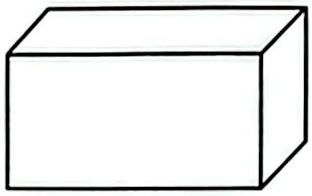
1 Solve the simultaneous equations

$$x - 3y = -2,$$

$$2x^2 = 3xy + 8.$$ [5]

2



A piece of wire is beat to make the rwelve edgee of the rectangular box.skown above.
The volume of the box, in cm^3, is given by $V = 720x^2 - 20x^3$.

(a) Find the value of x for which V has a stationary value. [3]

(b) Find the stationary value of V and determine if the stationary value of V is a maximum or a minimum. [4]

3 The equation of a curve is $y = -x^2 + 2x - 3$.

By completing the square,

(a) explain why y is always negative for all real values of x. [4]

(b) find the coordinates of the maximum point of the curve. [2]

4 (a) Show that $\dfrac{\operatorname{cosec}^2\theta}{\cot\theta} = 2\operatorname{cosec} 2\theta$. [3]

(b) Hence, solve the equation $\dfrac{\operatorname{cosec}^2\theta}{\cot\theta} = 4$ for $0° \leq \theta \leq 180°$. [4]

5 The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 - 11x + 30$.

(a) Find the remainder when $p(x)$ is divided by $x+1$. [1]

(b) Solve the equation $p(x) = 0$. [6]

6 The function f is defined, for all values of x, by

$$f(x) = 3\sin 2x + c\text{, where } c \text{ is a constant.}$$

The graph of $y = f(x)$ passes through the point $\left(\frac{3\pi}{4}, -7\right)$.

(a) Show that the value of c is -4. [1]

(b) State the amplitude and period of f. [2]

(c) Sketch the graph of $y = f(x)$ for $0 \leq x \leq \dfrac{3\pi}{2}$.

[3]

(d) State the value(s) or range of values of p such that the number of point(s) of intersection of the curve $y = f(x)$ and the line $y = p$ is

(i) 1, [1]

(ii) 4. [1]

7 A circle has centre (4, −3) and radius 7.

(a) State the equation of this circle in standard form. [2]

(b) The equation of another circle is $x^2 + y^2 - 10x = 0$.

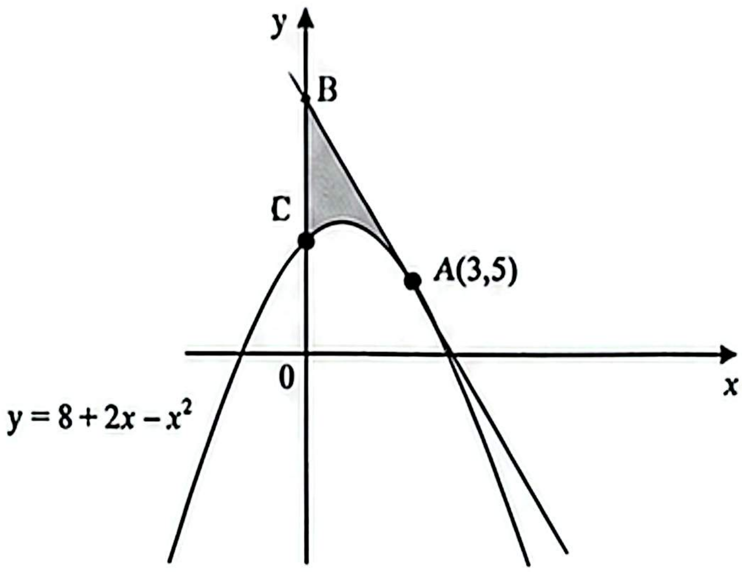
(i) Find the coordinates of the centre of this second circle, and its radius. [3]

(ii) $P(8, -4)$ is a point on the circle. Find the equation of the tangent to the circle at P. [4]

(iii) Show that the centre of the first circle lies within the second circle. [2]

8 The curve $y = 8 + 2x - x^2$ cuts the y-axis at C, $x = 0$

The tangent to the curve at $A(3,5)$ cuts the y-axis at B.



(a) Find the coordinates of C. [2]

(b) Show that the coordinates of B is (0,17). [4]

(c) Find the area of the shaded region. [4]

9 A curve has equation $y = \dfrac{3x}{\sqrt{2x+5}}$, for $x > -\dfrac{5}{2}$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{k(x+5)}{\sqrt{(2x+5)^3}}$, where k is a constant to be found. [4]

(b) Find the equation of the normal to the curve at the point where $x = 2$. [5]
Give your answer in the form $ax + by + c = 0$.

End of Paper