1 **(a)** Differentiate the following with respect to x and simplify your answer as a single fraction, where necessary.

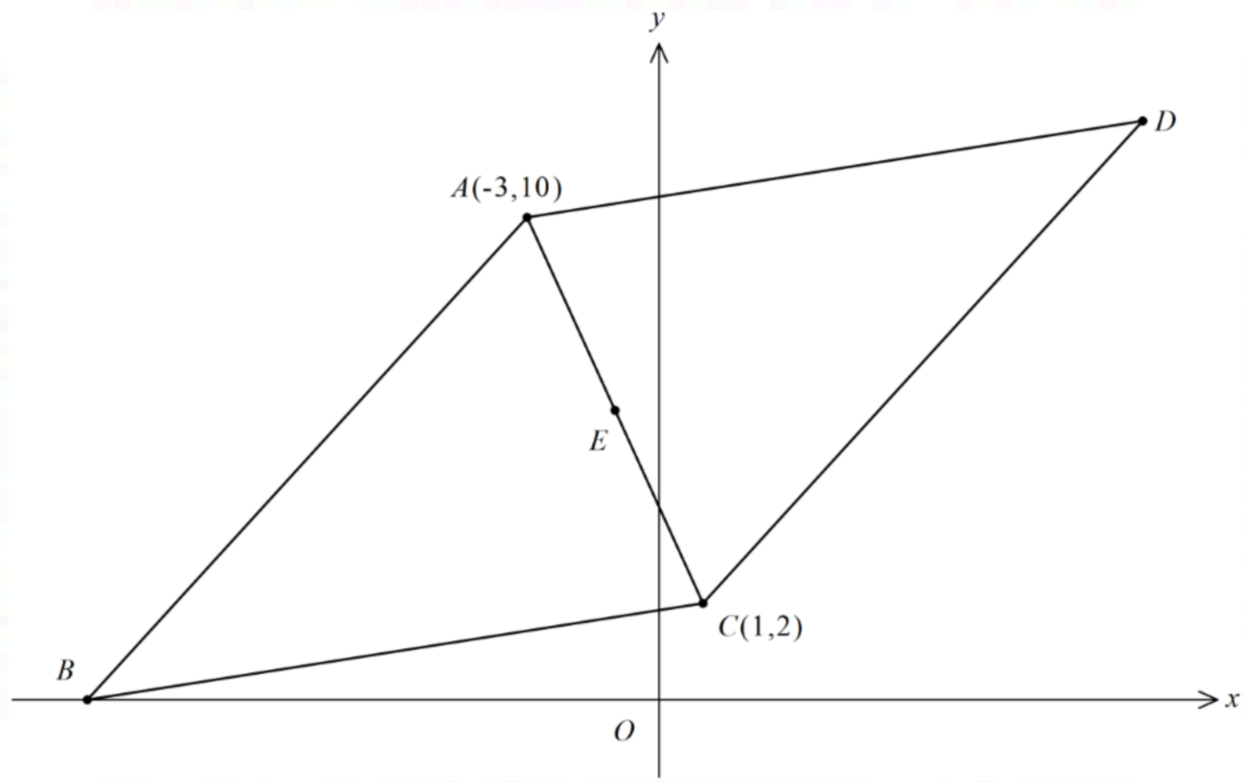
(i) $y = \ln\left(\dfrac{e^{4x^2}}{\sqrt{x+1}}\right)$ [3]

1 **(a)** **(ii)** $y = \dfrac{2x}{\sqrt{1+2x}}$ [3]

1 **(b)** Given that $y = x^3 e^{(x+1)^2}$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 e^{(x+1)^2}\left[\mathrm{f}(x)\right]$, find $\mathrm{f}(x)$. [3]

2 The points A $(-3, 10)$ and C $(1, 2)$ are vertices of a rhombus $ABCD$.

The point B lies on the x-axis and E is the mid-point of AC.



(a) Calculate the coordinates of E. [1]

2 **(b)** Calculate the coordinates of B. [3]

(c) Calculate the coordinates of D. [2]

3 **(a)** By completing the square, express $-2x^2 - 8x + c$ in the form $p(x+q)^2 + r$, where p and q are constants, and r is an expression in terms of c.

[2]

(b) The maximum value of $-2x^2 - 8x + c$ is 10.

(i) Find the value of c. [1]

(ii) State the value of x when the maximum value occurs. [1]

4 A polynomial can be expressed by $6x^3 - 13x^2 + Ax + 34 \equiv (2x^2 + Bx + 4)(3x + 7) + 5x + C$.

(a) Find the values of A, B and C.

[4]

(b) Find the remainder when $6x^3 - 13x^2 + Ax + 34$ is divided by $x - 4$. [2]

5 A curve has equation $y = \mathrm{f}(x)$ where $\mathrm{f}''(x) = 5 - \dfrac{5}{(x-2)^2}$.

Given that $\mathrm{f}(3) = \mathrm{f}'(3) = 5$, find $\mathrm{f}(x)$. [5]

6 The equation of the curve is $y = \dfrac{a\left(ax^2 - 1\right)}{3x + b}$, where a and b are non-zero constants.

The gradient of the tangent to the curve at $(0,\ 2)$ is 6.

(a) Find the value of a and of b. [5]

6 **(b)** Show that there is no tangent on the curve which is parallel to the line $y = 2$.

[2]

7 **(a)** Find $\displaystyle\int \frac{7e^{x} - 5e^{-x} + 2}{3e^{3x}}\,dx$. [2]

(b) Find $\displaystyle\int \frac{4}{\sqrt[3]{(2-5x)^2}}\,dx$. [2]

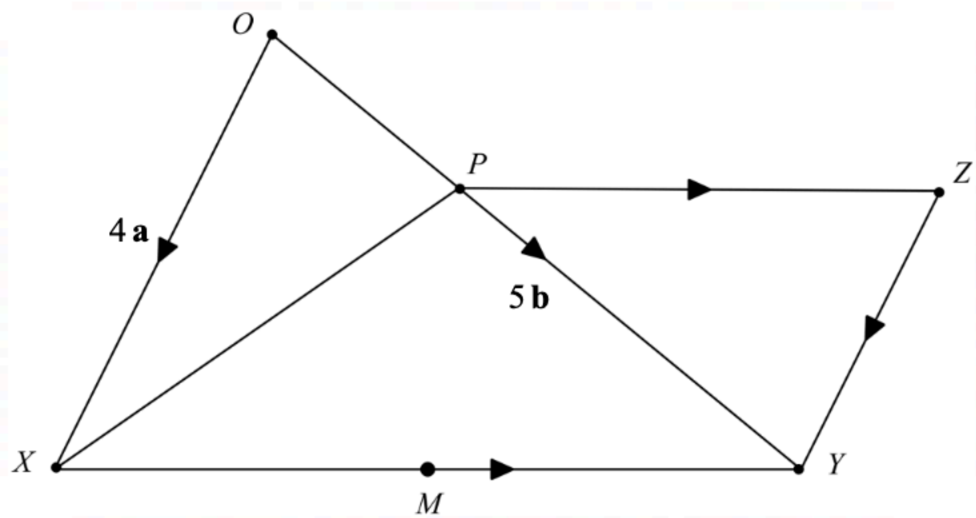
8 **(a)** Solve the equation $5^x = \sqrt{5e^{4x}}$. [3]

8 **(b)** Solve the equation to find x in terms of a, where $a > 0$.

$$2\ln(2x) = 2 + \ln a^2$$ [3]

9 In the diagram below, OX is parallel to ZY and PZ is parallel to XY.

O, P and Y lie on a straight line.
M is the midpoint of X and Y.
The position vectors of X and Y relative to O are $4\mathbf{a}$ and $5\mathbf{b}$ respectively.



(a) Show that $\overrightarrow{OP} = m\mathbf{b}$ where m is a real constant. [1]

(b) Given that $\overrightarrow{OP} = (2-n)\mathbf{a} + n\mathbf{b}$ where n is a real constant, find the value of n. [2]

9 **(c)** Find the numerical value of $\dfrac{\text{area of triangle } PYZ}{\text{area of triangle } YOX}$. [1]

(d) Find $\overrightarrow{PM}$. [3]

10 Let $\mathrm{f}(x) = 2x^2 + 3x - 3$ and $\mathrm{g}(x) = 4x^3 + ax^2 - 7x + b$, where a and b are constants.

When $\mathrm{g}(x)$ is divided by $\mathrm{f}(x)$, the quotient and remainder are equal.

(a) Find the quotient when $\mathrm{g}(x)$ is divided by $\mathrm{f}(x)$. [3]

(b) Show that $\mathrm{g}(x) = (2x-1)^2(x+2)$. [2]

10 **(c)** Express $\dfrac{2x^3-5x+1}{\mathrm{g}(x)}$ as partial fractions. [5]

11 A company has 3 outlets, A, B and C, selling chicken, mushroom and vegetable puffs.

The table shows the number of puffs sold on Friday and the selling price of each puff.

	Outlet A	Outlet B	Outlet C	Selling Price
Chicken	53	$2x-17$	72	\$3.50
Mushroom	33	x	42	\$3.20
Vegetable	x	50	75	\$2.80

The number of puffs of each type sold on Friday at each outlet is represented by a 3×3 matrix $\mathbf{N}=\begin{pmatrix} 53 & 2x-17 & 72 \\ 33 & x & 42 \\ x & 50 & 75 \end{pmatrix}$.

The selling price of each type of puff is represented by a row matrix **P**.

(a) A matrix **T** represents the product of matrices **N** and **P**.

The elements of **T** represent the total amount, in dollars, collected from selling all types of puffs on Friday by outlet A, B and C respectively.

Find the matrix **T**, leaving your answer in terms of x. [3]

11 **(b)** The total amount of money collected by outlets A and B is \$891.60.

Given that $\mathbf{TU} = (891.6)$, write down the matrix **U** and find the value of x. [3]

(c) On Saturday, there was a surge in sales of 20%, 30% and 15% in the outlets A, B and C respectively, compared to Friday.

Write down a matrix **S** such that the elements in **TS** give the extra amount collected by each outlet on Saturday compared to Friday. [1]

12 **(a)** Three numbers are chosen from the set $\{1,2,3,4,5,6,7,8,9\}$ at random and without replacement.

Find the probability that the sum of the three numbers is odd. [3]

(b) Alvin chooses two numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ at random and without replacement.

Independently, Ben also chooses two numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ at random and without replacement.

(i) Find the probability that the numbers chosen by Alvin and Ben have at least one number in common. [2]

12 **(b)** **(ii)** Find the probability that exactly two prime numbers are chosen by Alvin and none chosen by Ben. [2]

(iii) Find the probability that exactly two prime numbers are chosen by Alvin and Ben. [2]

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