



華僑中學

Hwa Chong Institution
Secondary 3 PAPER i

CANDIDATE
NAME

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CLASS

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REGISTER
NUMBER

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Integrated Mathematics

2 hours 30 minutes

Additional materials:

Writing papers

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work that you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 100.

This document consists of **8** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Mensuration

Arc Length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

2. TRIGONOMETRY

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

- 1** Solve the simultaneous equations

$$\begin{aligned}8^{x+3y} &= 4 \\ 2x - 3y - 1 &= 0.\end{aligned}\quad [4]$$

- 2** (a) Simplify $3\sqrt{216} - \sqrt{24} + \frac{4}{\sqrt{6}}$, expressing your answer in the form of

$$k\sqrt{6}, \text{ where } k \text{ is a constant.} \quad [3]$$

- (b) Solve the equation $8(5^{2x-1}) - 5^x = 2(5^{x+2})$. [4]

- 3** (i) Factorise $2x^3 - 128$ completely. [2]

- (ii) Hence or otherwise, solve the equation $2x^3 - 128 = 4 - x$. [3]

- 4** (a) Find the range of values of x for which $(3x - 2)(x + 6) - 3x > -2$. [3]

- (b) Given that the line $2y - b = ax$ is a tangent to the curve $4x^2 + 5y^2 - 20 = 0$,
show that $b^2 = 5a^2 + 16$. [4]

- 5** (a) Solve the equation $x - 5 = 4\sqrt{x}$. [3]

- (b) Solve the equation $\log_3 x^2 + 2 \log_x 3 = 5$. [5]

- 6** (i) On the same axes, sketch the graphs of $y = -x$ and $y = |-2x - 7|$, indicating
clearly the x -intercepts and y -intercepts. [4]

- (ii) Hence, or otherwise, solve the inequality $|-2x - 7| \leq -x$. [4]

[Turn over

- 7 (a) The points $A(3,0)$, $B(9,18)$ and $C(13,10)$ lie on a quadratic curve $y = f(x)$ and B is the maximum point on the curve. Find the equation of the curve in the form $y = ax^2 + bx + c$, where a , b and c are constants. [3]
- (b) Peter used 20 metres of fence to enclose two flower beds. One flower bed is in the shape of an equilateral triangle of side x metres. The other is in the shape of a circle of radius r metres.
- (i) Express r in terms of x and π . [1]
- (ii) Show that the total area, $A \text{ m}^2$ of the two flower beds is given by
$$A = \frac{\sqrt{3}\pi x^2 + (20 - 3x)^2}{4\pi}.$$
 [3]
- (iii) Hence, by completing the square, find the minimum total area of the two flower beds. [3]
- 8 $P(-3,3)$ and $T(0,2)$ lie on a circle with centre $Q(-1,4)$.
- (i) Given that PS is the diameter of the circle, find the coordinates of S . [2]
- (ii) Find the equation of the circle with centre Q in the general form. [3]
- (iii) $R(2,3)$ lies outside of the circle. By calculating the gradient of RS and PS , show that RS is a tangent to the circle in (i). [3]
- (iv) Find the area of the quadrilateral $PTRS$. [2]

- 9 (i) Given that $f(x) = 2e^x$ and $g(x) = 5 - 2e^x$, describe 2 successive transformations that will map the graph of $f(x)$ to the graph of $g(x)$. [2]
- (ii) Sketch the graph of $y = 5 - 2e^x$, indicating clearly the asymptote, the values of the x -intercept and y -intercept. [4]
- (iii) In order to solve the equation $\ln\left(\frac{x+6}{4}\right) = x$, a graph of a suitable straight line is drawn on the same axes as the graph of $y = 5 - 2e^x$. Find the equation of the straight line. [3]

- 10 (a) Given that $\mathbf{A} = \begin{pmatrix} -2 & -3 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 1 \\ -3 & 5 \end{pmatrix}$, find $3\mathbf{A} - \mathbf{B}$. [2]
- (b) E'Lite shop sells air-conditioning unit, instant water heater and humidifier. The number of air-conditioning unit, instant water heater and humidifier sold in 2020 and 2021 is given in the following table.

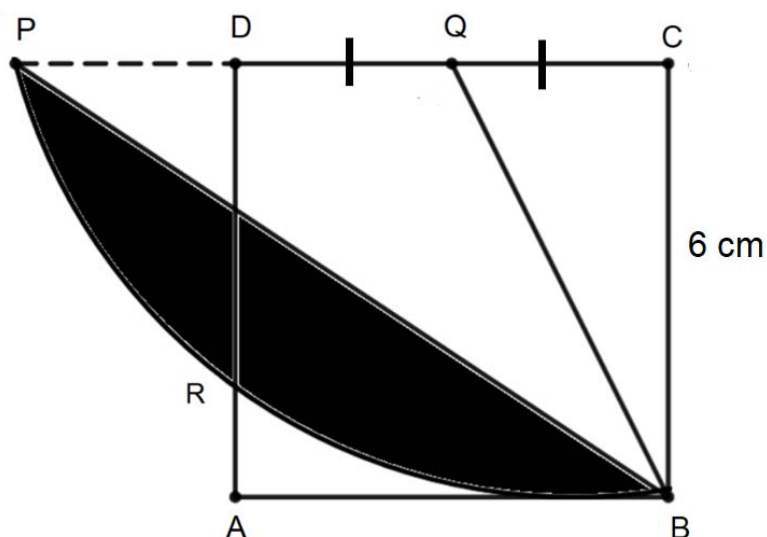
Type \ Year	air-conditioning	instant water heater	humidifier
2021	290	240	200
2020	235	362	117

- (i) Write down a 2×3 matrix $\mathbf{C}$ which represents the number of air-conditioning unit, instant water heater and humidifier sold in 2021 and 2020 respectively. [1]
- (ii) Given that the profit made from selling one air-conditioning unit is \$240, one instant water heater unit is \$95 and one humidifier unit \$110, write down a column matrix $\mathbf{D}$ that represents the profit made from selling each unit. [1]
- (iii) Hence, find matrix $\mathbf{E}$ where $\mathbf{E} = \mathbf{CD}$. [2]
- (iv) Explain what matrix $\mathbf{E}$ represents. [1]

[Turn over

- 11 The function $f(x) = ax^3 + 7x^2 - bx - 4$ where a and b are constants, is divisible by $x + 2$ and leaves a remainder of 185 when divided by $x - 3$.
- (i) Show that $a = 6$ and $b = 12$. [3]
- (ii) Use the value of a and b from (i), by **using long division**, find the remainder when $f(x)$ is divided by $2x^2 + 3$. [3]

- 12 $ABCD$ is a square of sides 6 cm. Q is the mid-point of DC .
An arc BRP is drawn with centre Q and P lies on DC produced.

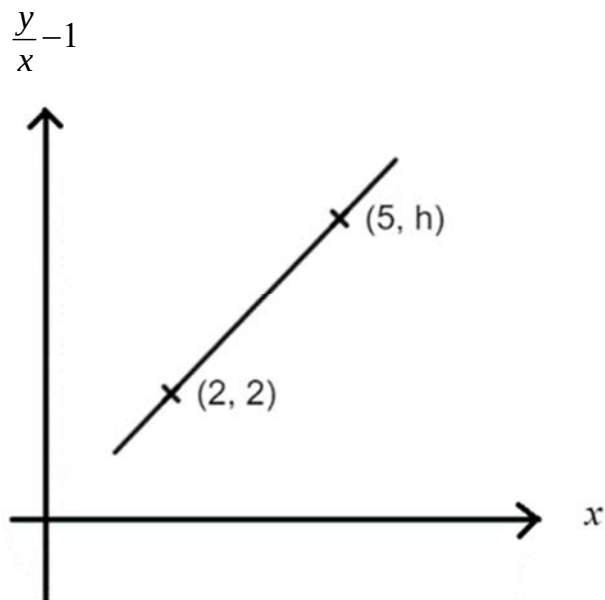


- (i) Show that the $\angle PQB = 2.03$. [2]
- (ii) Calculate the perimeter of $CPRBC$. [3]
- (iii) Calculate the area of the shaded region. [3]

- 13** The variables x and y are connected by the equation $2x^2 - ax - 3y = 0$, where a is a constant.

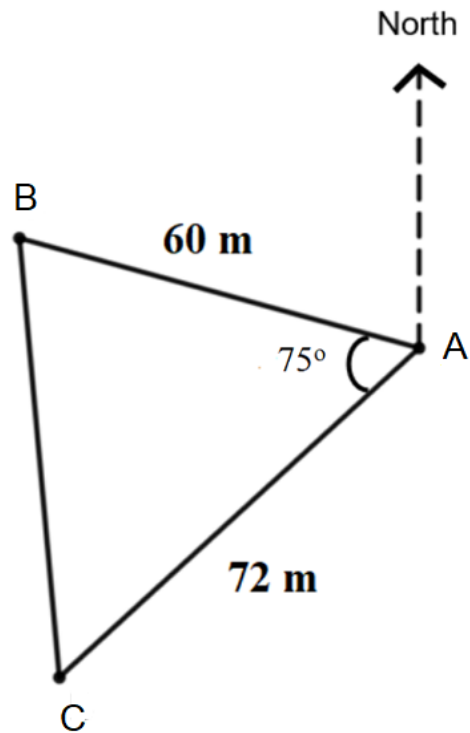
(i) Express $\frac{y}{x}$ in terms of x and a . [1]

(ii) When $\left(\frac{y}{x} - 1\right)$ is plotted against x , a straight line is obtained which passes through $(2, 2)$ and $(5, h)$. Find the value of a and of h . [4]



[Turn over

- 14 In the diagram, the points A , B and C are on level ground. The bearing of C from A is 217° and $\angle BAC = 75^\circ$.



Given that $AB = 60$ m and $AC = 72$ m, find

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|-------|-------------------------------|-----|
| (i) | the length BC , | [2] |
| (ii) | the angle ABC , | [2] |
| (iii) | the bearing of C from B . | [2] |