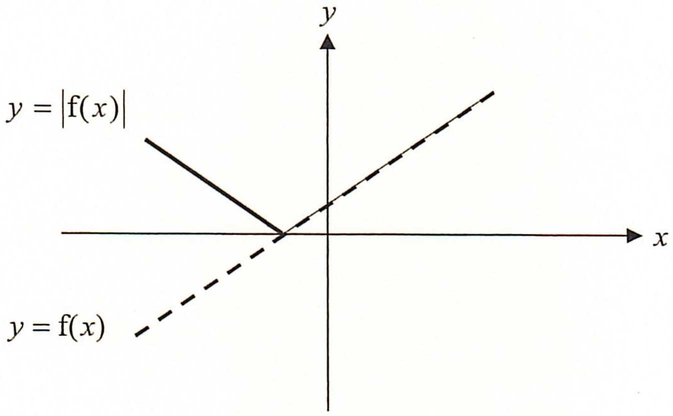
Hwa Chong Institution 华侨中学
Mathematics (SMTP)

Name: ______________________ (   ) Class: ______ Date: ________

MA302.4.X1 – More Geometrical Transformations

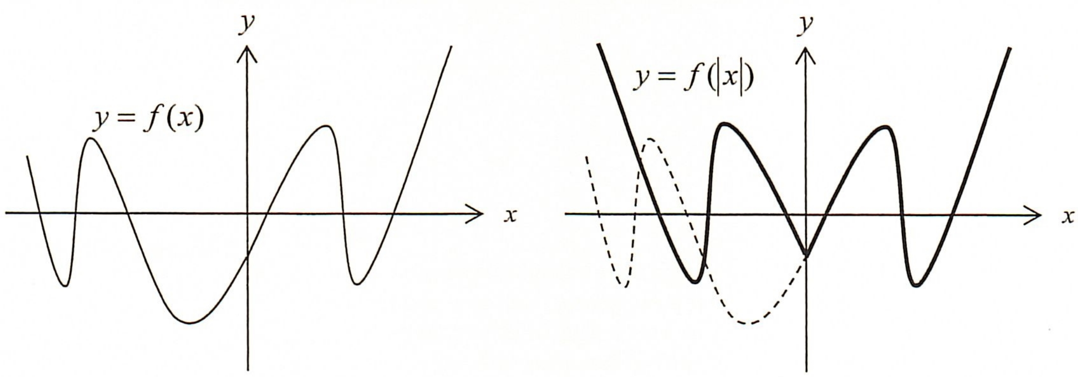
You have already encountered some basic transformation of graphs in the previous topics.

For example, the graph of $y=|\mathrm{f}(x)|$ is derived from that of $y=\mathrm{f}(x)$ by **reflecting** the portion of the graph which lies below the x-axis **about the x-axis**.



The case of $y=\mathbf{f}(|x|)$

Here notice that $\mathrm{f}(|-x|)=\mathrm{f}(|x|)$ so we would expect the graph to be symmetric about the y-axis. The portions of the original graph where $x<0$ are totally ignored under the transformation.



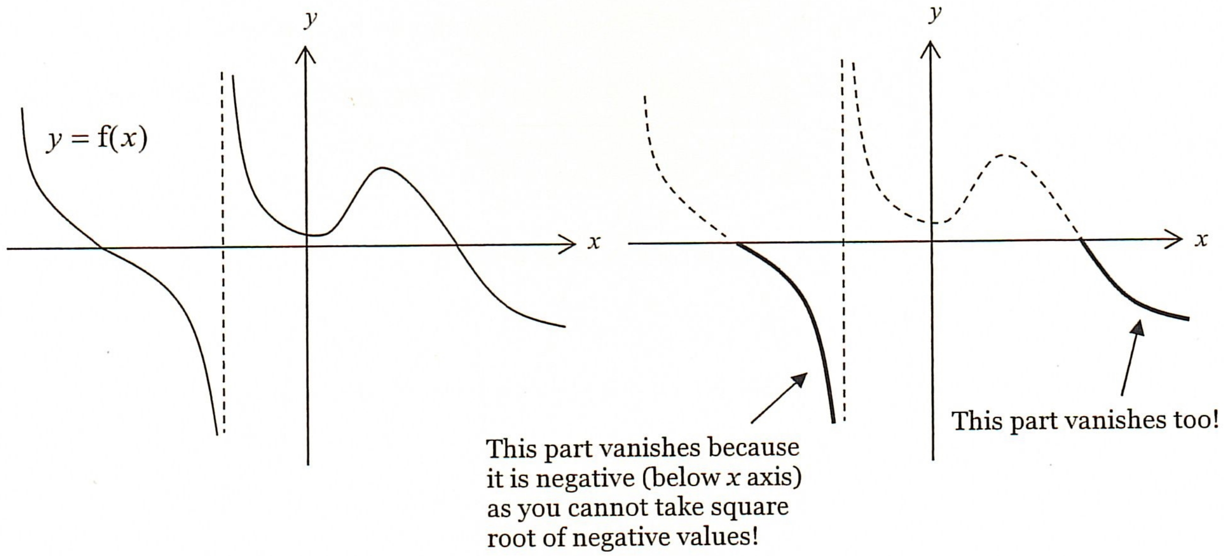
E1. Draw the graph of $y=2(x-1)^2-3$. Hence or otherwise, draw the graphs of $y=2(|x|-1)^2-3$ and $y=\left|2(|x|-1)^2-3\right|$, labelling your graphs clearly.

The case of $y^2=\mathrm{f}(x)$

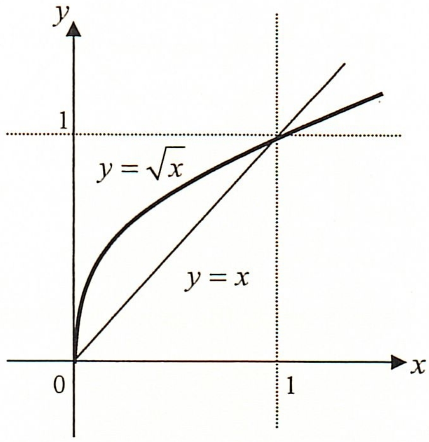
This can be rewritten as $y=\pm\sqrt{\mathrm{f}(x)}$. So for each x, there will have two values of y, and they are negates of each other.

$$(x,y)\mapsto(x,+\sqrt{y})\ \&\ (x,-\sqrt{y})$$

Since $\sqrt{y}$ is not real when $y<0$, we would not expect to see anything happening to the parts of the graph which lie below the x-axis. After the transformation, these parts simply vanish.

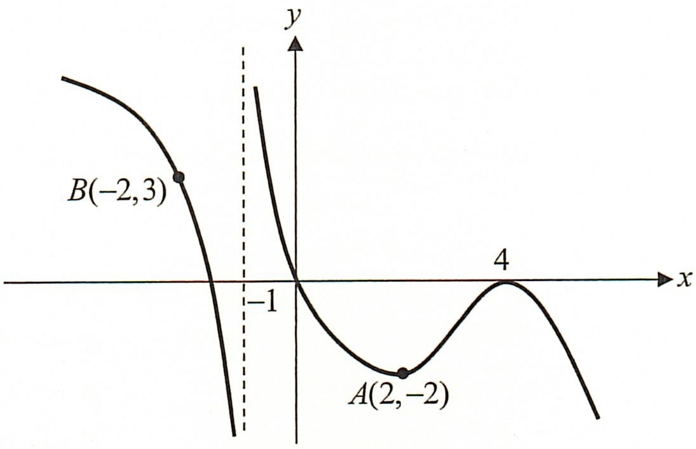


Let us look at the function $f : y \mapsto \sqrt{y}$ closely. We know that if $y > 1$, $\sqrt{y} < y$, whereas for $y = 1$, $\sqrt{y} = y$ (invariant point). For $0 < y < 1$, however, $\sqrt{y} > y$. Hence, while we expect the graph to be 'compressed' when the function undergoes the transformation $f : y \mapsto \sqrt{y}$, the graph actually takes a different 'turn' for values of y between 0 and 1.



S1. Sketch the graph of $y = e^{2x} - 1$. Hence sketch the graph of $y^2 = e^{2x} - 1$ on the same axes, showing clearly any intercepts with the axes and asymptotes.

S2. Below is a graph of $y = \mathrm{f}(x)$.



(a) Determine the images of A and B under the graph of $y = |\mathrm{f}(x)| + 1$.

(b) Describe what happens to the asymptote $x = -1$ under the graph of $y = 2\,\mathrm{f}(-x)$.

(c) Sketch the graph of $y = 1 - \mathrm{f}(x)$.