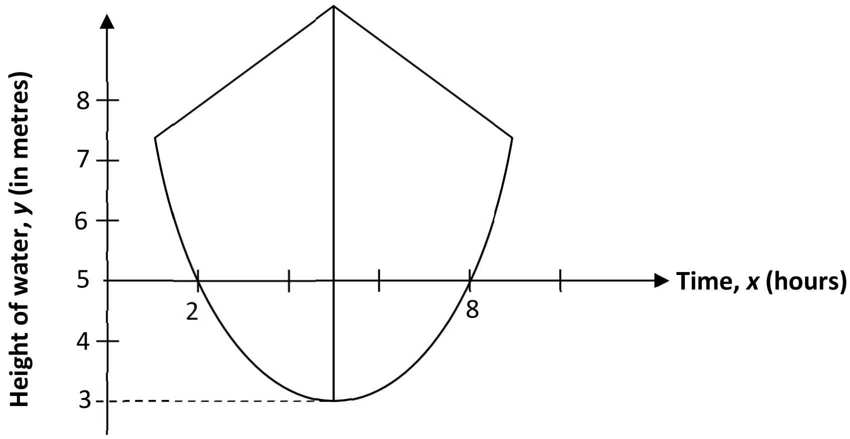
Topical Revision Worksheet 6

Topic: Coordinate Geometry & Standard Graphs

Question 1 [2008 EOY, Q9]	marks

The graph below shows part of a quadratic curve which represents the water level, y metres, of a river over a period of time in hours during the operation of a water dam nearby. At 2 pm and 8 pm, the water level was at the recommended level of 5 metres. Let x represents the time in hours since 12pm.



(i) At what time did the water level reaches its minimum? [1]

Solution:

$\frac{2+8}{2}=5$; at 5pm

(ii) Find the equation of the curve, expressing it in the form $y=a(x-h)^2+k$ where a, h and k are real numbers. [4]

Solution:

The vertex is at (5, 3) so $y=a(x-5)^2+3$

Sub (2, 5) into the equation to get $5=9a+3 \Rightarrow a=\frac{2}{9}$

Hence $y=\frac{2}{9}(x-5)^2+3$

(iii) Hence, or otherwise, determine the height of the water level at 12 pm. [2]

Solution:

Sub $x=0$ the height at 12 pm is $y=\frac{2}{9}(25)+3=8\frac{5}{9}$ metres

Question 2 [2008 EOY, Q10]	marks

Find the range of values of k if the line $2y = x + k$ intersects the curve $y = 2x + \frac{6}{x}$ more than once. [5]

Solution:

Equate the two equations to get

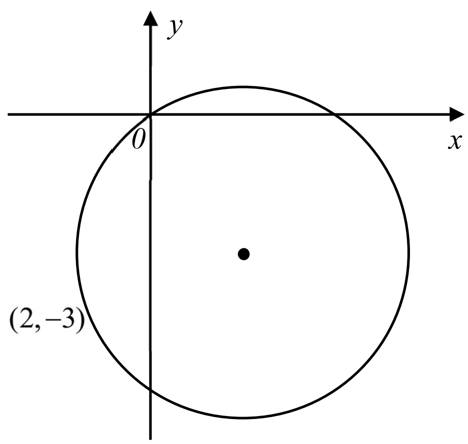
$$\frac{x+k}{2} = 2x + \frac{6}{x} \Rightarrow x^2 + kx = 4x^2 + 12 \Rightarrow 3x^2 - kx + 12 = 0$$

For more than one root, $(-k)^2 - 4(3)(12) > 0 \Rightarrow k^2 - 144 > 0$

Hence $(k-12)(k+12) > 0 \Rightarrow k < -12$ or $k > 12$

Question 3 [2008 EOY, Q12]	marks

The diagram below shows a circle with radius r units and its centre is $(2,-3)$.



(i) Given that the circle passes through the origin, find the exact value of r.

Solution:

$r = \sqrt{2^2 + (-3)^2} = \sqrt{13}$ [2]

(ii) The circle is reflected about the y-axis and then moved 5 units downwards. Write down the coordinates of the new centre of the circle. [1]

Solution:

$(-2, -3-5) = (-2, -8)$

(iii) Find the equation of the new circle in the form $x^2 + y^2 + 2gx + 2fy + c = 0$ where g, f and c are real constants. [3]

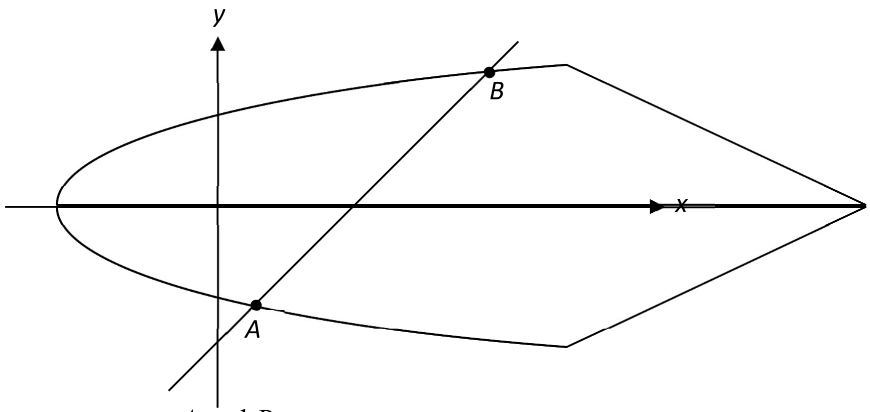
Solution:

$(x+2)^2 + (y+8)^2 = 13$

$\Rightarrow x^2 + 4x + 4 + y^2 + 16y + 64 = 13 \Rightarrow x^2 + y^2 + 4x + 16y + 55 = 0$

Question 4 [2008 EOY, Q15] — marks

The figure below shows the graphs of $2y^2 = x+7$ and $y = x-3$.



The two curves meet at A and B.

(i) Show that the coordinates of A are $(1,-2)$ and find the coordinates of B.

Solution: [3]

$$2(x-3)^2 = x+7 \Rightarrow 2x^2 - 12x + 18 = x+7 \Rightarrow 2x^2 - 13x + 11 = 0$$

Solve to get $(2x-11)(x-1) = 0 \Rightarrow x = 1$ or $\dfrac{11}{2}$

Sub $x=1$ to get $y=-2$ hence A is $(1,-2)$ (shown) and

sub $x = \dfrac{11}{2}$ to get $y = \dfrac{5}{2}$ so B is $\left(\dfrac{11}{2}, \dfrac{5}{2}\right)$

(ii) Find the length of AB.

Solution: [2]

$$AB = \sqrt{\left(\frac{11}{2}-1\right)^2 + \left(\frac{5}{2}+2\right)^2} = \sqrt{\frac{2(81)}{4}} = \frac{9\sqrt{2}}{2}$$ or 6.36 units

(iii) Find the equation of the perpendicular bisector of AB.

Solution: [3]

Midpoint $= \dfrac{1}{2}\left(1+\dfrac{11}{2}, -2+\dfrac{5}{2}\right) = \left(\dfrac{13}{4}, \dfrac{1}{4}\right)$

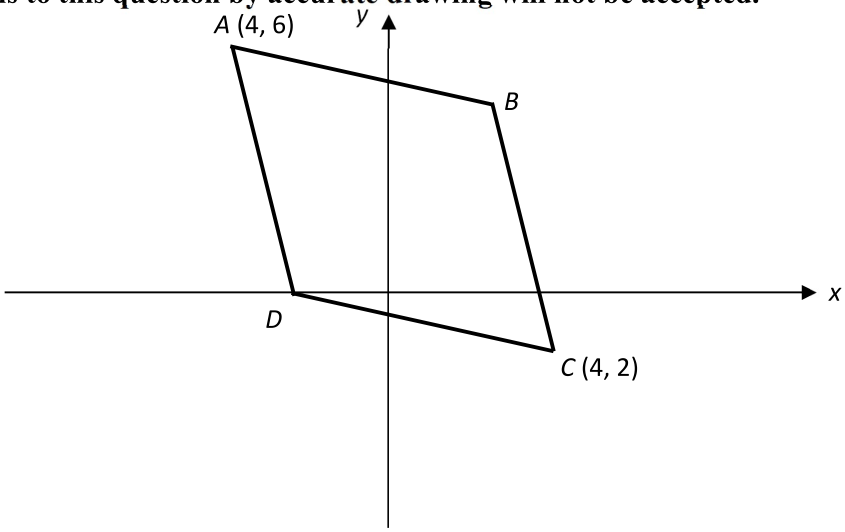
Gradient of perpendicular $= -1 \div \dfrac{\frac{5}{2}+2}{\frac{11}{2}-1} = -1$ hence

Perpendicular bisector of AB is $y - \dfrac{1}{4} = -\left(x - \dfrac{13}{4}\right) \Rightarrow y = \dfrac{7}{2} - x$

Question 5 [2009 EOY, Q1]	marks
Find the coordinates of the points at which the straight line $y=-x-5$ intersects the curve $x^2+(y+2)^2=5$. **Solution:** Sub $y=-x-5$ into $x^2+(y+2)^2=5$: $x^2+(-x-3)^2=5$ $x^2+x^2+6x+9=5$ $2x^2+6x+4=0$ $x^2+3x+2=0$ $(x+2)(x+1)=0$ $x=-1$ or -2 $\therefore y=-4$ or -3 The coordinates of the points are (−1, −4) and (−2, −3).	[4]

Question 6 [2009 EOY, Q9]	marks
The equation of a circle *A* is $x^2+y^2+6x-4y+4=0$. Find the coordinates of the centre of circle *A* and state its radius. **Solution:** $(x+3)^2-9+(y-2)^2-4+4=0$ $(x+3)^2+(y-2)^2=3^2$ A has centre (− 3, 2), radius = 3	[3]

Question 7 [2009 EOY, Q11]	marks
Solutions to this question by accurate drawing will not be accepted.	



In the figure, *ABCD* is a rhombus with vertices *A* (−4, 6) and *C* (4, −2). [3]

Given that the vertex *B* lies on the line $3x+2y=14$, calculate the

(i) equations of *AC* and *BD*,

<u>Solution:</u>

midpoint of *AC* = midpoint of *BD* = (0, 2)

gradient of *AC* = $\dfrac{6-(-2)}{-4-4}=-1$

Equation of *AC*: $y-6=-1(x+4)$

$y=-x+2$

Gradient of *BD* = 1

Equation of *BD*: $y-2=1(x-0)$ $\quad y=x+2$

(ii) coordinates of *B* and *D*,

<u>Solution:</u> [3]

Coordinates of *B*: Solve $y=x+2$ and $3x+2y=14$ simultaneously.

$y=x+2\rightarrow(1)$

$3x+2y=14\rightarrow(2)$

Solving, x = 2, y = 4 $\quad$ *B* = (2, 4)

Use midpoint to calculate coordinates of *D*:

$\left(\dfrac{x_1+2}{2},\dfrac{y_1+4}{2}\right)=(0,2)$ $\quad D=(-2,0)$

(iii) equation of AB, **Solution:** Gradient of $AB = \frac{6-4}{-4-2} = -\frac{1}{3}$ Equation of AB: $y - 6 = -\frac{1}{3}(x+4)$ $y = -\frac{1}{3}x + \frac{14}{3}$	[2]
(iv) area of the rhombus $ABCD$ and hence find the perpendicular distance from B to the line CD. **Solution:** Area of $ABCD = \frac{1}{2}\begin{vmatrix} -4 & 2 & 4 & -2 & -4 \\ 6 & 4 & -2 & 0 & 6 \end{vmatrix}$ $= \frac{1}{2}\left\|-16-4+0-12-(0+4+16+12)\right\|$ $= \frac{1}{2}\left\|-64\right\|$ $= 32$ units^2. Let perpendicular distance from B to line CD be h. $AB \times d = 32$ $d = \frac{32}{\sqrt{(6-4)^2 + (-4-2)^2}}$ $= \frac{32}{\sqrt{40}}$ $= 1.6\sqrt{10}$ or 5.06 units	[4]

Question 8 [2010 EOY, Q4]	marks
Find the range of values of k such that the curve $y+2x^2=k(x-1)$ meets the line $2y=6kx+2k^2-3$ at two distinct points. **Solution:** Substitute y: $2(kx-k-2x^2)=6kx+2k^2-3 \Rightarrow 4x^2+4kx+2k^2-3+2k=0$ For two distinct roots, $(4k)^2-4(4)(2k^2+2k-3)>0 \Rightarrow k^2+2k-3<0$ Hence $(k+3)(k-1)<0 \Rightarrow -3<k<1$	[5]

Question 9 [2010 EOY, Q5]	marks
A quadratic curve is such that it is symmetrical about the line $x=2k$, where $k>1$. Given that it cuts the x-axis when $x=2$, and the maximum value of the curve is $k-1$, (i) explain why the other x-intercept is $4k-2$, and **Solution:** The roots are equidistant from the line of symmetry, so the other x-intercept is at $2k+(2k-2)=4k-2$	[1]
(ii) find, in terms of x, y and k, the equation of the curve **Solution:** Vertex form: $y=a(x-2k)^2+(k-1)$ Sub $(2,0)$ to get $a=\frac{1}{4(1-k)}$ [A1], hence $y=\frac{(x-2k)^2}{4(1-k)}+(k-1)$ **OR** Product form: $y=p(x-2)(x-4k+2)$ Sub $(2k,k-1)$ to get $p=\frac{1}{4(1-k)}$, hence $y=\frac{(x-2)(x-4k+2)}{4(1-k)}$	[3]
(cont'd) Given further that the distance between the two x-intercepts is 6 units, find the coordinates of the y-intercept. **Solution:** $4k-2-2=6 \Rightarrow k=\frac{5}{2}$ [B1] hence $y=-\frac{(x-5)^2}{6}+\frac{3}{2}$ (using vertex form) Hence y-intercept $=-\frac{25}{6}+\frac{3}{2}=-\frac{8}{3}$	[3]

Question 10 [2010 EOY, Q6]	marks

On separate axes, sketch the graphs of

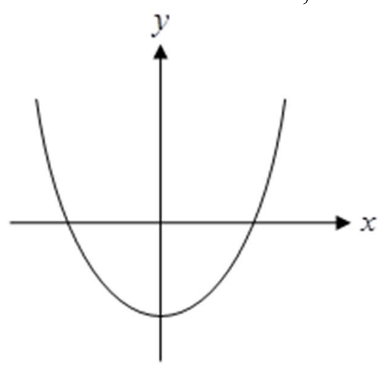
(i) $y = a(x-h)^2 + k$ for $a < 0$, $h < 0$ and $k > 0$ [2]

Solution:

[G1] – Open downwards [G1] – Vertex in 2nd quadrant

(ii) $y = k(x-\alpha)(x-\beta)$ for $\alpha\beta < 0$, $|\alpha| = |\beta|$ and $k > 0$ [2]

Solution:

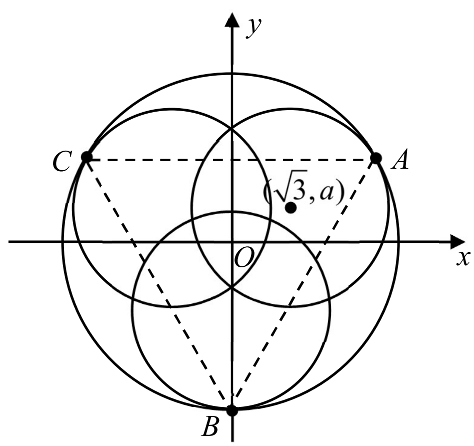


[G1] – Open upwards

[G1] – Vertex lies on y-axis

Question 11 [2010 EOY, Q7]	marks

The diagram below shows three small identical circles inscribed in a big circle centred at the origin. Each small circle has radius r units (where $r > 2$) and one of the small circles has its centre at the point $(\sqrt{3}, a)$. The small circles are arranged with a rotational symmetry about the origin of order 3.



(i) By considering the rotational symmetry of the diagram, explain fully why $a = 1$.

Solution:

Due to rotational symmetry of order 3, OA (the radius of the bigger circle) is part of the line of symmetry of the equilateral triangle ABC, and hence makes an angle of 30° [M1] with the x-axis (alternate angle with $\measuredangle OAB$). Hence, $\tan 30° = \dfrac{a}{\sqrt{3}} \Rightarrow a = \sqrt{3} \times \dfrac{1}{\sqrt{3}} \Rightarrow a = 1$ [2]

(ii) Write down the equation of the bigger circle, in terms of *x*, *y* and *r*. **Solution:** Radius of big circle $=r+2$ units [B1] Hence $x^2+y^2=(r+2)^2$ or $x^2+y^2=r^2+4r+4$	[2]
Given further that $r=3$, (iii) find the coordinates of *A*, the point of contact between the smaller circle and the bigger circle as shown in the diagram, and **Solution:** When $r=3$, $OA=5$ units [B1] hence coordinates of *A* are $(5\cos 30°, 5\sin 30°)=\left(\frac{5\sqrt{3}}{2},\frac{5}{2}\right)$	[3]
(iv) show that the area of the triangle *ABC* is $\frac{75\sqrt{3}}{4}$ units². **Solution:** Due to symmetry, $AB=\frac{5\sqrt{3}}{2}\times 2$ units $=5\sqrt{3}$ units Hence area of triangle $=\frac{1}{2}\times 5\sqrt{3}\times 5\sqrt{3}\times \sin 60°$ units^2 $=\frac{75\sqrt{3}}{4}$ units^2	[3]
Question 12 [2011 EOY, Q6]	marks
The equation of a circle is given by $x^2+y^2=6x-8y$. (i) Find the centre and the radius of the circle. **Solution:** $\left(x^2-6x+9\right)+\left(y^2+8y+16\right)=25$ $(x-3)^2+(y+4)^2=25$ Centre $(3,-4)$, radius 5 .	[4]
(ii) Hence or otherwise, find the greatest possible value of y. **Solution:** By sketching the circle, or show $y=-4+5$: Greatest *y* is 1	[2]

Question 13 [2011 EOY, Q10a]	marks
A quadratic curve has a minimum point at $(1,\ 5)$ and passes through $(0,\ 7)$. If the curve does not meet the line $2(y-mx)=13$, find the range of values of *m*.	[5]

Solution:

Equation of the curve is $y=a(x-1)^2+5$

At $(0,7)$, $\quad 7=a(-1)^2+5 \ \Rightarrow a=2$

$y=2(x-1)^2+5........(1)$

$y=mx+6.5......(2)$

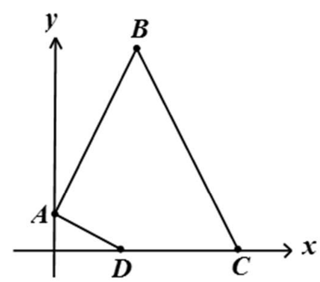
$2(x-1)^2+5=mx+6.5$

$2x^2-x(4+m)+0.5=0$

Since line does not meets the curve, $b^2-4ac<0 \qquad (4+m)^2-4(2)(0.5)<0$

$m^2+8m+12<0 \qquad (m+2)(m+6)<0 \qquad -6<m<-2$

Question 14 [2011 EOY, Q14]	marks
Solutions to this question by accurate drawing will not be accepted. The diagram shows the quadrilateral *ABCD*. The coordinates of *A* and *B* are $(0,1)$ and $(3,7)$ respectively. Points *C* and *D* lie on the *x*-axis and $\angle BAD=90°$. (i) Find the equation of the perpendicular bisector of line *AB*.	[3]



Solution:

Midpoint of *AB*, $M(1.5,4)$

Gradient of *AB* $=\dfrac{7-1}{3-0}=2$

Equation of perpendicular of *AB* is:

$y-4=-0.5(x-1.5)$

$y=-0.5x+4.75$

(ii) Find the equation of *AD* and hence find the coordinates of *D*.	[2]

Solution:

Equation of *AD* is $y=-0.5x+1$

At *D*, $y=0$, $\therefore x=2 \qquad D(2,0)$

(iii) If the area of ΔBCD is 14 square units, find the coordinates of *C*	[1]

Solution:

$\dfrac{1}{2}\times CD\times 7=14 \qquad CD=4 \qquad \therefore C(2+4,0), \quad i.e.\ C(6,0)$

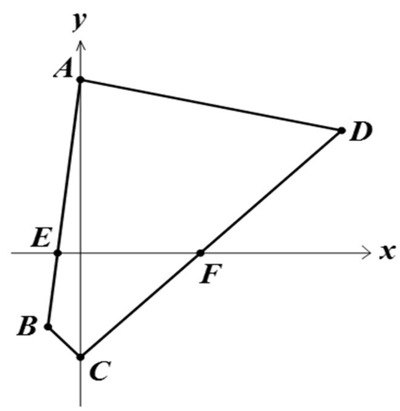
(iv) Find all the possible coordinates of E for which A, B, C and E are vertices of a parallelogram. **Solution:** $E(6-3,0-6),\ \ i.e.\ E(3,-6)$ or $E(6+3,0+6),\ \ i.e.\ E(9,6)$ $or\ \ E(3-6,8-0),\ \ i.e.\ \ E(-3,8)$	[3]
Question 15 [2012 EOY, Q5]	marks
A set of points with coordinates (x, y) lies on the graph whose equation is $\frac{x^2+2x}{4}+\left(\frac{y}{2}-1\right)^2=2$. The points are d units from a fixed point P. Find the coordinates of P and the value of d. **Solution:** $\frac{x^2+2x}{4}=2-\left(\frac{y}{2}-1\right)^2\ \ (x+1)^2+(y-2)^2=9$ $\therefore P(-1,2)$ and $d=3$	[4]
Question 16 [2012 EOY, Q10 c]	marks
Find the coordinates of the points of intersection of the line $10x+5y+4=0$ and the curve $4x^2+y^2=16$. **Solution:** $4x^2+y^2=16\rightarrow(1)$ $10x+5y=-4\rightarrow(2)$ From (2): $y=\frac{-4-10x}{5}\rightarrow(3)$ Sub (3) into (1): $4x^2+\left(\frac{-4-10x}{5}\right)^2=16$ $100x^2+(-4-10x)^2=400$ $100x^2+16+80x+100x^2=400$ $200x^2+80x-384=0$ $25x^2+10x-48=0$ $(5x-6)(5x+8)=0$ $x=\frac{6}{5}\,or-\frac{8}{5}$ $y=-\frac{16}{5}\,or\,\frac{12}{5}$ $\therefore$ the points of intersection are $\left(\frac{6}{5},-\frac{16}{5}\right)$ and $\left(-\frac{8}{5},\frac{12}{5}\right)$	[5]

Question 17 [2012 EOY Q11] marks

Solutions to this question by accurate drawing will not be accepted.

The diagram shows a quadrilateral *ABCD* with $A(0,6)$, $B(-1,-3)$, $C(0,-4)$, and $D(9,\ 5)$. *AB* and *CD* intersect the *x*-axis at points $E\left(-\frac{2}{3},0\right)$ and $F\ (4,0)$ respectively.

(i) Show that angle $BAD = 90°$.



Solution:

gradient of $AB = \frac{6-(-3)}{0-(-1)} = 9$

gradient of $AD = \frac{6-5}{0-9} = -\frac{1}{9}$

gradient of $AB \times$ gradient of $AD = 9 \times \left(-\frac{1}{9}\right) = -1$

$\therefore AB \perp AD$

[2]

(ii) Find the equation of the perpendicular bisector of *AD*. [3]

Solution:

midpoint of $AD = \left(\frac{0+9}{2}, \frac{6+5}{2}\right) = (4.5, 5.5)$

the equation of the perpendicular bisector of line AD:

$\frac{y-5.5}{x-4.5} = 9 \qquad y - 5.5 = 9x - 40.5$

$y = 9x - 35$

(iii) Given that *ABCD* is a cyclic quadrilateral, find the exact length of the diameter of the circle passing through *A*, *B*, *C* and *D*. [2]

Solution:

$\because \angle BAD = 90°$,

BD is the diameter of the circle ($\angle$in semi-circle)

$BD \quad = \sqrt{(-1-9)^2 + (-3-5)^2}$

$\quad = \sqrt{164} = 2\sqrt{41}$

(iv) A point *P* is such that *BCDP* is a rectangle, find the coordinates of *P*. [2]

Solution:

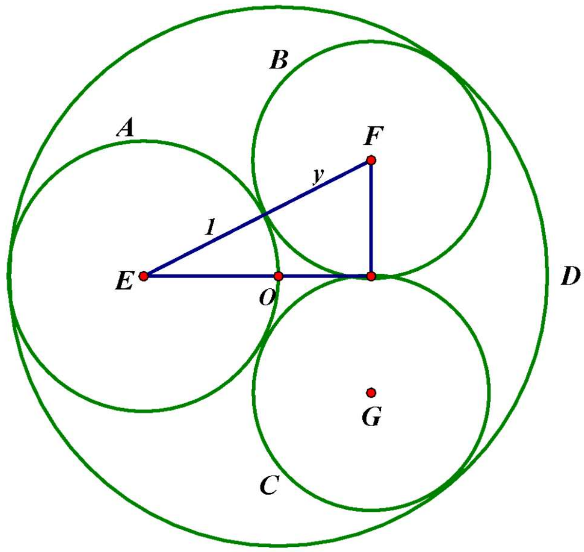
Mid-point of CP = Mid-point of BD

$\left(\frac{0+x}{2}, \frac{-4+y}{2}\right) = \left(\frac{-1+9}{2}, \frac{-3+5}{2}\right)$

$(x, y) = (8, 6)$

(v) Find the area of the quadrilateral $AEFD$. **Solution:** Area of $AEFD$ $= \frac{1}{2}\begin{vmatrix} 0 & -\frac{2}{3} & 4 & 9 & 0 \\ 6 & 0 & 0 & 5 & 6 \end{vmatrix}$ $= \frac{1}{2}(20+54+4) = 39$	[2]
Question 18 [2013 EOY, Q13]	marks
(i) The equation of a circle is given by $x^2 + y^2 - 10x + 2y + 1 = 0$. Find the centre and the radius of the circle. **Solution:** Centre is (5, -1) and the radius is $r = \sqrt{5^2 + (-1)^2 - 1} = 5$ units	[2]
(ii) The tangent to the circle at a point A is parallel to the straight line $x - y + 2 = 0$. Find the equation of the diameter of the circle through A. **Solution:** The gradient of the straight line $x - y + 2 = 0$ is $m = 1$ so the gradient of the diameter through A is -1. The equation of the diameter passes through A is $y - 5 = -1(x+1) \quad \Rightarrow \quad x + y = 4$	[2]
(iii) The circle is then reflected on the line $y = x$. State the equation of the new circle formed. **Solution:** The coordinates of the centre of the circle after the reflection on the line $y = x$ are (-1, 5) Hence the equation of the new circle formed is $(x+1)^2 + (y-5)^2 = 5^2$ $\Rightarrow x^2 + y^2 + 2x - 10y + 1 = 0$	[1]

Question 19 [2013 EOY, Q14]	marks
Circles A, B and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. E, F and G are the centres of circles A, B and C respectively. Circle A has radius 1 and passes through O, the centre of circle D. Both circles B and C have radii y. By considering two right-angled triangles, find the radius y of the circle B.	[6]



Solution:

Let P be the contact point of circle B and circle C.

Let $OP = x$ and join OF, $OF = 2 - y$.

In right-angled triangle ΔEFP,

$$(1+y)^2 = y^2 + (1+x)^2$$

$$\Rightarrow\ y = x + \frac{x^2}{2} \quad \text{---- (1)}$$

In right-angled triangle ΔFOP,

$$(2-y)^2 = x^2 + y^2$$

$$\Rightarrow\ y = 1 - \frac{x^2}{4} \quad \text{---- (2)}$$

Solving eqns (1) and (2)

$$x + \frac{x^2}{2} = 1 - \frac{x^2}{4}$$

$$\Rightarrow\ 3x^2 + 4x - 4 = 0$$

$$(3x-2)(x+2) = 0$$

i.e. $x = \frac{2}{3}$ or $x = -2$ (rej)

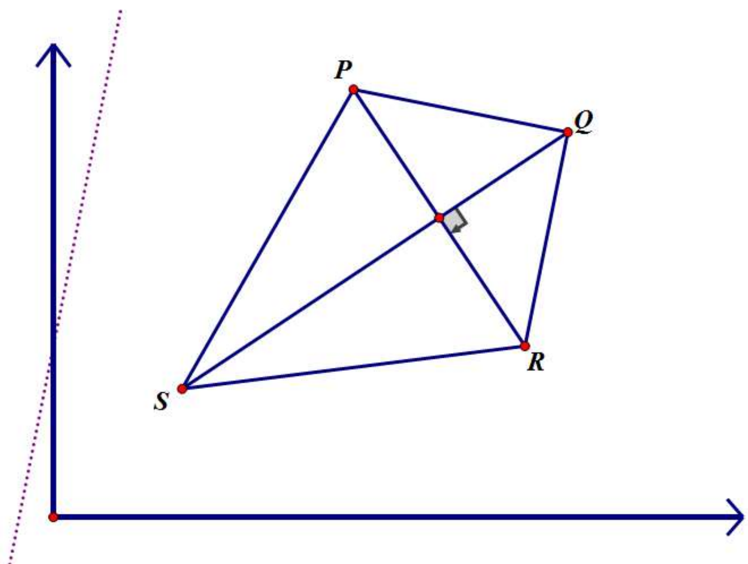
hence $y = 1 - \frac{(\frac{2}{3})^2}{4} = 1 - \frac{1}{9} = \frac{8}{9}$ (the radius of circle B).

Question 20 [2013 EOY, Q16]	marks

Solutions to this question by accurate drawing will not be accepted

The diagram shows a quadrilateral $PQRS$ in which QS is the perpendicular bisector of PR. The mid-point of PR is M.

The coordinates of R and M are $(11, 4)$ and $(9, 7)$ respectively. RQ is parallel to the line $y = 5x + 4$ and the equation of SR is $x - 8y + 21 = 0$.



Find

(i) the coordinates of P,

[2]

Solution:

let $P = (s, t)$, since M is the mid-point PR

$$\left(\frac{s+11}{2}, \frac{t+4}{2}\right) = (9, 7) \quad \Rightarrow \quad s = 7 \text{ and } t = 10$$

$P(7,10)$

(ii)

the equations of QR and QS,

[4]

Solution:

Gradient of the given line is 5 and RQ//to the given line

So the equation of QR is $y - 4 = 5(x - 11)$ or $y = 5x - 51$

Gradient of $RM = \frac{7-4}{9-11} = -\frac{3}{2}$

Gradient of $QS = \frac{2}{3}$.

So the equation of QS is

$y - 7 = \frac{2}{3}(x - 9)$ or $2x - 3y + 3 = 0$

(iii)

the coordinates of Q and of S, [3]

Solution:

Coordinates of Q : Solve Equation QR : $y = 5x - 51$ and Equation QS : $2x-3y+3=0$

$\Rightarrow \quad x=12$ and $y=9 \quad \therefore \ Q(12,9)$

Coordinates of S : Solve Equation RS : $x-8y+21=0$ and Equation QS :

$2x-3y+3=0$

or $y=\frac{2}{3}x+1$

$\Rightarrow \quad x=3$ and $y=3 \quad \therefore \ S(3,3)$

(iv)

the area of $PQRS$. [2]

Solution:

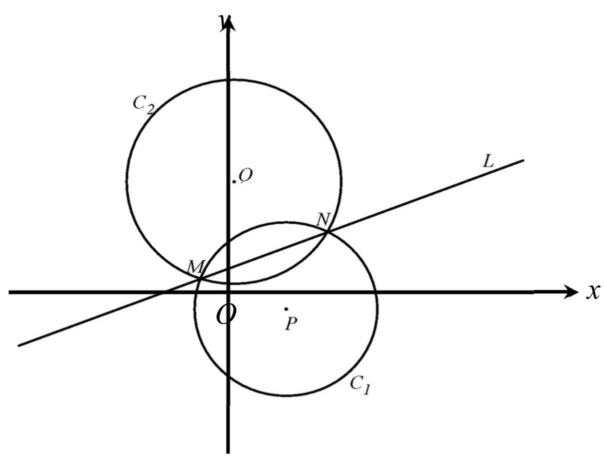
Area of $PQRS$ $=\frac{1}{2}\times\begin{vmatrix}3 & 11 & 12 & 7 & 3\\ 3 & 4 & 9 & 10 & 3\end{vmatrix}$

$=\frac{1}{2}\times(12+99+120+21-33-48-63-30)$

= 39 square unit

Question 21 [2014 EOY, Q16]	marks

The diagram shows two intersecting circles C_1 and C_2 with centres P and Q respectively. A straight line L passes through M and N, which are the points of intersections of the two circles.



(i) Given that the equation of the circle C_1 is $x^2+y^2-6x+4y-21=0$, find the coordinates of P and the radius of C_1.

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Solution:

$x^2+y^2-6x+4y-21=0$

$(x-3)^2+(y+2)^2=34$

Hence, the coordinates of *P* are $(3,-2)$, and the radius of C_1 is $\sqrt{34}$ units.

</td><td>[3]</td></tr>
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(ii) It is also given that the equation of the line *L* is $4y-x=6$, find the coordinates of *M* and of *N*.

Solution:

$4y-x=6 \quad ...(1)$

$x^2+y^2-6x+4y-21=0 \quad ...(2)$

From (1), $x=4y-6 \quad ...(3)$

Sub (3) into (2):

$(4y-6)^2+y^2-6(4y-6)+4y-21=0$

$17y^2-68y+51=0$

$y^2-4y+3=0$

$(y-1)(y-3)=0$

$y=1$ or 3

$x=-2$ or 6

$\therefore M=(-2,1),\ N=(6,3)$

</td><td>[4]</td></tr>
<tr><td>

(iii) Find the equation of the perpendicular bisector of *MN*.

Solution:

Equation of $MN: 4y-x=6$

$\Rightarrow y=\frac{1}{4}x+\frac{3}{2}$

$\therefore$ Gradient of the perpendicular bisector

$=\frac{-1}{1/4}=-4$

Midpoint of $MN=\left(\frac{6-2}{2},\frac{3+1}{2}\right)=(2,2)$

$\therefore$ Equation of the perpendicular bisector:

$y-2=-4(x-2)$

$y=-4x+10$

</td><td>[3]</td></tr>
<tr><td>

(iv) Given that the radius of circle C_2 is $\frac{1}{2}\sqrt{221}$ units, find the coordinates of *Q*.

Solution:

Since *Q* is the centre of C_2, it lies on the perpendicular bisector of chord *MN*.

Hence, the coordinates of *Q* satisfy the equation

</td><td>[4]</td></tr>
</table>

$y = -4x + 10 \quad ...(1)$

Also, since the radius of $\frac{1}{2}\sqrt{221}$ units, $QM = \frac{1}{2}\sqrt{221}$.

$\sqrt{(x+2)^2 + (y-1)^2} = \frac{1}{2}\sqrt{221}$

$(x+2)^2 + (y-1)^2 = \frac{221}{4} \quad ...(2)$

Substitute (1) into (2):

$(x+2)^2 + (-4x+9)^2 = \frac{221}{4}$

$17x^2 - 68x + \frac{119}{4} = 0$

$4x^2 - 16x + 7 = 0$

$(2x-1)(2x-7) = 0$

$x = \frac{1}{2}$ or $\frac{7}{2}$

$y = 8$ or -4

Since Q lies above the x-axis, only the pair of solution $\left(\frac{1}{2}, 8\right)$ is accepted.

Hence, coordinates of Q are $\left(\frac{1}{2}, 8\right)$.

(v)

Calculate the area of the quadrilateral $PMQN$. [2]

Solution:

Area of $PMQN$

$$= \frac{1}{2}\begin{vmatrix} 3 & 6 & \frac{1}{2} & -2 & 3 \\ -2 & 3 & 8 & 1 & -2 \end{vmatrix}$$

$$= 42\frac{1}{2} \text{ units}^2$$

Question 22 [2015 EOY, Q10]	marks
Two lines l_1 and l_2 are such that they are parallel, and l_2 is the perpendicular bisector of two points $A(-2,4)$ and $B(3,-6)$. (i) Find the equation of l_2. **<u>Solution:</u>** Midpoint of $AB = M\left(\frac{1}{2},-1\right)$ and gradient of $AB = -2$ Hence equation of l_2: $y-(-1)=\frac{1}{2}\left(x-\frac{1}{2}\right) \Rightarrow 4y+5=2x$ or $y=\frac{1}{2}x-\frac{5}{4}$	[3]
(ii) Hence find the equation of l_1 if it passes through the point $C(5,8)$. **<u>Solution:</u>** Equation of l_1: $y-8=\frac{1}{2}(x-5) \Rightarrow 2y=x+11$ or $y=\frac{1}{2}x+\frac{11}{2}$	[2]
(iii) Find the distance between the two lines. **<u>Solution:</u>** Equation of perpendicular to l_1 at $\left(0,\ \frac{11}{2}\right)$: $y=-2x+\frac{11}{2}$ Intersecting with l_2: $-2x+\frac{11}{2}=\frac{1}{2}x-\frac{5}{4}$	[4]

$\Rightarrow x = 2.7$ and $y = 0.1$ Hence distance between the two lines $= \sqrt{2.7^2 + (5.5 - 0.1)^2}$ units $= 2.7\sqrt{5}$ or 6.04 (3 sf.) units **OR** Equation of AB: $y - 4 = -2\left(x - (-2)\right) \Rightarrow y = -2x$ Intersecting with l_1: $\frac{1}{2}x + \frac{11}{2} = -2x$ $\Rightarrow x = -2.2$ and $y = 4.4$ Hence distance between the two lines $= \sqrt{(0.5 + 2.2)^2 + (4.4 + 1)^2}$ units $= 2.7\sqrt{5}$ or 6.04 (3 sf.) units	