

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

- 1 (a) A polynomial $P(x)$ leaves a remainder of 1 when divided by $x-1$ and a remainder of -1 when divided by $2x-1$. Find the remainder when $P(x)$ is divided by $2x^2-3x+1$. [3]

- (b) In the expansion of $(p-3x)\left(1-\frac{1}{4}x\right)^8$, the coefficient of x is equal to the coefficient of x^3 . Find the value of the constant p .

[5]

2 Do not use a calculator in this question.

(a) (i) By using $\tan 2A$, show that $\cot 67.5^\circ = -1 + \sqrt{2}$. [6]

(ii) Hence, find the exact value of $\operatorname{cosec}^2 67.5^\circ$. [2]

[Turn over

- (b) Given that $\frac{\sin(A+B)}{\sin(A-B)} = \frac{2}{3}$, prove that $5 \tan B + \tan A = 0$. [4]

- 3 (a) Show that $x - 1$ is a factor of $2x^3 - 5x^2 + x + 2$.
Hence, factorise $2x^3 - 5x^2 + x + 2$ completely.

[5]

- (b) Solve the equation $2(e^{2y-2}) - 5(e^{y-1}) + 2e^{1-y} + 1 = 0$.
Leave your answers in exact form.

[5]

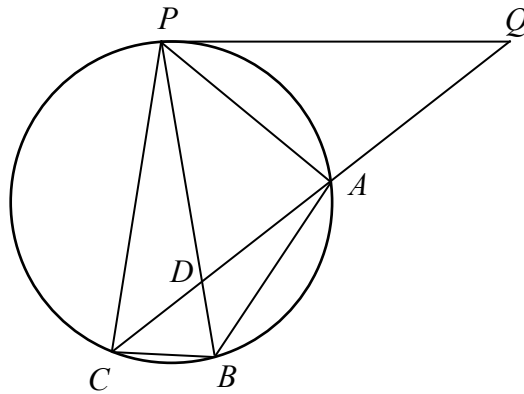
- 4 The graph of $y = a \sin bx + c$ has a turning point at $\left(\frac{\pi}{4}, 0\right)$ and the next turning point after this has coordinates $\left(\frac{3\pi}{4}, 6\right)$.

(a) Find the values of the constants a , b and c . [3]

(b) Hence, sketch the graph of $y = a \sin bx + c$ for $0 \leq x \leq 2\pi$. [2]

(c) By drawing a suitable straight line on the same axes, find the number of solutions of the equation $a \sin bx = -\frac{3}{2\pi}x$. [3]

- 5 The diagram shows a point P on a circle. PQ is a tangent to the circle at P . Points A , B and C lie on the circle such that PA bisects angle QPB . QAC is a straight line. The lines QC and PB intersect at D .



- (a) Prove that $AB = AP$.

[2]

- (b) By first showing that triangle CDP is similar to triangle CBA , show that $PA \times CD = CB \times DP$.

[5]

- 6 A chicken is taken out from a freezer to defrost. The temperature, $T^{\circ}\text{C}$, of the chicken, t hours after it is taken out from the freezer, is given by the formula $T = 20 - 40e^{kt}$, where k is a constant. The temperature of the chicken is -1.95°C after it has been taken out from the freezer for 1 hour.

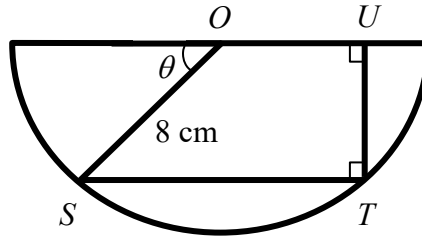
(a) Find the value of k .

[2]

- (b) Find the time taken for the temperature of the chicken to increase by 12°C from the time when it was taken out from the freezer. [3]

- (c) Explain, with working, why the temperature of the chicken can never reach 20°C . [2]

- 7 The diagram shows a trapezium $STUO$ inscribed in a semicircle with centre O and radius 8 cm. OS makes an angle θ radians with the diameter. ST is parallel to the diameter. UT is perpendicular to lines ST and OU .



- (a) Show that the perimeter, P cm, of the trapezium is given by $P = 8 + 24 \cos \theta + 8 \sin \theta$.

[3]

- (b) By expressing the perimeter of the trapezium in the form $P = a + R \sin(\theta + \alpha)$, where $0 < \alpha < \frac{\pi}{2}$, find the maximum possible perimeter of the trapezium and the corresponding value of θ . [4]

- 8** A particle, travelling in a straight line, has velocity v m/s, at time t seconds, $t \geq 0$, given by $v = 4\cos^2\left(\frac{t}{2}\right) - 3$.

Find

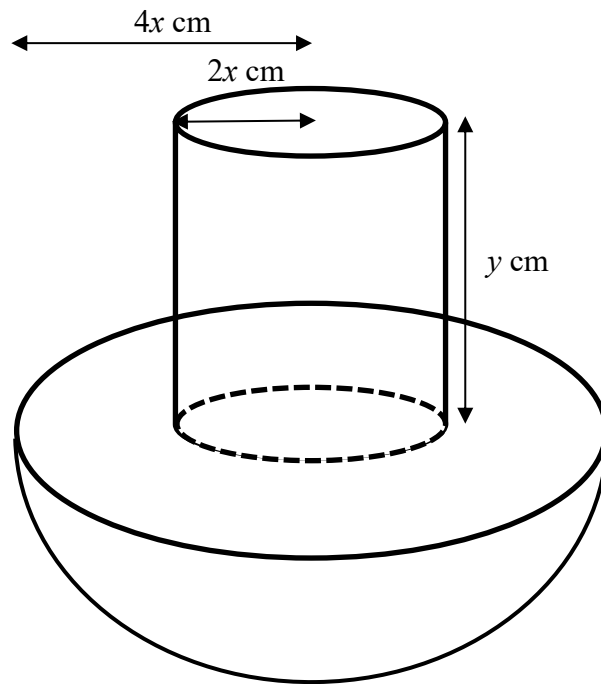
- (a)** the initial velocity of the particle. [1]

- (b)** the time when the acceleration first reaches -2 m/s². [3]

- (c) the total distance travelled by the particle in the first 4 seconds.

[7]

- 9 The diagram shows a solid which consists of a cylinder placed on top of a hemisphere. The radius of the cylinder is $2x$ cm and its height is y cm. The hemisphere has a radius of $4x$ cm. The total volume of the solid is 200π cm³.



- (a) Express y in terms of x , simplifying the expression.

[3]

- (b) Show that the total surface area, $A \text{ cm}^2$, of the solid is $A = 40\pi\left(\frac{5}{x} + \frac{2x^2}{15}\right)$. [3]

- (c) Given that x can vary, find the stationary value of A .

[4]

- (d) Mr Chan intends to paint the solid with a total surface area found in part (c). Determine if the amount of paint needed is a maximum or a minimum. [2]

10 **(a)** The equation of a curve is given by $y = P - Q \cos 4x - \frac{1}{2} \sin 2x$.

Given that $\frac{d^2 y}{dx^2} + 4y = 15 \cos 4x - 8$, find the values of the constants P and Q . [4]

- (b) With the values of the constants P and Q found in part (a), find the equation of the normal to the curve at $x = \frac{\pi}{2}$. Leave your equation in terms of π . [4]

