Practice Paper B

Sec 3 Math Answers

Qn	Solutions
1(a)	$(x-3)^3+(x-1)^3$ $=(x-3+x-1)[(x-3)^2-(x-3)(x-1)+(x-1)^2]$ $=(2x-4)(x^2-6x+9-x^2+4x-3+x^2-2x+1)$ $=2(x-2)(x^2-4x+7)$
1(b)	$=\dfrac{\frac{2x-x+1}{2}}{\frac{2x-x+1}{x}}$ $=\dfrac{x}{2}$ or $\dfrac{x-\frac{x-1}{2}}{2-\frac{x-1}{x}}$ $=\dfrac{2x^2-x(x-1)}{4x-2(x-1)}$ $=\dfrac{x^2+x}{2x+2}$ $=\dfrac{x(x+1)}{2(x+1)}$ $=\dfrac{x}{2}$
2	Length of the side BC $=\dfrac{1+3\sqrt{2}}{2+\sqrt{8}}$ $=\dfrac{(1+3\sqrt{2})(2-2\sqrt{2})}{(2+\sqrt{8})(2-2\sqrt{2})}$ $=\dfrac{2-2\sqrt{2}+6\sqrt{2}-12}{-4}$ $=\dfrac{-10+4\sqrt{2}}{-4}$ $=\left(\dfrac{5}{2}-\sqrt{2}\right)$ m
3(a)	$\log_{(x-1)}(3x^2-7x-2)=2$ $\log_{(x-1)}(3x^2-7x-2)=\log_{(x-1)}(x-1)^2$ $3x^2-7x-2=x^2-2x+1$ $2x^2-5x-3=0$ $(2x+1)(x-3)=0$ $x=-\dfrac{1}{2}$ (rejected) or 3

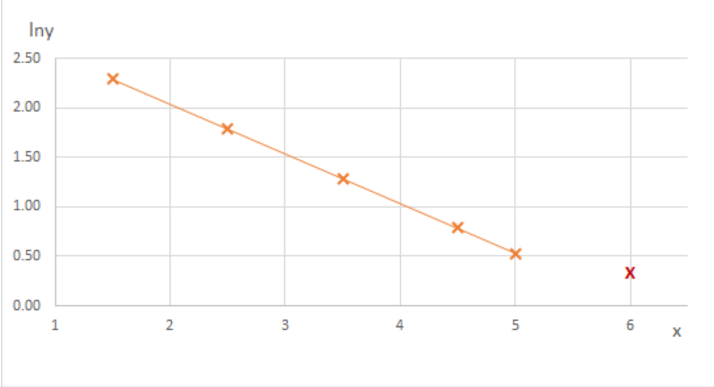
3(b)	$\ln\left\|\sqrt{x}-2\right\|+\frac{1}{2}\ln x=0,$ where $x>0$ $\ln\left\|\sqrt{x}-2\right\|+\ln\sqrt{x}=0$ $\ln\left\|x-2\sqrt{x}\right\|=0$ $\left\|x-2\sqrt{x}\right\|=1$ $x-2\sqrt{x}=1$ or -1 (Two cases) Case 1: $x-2\sqrt{x}=1$ $2\sqrt{x}=x-1$ $x^2-2x+1=4x$ $x^2-6x+1=0$ $x=3\pm2\sqrt{2}$ (reject $3-2\sqrt{2}$) OR $x=5.83$ or 0.172 (rejected, as it does not satisfy the Case 1 equation) Case 2: $x-2\sqrt{x}=-1$ $2\sqrt{x}=x+1$ $x^2+2x+1=4x$ $(x-1)^2=0$ $x=1$ or $x-2\sqrt{x}+1=0$ $(\sqrt{x}-1)^2=0$ $x=1$ Hence $x=3+2\sqrt{2}$ or 1
4(i)	$\mathbf{P(R-Q)}$ or $\begin{pmatrix}100 & 120\\ 70 & 85\end{pmatrix}\begin{pmatrix}x-5\\ y-6\end{pmatrix}$ or $\begin{pmatrix}100 & 120\\ 70 & 85\end{pmatrix}\left[\begin{pmatrix}x\\ y\end{pmatrix}-\begin{pmatrix}5\\ 6\end{pmatrix}\right]$
4(ii)	$\begin{pmatrix}100 & 120\\ 70 & 85\end{pmatrix}\begin{pmatrix}x-5\\ y-6\end{pmatrix}=\begin{pmatrix}1710\\ 1205\end{pmatrix}$ $\begin{pmatrix}x-5\\ y-6\end{pmatrix}=\begin{pmatrix}100 & 120\\ 70 & 85\end{pmatrix}^{-1}\begin{pmatrix}1710\\ 1205\end{pmatrix}$ $\begin{pmatrix}x-5\\ y-6\end{pmatrix}=\frac{1}{100}\begin{pmatrix}85 & -120\\ -70 & 100\end{pmatrix}\begin{pmatrix}1710\\ 1205\end{pmatrix}$ $\begin{pmatrix}x-5\\ y-6\end{pmatrix}=\begin{pmatrix}7.5\\ 8\end{pmatrix}$ $\therefore x=12.5, y=14$ The price of pandan cake is \$12.50 each, and the price of chocolate cake is \$14 each.
5	$y=mx^2-(m+1)x+m$ is always positive $\Rightarrow m>0$ and $\Delta<0$

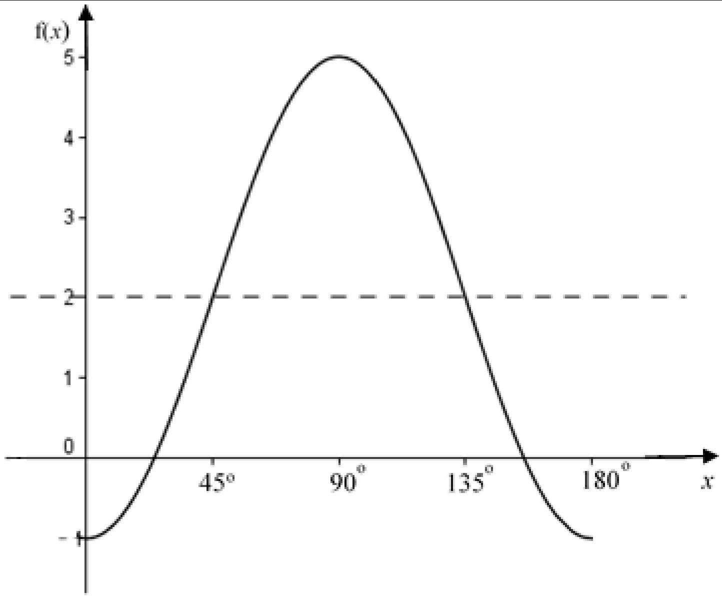
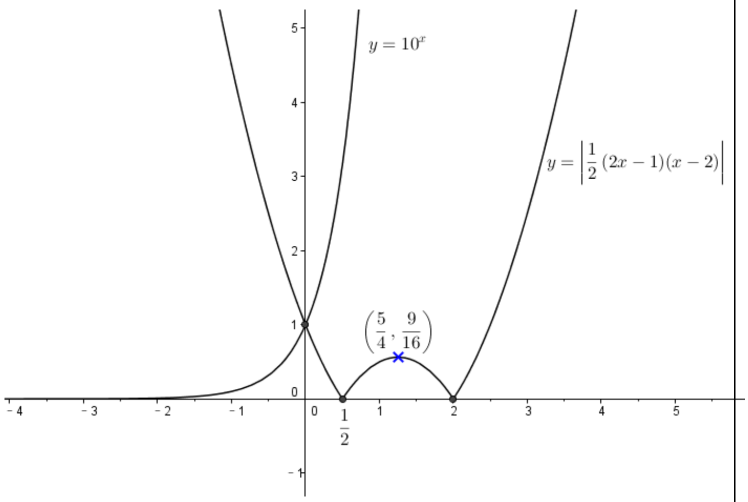
	$\Delta=(m+1)^2-4m^2<0$ $m^2+2m+1-4m^2<0$ $3m^2-2m-1>0$ $(3m+1)(m-1)>0$ $m<-\frac{1}{3}$ or $m>1$ $\because m$ is positive, $\therefore m>1$
6(i)	$x^2+2k+1=(k+2)x$ $x^2-(k+2)x+(2k+1)=0$ $\Delta=(k+2)^2-4(2k+1)\geq 0$ $k^2-4k\geq 0$ $k(k-4)\geq 0$ $k\leq 0$ or $k\geq 4$
6(ii)	$\left.\begin{aligned}\alpha+\beta&=k+2\\ \alpha\beta&=2k+1\end{aligned}\right\}$ $\alpha^2+\beta^2=11$ $\Rightarrow(\alpha+\beta)^2-2\alpha\beta=11$ $(k+2)^2-2(2k+1)=11$ $k^2=9$ $k=\pm 3$ $\because \alpha,\beta$ are real, range of values for $k: k\leq 0$ or $k\geq 4$ $\therefore k=-3$
7(a)	$2x^3=5x^2-x-2$ $2x^3-5x^2+x+2=0$ Let $f(x)=2x^3-5x^2+x+2$ $\because f(1)=0,\ \therefore (x-1)$ is a factor of $f(x)$. Using long division, $(x-1)(2x^2-3x-2)=0$ $(x-1)(2x+1)(x-2)=0$ $x=1,-\frac{1}{2}$ or 2

7(b)	Let $f(x)=(x-3)^{200}-(x-2)^{100}$. When divided by x^2-5x+6, $f(x)=(x^2-5x+6)Q(x)+ax+b$, where a and b are real constants. $f(x)=(x-2)(x-3)Q(x)+ax+b$ By Remainder Theorem, $f(2)=(2-3)^{200}-(2-2)^{100}=1$ $\Rightarrow 2a+b=1$ $f(3)=(3-3)^{200}-(3-2)^{100}=-1$ $\Rightarrow 3a+b=-1$ Solving the above two equations, $a=-2, b=5$ $\therefore$ The remainder is $-2x+5$.
8	$S=ax^2+\frac{b}{y}$ When $x=2$ and $y=3$, $S=1$, $\Rightarrow 4a+\frac{b}{3}=1 \quad ...(1)$ When $x=4$ and $y=6$, $S=7\frac{1}{2}$, $\Rightarrow 16a+\frac{b}{6}=7\frac{1}{2} \quad ...(2)$ Solvng equations (1) and (2) simultaneously, $a=\frac{1}{2}, b=-3$ $\therefore S=\frac{1}{2}x^2-\frac{3}{y}$

9(a)	$0° \le x \le 720°$ $\Rightarrow 30° \le \frac{x}{2} + 30° \le 390°$ $6 - 4\cos^2\left(\frac{x}{2}+30°\right) = 9\sin\left(\frac{x}{2}+30°\right)$ $6 - 4 + 4\sin^2\left(\frac{x}{2}+30°\right) = 9\sin\left(\frac{x}{2}+30°\right)$ $4\sin^2\left(\frac{x}{2}+30°\right) - 9\sin\left(\frac{x}{2}+30°\right) + 2 = 0$ $\left[4\sin\left(\frac{x}{2}+30°\right)-1\right]\left[\sin\left(\frac{x}{2}+30°\right)-2\right] = 0$ $\sin\left(\frac{x}{2}+30°\right) = \frac{1}{4}$ or 2 (rejected) $\left.\begin{array}{l} \text{B.A.} = 14.48° \\ \frac{x}{2} + 30° = 165.52° \text{ or } 374.48° \end{array}\right\}$ $x = 271.0°$ or $689.0°$
9(b)	$\text{LHS} = 2\sin^2 x - \frac{\tan x - \cot x}{\tan x + \cot x}$ $= 2\sin^2 x - \frac{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$ $= 2\sin^2 x - \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x}$ $= 2\sin^2 x - (\sin^2 x - \cos^2 x)$ $= \sin^2 x + \cos^2 x$ $= 1 = \text{RHS}$
10(i)	$y = ae^{-bx} \Rightarrow \ln y = \ln a - bx$

x	1.5	2.5	3.5	4.5	5.0	6.0
y	9.9	6.0	3.6	2.2	1.7	1.4
$\ln y$	2.29	1.79	1.28	0.79	0.53	0.34

	
10(ii)	$\ln a \approx 3.04$ $\Rightarrow a \approx 20.9$ (accept 20.0 to 21.8) $b \approx 0.500$ (accept 0.47 to 0.53)
10(iii)	The abnormal reading is 1.4. When x = 6.0, $\ln y$ is estimated to be 0.04. Hence, the correct value of $y \approx e^{0.04} \approx 1.04$. (accept 1.00 to 1.04)
11	$\mathrm{f}(x) = \frac{a}{x} + 1$ Let $y = \mathrm{f}^{-1}(x)$ $\Rightarrow \mathrm{f}(y) = x$ $\frac{a}{y} + 1 = x$ $y = \frac{a}{x-1}$ $\Rightarrow \mathrm{f}^{-1}(x) = \frac{a}{x-1}, \ x \neq 1$ $2\,\mathrm{f}(-1) + \mathrm{f}^{-1}(2) = 4$ $2\left(\frac{a}{-1} + 1\right) + \frac{a}{2-1} = 4$ $-2a + 2 + a = 4$ $a = -2$

12	 Hence, the range of the function is $-1 \le \mathrm{f}(x) \le 5$.
13(a)	 Hence, the range of values of x such that $10^x < \left\lvert \frac{1}{2}(2x-1)(x-2) \right\rvert$ is $x < 0$.
13(b)	$y = \mathrm{e}^x + 1 \Rightarrow \mathrm{e}^x = y - 1 \Rightarrow x = \ln(y-1)$ First transformation: reflection in the line $y = x$. Second transformation: stretching / scaling parallel to the y-axis by a factor of 2.
14(i)	Height of the flagpole XY $= 30 \tan 25°$ $\approx 13.989 = 14.0$ m (3 s.f.)
14(ii)	The bearing of T from S and the bearing of T from Y are $042°$ and $075°$ respectively.

	$\Rightarrow \angle YST = 42°, \angle SYT = 180° - 75° = 105°$ $\therefore \angle YTS = 180° - 42° - 105° = 33°$ Alternatively, by stating that $\angle NYT = 75°$, $\angle YTS = 75° - 42° = 33°$ $\dfrac{ST}{\sin 105°} = \dfrac{SY}{\sin 33°}$ $\therefore ST = \dfrac{30}{\sin 33°} \times \sin 105°$ $\approx 53.205 = 53.2$ m (3 s.f.)
14(iii)	Let the midpoint of S and T be M. $SM = \dfrac{1}{2} \times ST \approx 26.6025$ Using Cosine Rule, $MY^2 = SY^2 + SM^2 - 2 \times SY \times SM \times \cos \angle YSM$ $\therefore MY = \sqrt{30^2 + 26.6025^2 - 2 \times 30 \times 26.6025 \times \cos 42°}$ ≈ 20.531 Angle of elevation of X from the man $= \tan^{-1} \dfrac{XY}{MY} = \tan^{-1} \dfrac{13.989}{20.531}$ $= 34.3°$
15	$\dfrac{1}{\cos\theta\sqrt{1+\tan^2\theta}} + \dfrac{2\cot\theta}{\sqrt{\dfrac{1}{\sin^2\theta} - 1}}$ $= \dfrac{1}{\cos\theta\sqrt{\sec^2\theta}} + \dfrac{2\cot\theta}{\sqrt{\cot^2\theta}}$ $= \dfrac{1}{\cos\theta\lvert\sec\theta\rvert} + \dfrac{2\cot\theta}{\lvert\cot\theta\rvert}$ $\because \theta$ lies in the 4th quadrant, $\sec\theta > 0$, $\cot\theta < 0$, $\therefore \lvert\sec\theta\rvert = \sec\theta$, $\lvert\cot\theta\rvert = -\cot\theta$ Hence, the expression $= \dfrac{1}{\cos\theta\sec\theta} + \dfrac{2\cot\theta}{-\cot\theta}$ $= 1 - 2 = -1$
16(i)	$x^2 + y^2 - 6x + 4y - 21 = 0$ $(x-3)^2 + (y+2)^2 = 34$ Hence, the coordinates of P are $(3, -2)$, and the radius of C_1 is $\sqrt{34}$ units.

16(ii)	$4y - x = 6 \quad ...(1)$ $x^2 + y^2 - 6x + 4y - 21 = 0 \quad ...(2)$ From (1), $x = 4y - 6 \quad ...(3)$ Sub (3) into (2): $(4y-6)^2 + y^2 - 6(4y-6) + 4y - 21 = 0$ $17y^2 - 68y + 51 = 0$ $y^2 - 4y + 3 = 0$ $(y-1)(y-3) = 0$ $y = 1$ or 3 $x = -2$ or 6 $\therefore M = (-2,1),\ N = (6,3)$
16(iii)	Equation of $MN : 4y - x = 6$ $\Rightarrow y = \frac{1}{4}x + \frac{3}{2}$ $\therefore$ Gradient of the perpendicular bisector $= \frac{-1}{1/4} = -4$ Midpoint of $MN = \left(\frac{6-2}{2}, \frac{3+1}{2}\right) = (2,2)$ $\therefore$ Equation of the perpendicular bisector: $y - 2 = -4(x-2)$ $y = -4x + 10$
16(iv)	Since Q is the centre of C_2, it lies on the perpendicular bisector of chord *MN*. Hence, the coordinates of Q satisfy the equation $y = -4x + 10 \quad ...(1)$ Also, since the radius of $\frac{1}{2}\sqrt{221}$ units, $QM = \frac{1}{2}\sqrt{221}$. $\sqrt{(x+2)^2 + (y-1)^2} = \frac{1}{2}\sqrt{221}$ $(x+2)^2 + (y-1)^2 = \frac{221}{4} \quad ...(2)$ Substitute (1) into (2):

	$(x+2)^2+(-4x+9)^2=\frac{221}{4}$ $17x^2-68x+\frac{119}{4}=0$ $4x^2-16x+7=0$ $(2x-1)(2x-7)=0$ $x=\frac{1}{2}$ or $\frac{7}{2}$ $y=8$ or -4 Since Q lies above the x-axis, only the pair of solution $\left(\frac{1}{2},8\right)$ is accepted. Hence, coordinates of Q are $\left(\frac{1}{2},8\right)$.
16(v)	Area of *PMQN* $=\frac{1}{2}\begin{vmatrix} 3 & 6 & \frac{1}{2} & -2 & 3 \\ -2 & 3 & 8 & 1 & -2 \end{vmatrix}$ $=42\frac{1}{2}$ units^2