinda stuff — find U_n by finding $S_n - S_{n-1}$ (formula) and must prove separately for U_1
- Proving GP, find U_{n+1} / U_n , show it's a constant term
- for sigma (oh..)

- If summation has only r inside with end value stated, use GC
 - if not, eg if got unknown final term n , must evaluate
 - and look out for AP GP!!
- For those interest etc questions, do a table (really it helps) then factorise and find the gp pattern

	Beginning of month	End of month
Jan	100	$1.005(100)$
Feb	$1.005(100) + x$	$1.005[1.005(100) + x]$ $= 1.005^2(100) + 1.005x$
Mar	$1.005^2(100) + 1.005x + x$	$1.005[1.005^2(100) + 1.005x + x]$ $= 1.005^3(100) + 1.005^2x + 1.005x$

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- for stuff asking what value will it approach, if they give an n expression (eg for convergent series), try to use the n expression not the r expression to evaluate, and try to factorise a whole number (the sum to infinity) out

$$\sum_{r=1}^n \frac{1}{4r^2-1} = \frac{n}{2n+1}$$

$$= \frac{1}{2} - \frac{1}{2(2n+1)}$$

$$\left. \begin{array}{l} \text{As } n \rightarrow \infty \\ \frac{1}{2(2n+1)} \rightarrow 0 \end{array} \right\}$$

$$\begin{aligned} \text{Var}(X) &= E(X-\mu)^2 \\ &= \sum (x-\mu)^2 P(X=x) \end{aligned}$$

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- (b) An arithmetic series has a positive first term v_1 and common difference d . Given that $3v_{10} = 5v_{19}$, find the largest possible value of the sum of the first n terms, leaving your answer in terms of d . [4]

- for this type thing (above) you need to find the S_n expression as a quadratic or something and then do the y3 stuff about finding min/max of quadratic equation using completing the square or something ...
- Sometimes you just gotta expand out the expression and see what you get

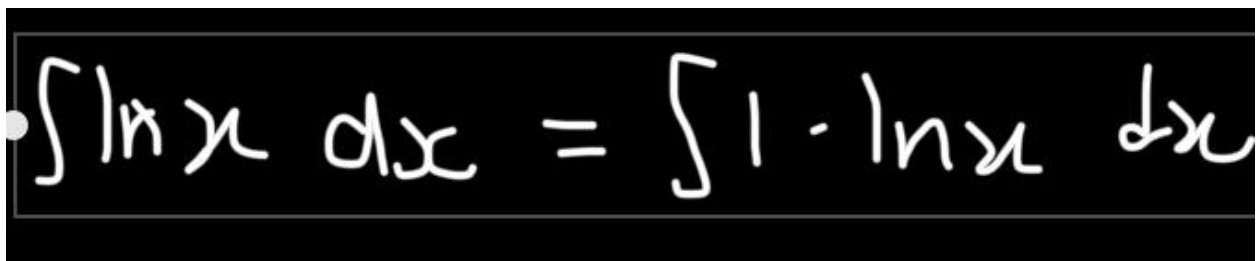
Complex Numbers

- possible techniques:

- Compare real and imaginary parts
 - Negative numbers can always potentially be i^2
- Possible techniques for simultaneous:
 - Multiply and then add/subtract to eliminate, like the typical easy kind
 - Harder kind: Make one unknown the subject of the other, then sub back into the other equation (works better if there are unknowns multiplied eg one equation has xy , then find x in terms of y and sub back)
 - Take note if any equation has mod elements. Eg $|x| = y + \text{something}$. THIS MEANS y is REAL.
- Cartesian form is $a+ib$. Try: rationalise (like a surd),
- if there's ever complex number in denominator first try rationalise using the denominator even if it's a weird weird trigo kind of thing (2019 p1)

Integration

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2(x)$
- if integrating inverse trig or logarithmics BY ITSELF, usually it's integration by parts but you got to add a 1 before it yourself then integrate by parts (follow LIATE) it becomes like



A handwritten equation on a black background with white text. The equation is $\int \ln x \, dx = \int 1 \cdot \ln x \, dx$. The '1' is written as a simple vertical line.

- if there's a mod when integrating, esp def integrals, see what the function really is within the range of the def integral and use that to remove the mod, eg if the function is $\text{abs } x-4$ but you find that the graph within that interval is actually $4-x$, then just change the expression to $4-x$ while removing the mod. Cannot integrate with a mod.
- For trig integration identities, if the bottom part is not nice, try completing the square and fit it in to one of the $1-x^2$ type of identities
- Integration substitution with trigo - may have to draw trig triangle

Statistics

- If they ask if probability of a certain event is lower than probability of another event (without further calculation) and the answer is that it's lower because the first event is a subset of the second one, must give EXAMPLE of an instance when the first event is a subset of the second one. Esp if 2 marks
- Hypo: if the qn didn't ask you to calculate unbiased estimate (phrasing), must write working as s^2 instead as s^2 is used to estimate sigma. Hence must also write normal dist "approximately" even tho this isn't CLT.

let μ be population mean power consumption of a laptop.

To test $H_0: \mu = 100$ vs $H_1: \mu > 100$

Perform a 1-tail test at 3% level of significance.

Under H_0 , $\bar{w} \sim N(100, \frac{1}{50}(\frac{6489}{1225}))$ approximately.

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- mode of binomial distribution is $(n+1)p$. If result is integer, there are two modes: the result and the result minus one. If the result is not an integer, mode is the greatest integer less than or equal to the result.
- how to find sum of squares of resi. Type in the regression line as usual then type in the best fit line equation (using L1 as x). Then find the difference between y values originally given in scatter table (L2) and the resultant y values from line (L3). Then do 1-Var for L4, and scroll to find sum
- CLT is applied so SAMPLE MEAN is estimated to be normal!!!
- Rmb the hidden conditional probs...

!! "why is this model unsuitable"

- for binom: usually about probability not constant, events not indep
- For norm: usually some impossible situation eg time modelled by normal. But $P(X < 0)$ is a considerable amount, which is not possible

$$\begin{aligned}\text{Var}(X) &= E(X - \mu)^2 \\ &= \sum (x - \mu)^2 P(X = x)\end{aligned}$$

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Statistics – Hypo Testing

- level of significance : probability of rejected H_0 when it is in fact true
- Test statistic: ($\bar{X}$) the random variable used to make the decision to reject H_0 or not
- Critical value: boundary, cut off point of the region where values would be deemed too far from (pop mean)
- Critical region: the range of sample mean (time, mass, whatever) (and define in context) such that there is sufficient evidence at the x% significance level to conclude that... reject H_0 .