

Circuits

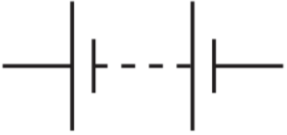
Guiding Questions	Learning Objectives
1. How are symbols and diagrams used to represent real circuits?	(a) recall and use appropriate circuit symbols. (b) draw and interpret circuit diagrams containing sources, switches, resistors (fixed and variable), ammeters, voltmeters, lamps, thermistors, light-dependent resistors, diodes, capacitors and/or any other type of component referred to in the syllabus
2. How are current, voltage, and resistance related in an electrical circuit?	(c) define the resistance of a circuit component as the ratio of the potential difference across the component to the current in it, and solve problems using the equation $V = IR$. (d) recall and solve problems using the equation relating resistance to resistivity, length and cross-sectional area, $R = \frac{\rho l}{A}$. (e) sketch and interpret the I-V characteristics of various electrical components in a d.c. circuit, such as an ohmic resistor, a semiconductor diode, a filament lamp and a negative temperature coefficient (NTC) thermistor. (f) explain the temperature dependence of the resistivity of typical metals (e.g. in a filament lamp) and semiconductors (e.g. in an NTC thermistor) in terms of the drift velocity and number density of charge carriers respectively. (g) show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.
3. How are the principles of charge and energy conservation applied to analyse circuits?	(h) solve problems using the formula for the combined resistance of two or more resistors in series. (i) solve problems using the formula for the combined resistance of two or more resistors in parallel. (j) solve problems involving series and parallel arrangements of resistors for one source of e.m.f., including potential divider circuits which may involve NTC thermistors and light-dependent resistors. (k) solve problems using the formulae for the combined capacitance of two or more capacitors in series and in parallel.

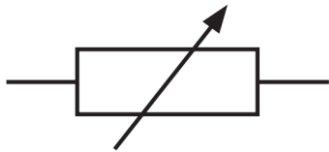
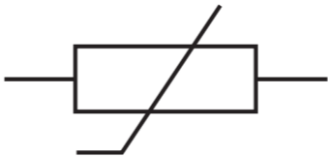
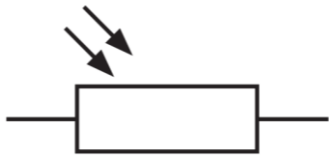
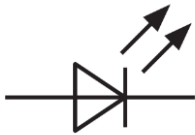
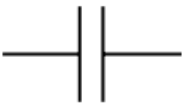
	(I) describe and represent the variation with time, of quantities like current, charge and potential difference, for a capacitor that is charging or discharging through a resistor, using equations of the form $x = x_0 \exp(-t/\tau)$ or $x = x_0[1 - \exp(-t/\tau)]$, where $\tau = RC$ is the time constant.
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How are symbols and diagrams used to represent real circuits?

1. Circuit Symbols and Diagrams

A standard set of common symbols is used to draw a circuit diagram. Groupings of symbols should improve clarity and aid in the understanding of the electric circuit.

cell		switch	
battery of cells	 or 	earth	
power supply		alternating current (a.c.) power supply	
junction of wires		lamp	

fixed resistor		variable resistor	
thermistor		light-dependent resistor (LDR)	
potentiometer		diode	
voltmeter		light-emitting diode (LED)	
ammeter		galvanometer	
capacitor			

How are current, voltage, and resistance related in an electrical circuit?

2. Resistance

2.1 Electrical Resistance



The **resistance** R of a conductor is defined as the ratio of potential difference V across the conductor and current I in it.

$$R = \frac{V}{I}$$

The SI unit of resistance is ohm (Ω).

The **ohm** is defined as the electrical resistance between two points of a conductor through which a steady current of 1 A flows when a constant potential difference of 1 V is maintained across it. Hence, $1 \Omega = 1 \text{ V A}^{-1}$.

2.1.1 Ohm's Law



Ohm's law states that the current I flowing through a conductor is directly proportional to the potential difference V applied across the conductor provided that the physical conditions (such as temperature, stress etc) remain constant.

By Ohm's law,

$$I \propto V$$

$$I = kV \quad \text{where graph } I - V \text{ is a straight line that passes through the origin}$$

By definition of resistance,

$$R = \frac{V}{I}$$

$$= \frac{1}{k} \quad \text{where } k \text{ is a constant}$$

2.1.2 Resistivity

Resistance of a conductor R is directly proportional to its length, l and inversely proportional to its cross-sectional area A .

$$R = \frac{\rho l}{A}$$

where ρ is the resistivity of the material

$$\rho = \frac{RA}{l}$$

The SI unit of resistivity is ohm metre ($\Omega \text{ m}$)



Resistivity is an electrical property of a material, and its value gives the electrical resistance of a conductor of unit cross-sectional area and unit length, made from that material.

At room temperature,

- Metal: $\rho = 10^{-13} \Omega \text{ m}$
- Semiconductor: $\rho = 10^{-4} \text{ to } 10^5 \Omega \text{ m}$
- Insulator: $\rho = 10^{13} - 10^{16} \Omega \text{ m}$

Example 2.1

Two wires A and B of circular cross section are made of the same metal and have equal lengths, but the resistance of wire A is three times greater than that of wire B.

- Determine the ratio of their cross-sectional areas.
- Determine the ratio of their radii.

Solution:

(a)

$$R = \rho \frac{l}{A} \quad \text{where } \rho, l \text{ are constants}$$

$$\frac{R_A}{R_B} = \frac{A_B}{A_A} \quad \text{where } A = \pi r^2$$

$$\frac{A_B}{A_A} = \frac{3R}{R} = 3$$

(b)

$$\text{Since } \frac{A_B}{A_A} = \frac{r_B^2}{r_A^2},$$

$$\frac{r_B}{r_A} = \sqrt{3} = 1.73$$

Example 2.2

A car headlight filament is made of tungsten and has a resistance of $0.35 \, \Omega$. The resistivity of tungsten is $5.6 \times 10^{-8} \, \Omega \, \text{m}$.

If the filament is a cylinder 4.0 cm long (it may be coiled to save space), what is its diameter?

Solution:

$$R = \frac{\rho l}{A}$$

$$A = \frac{\rho l}{R}$$

Assuming that the wire is perfectly cylindrical so A can be expressed as $\frac{\pi D^2}{4}$.

$$\begin{aligned} \frac{\pi D^2}{4} &= \frac{\rho l}{R} \\ D &= \sqrt{\frac{4\rho l}{\pi R}} = \sqrt{\frac{4(5.6 \times 10^{-8})(4.0 \times 10^{-2})}{\pi(0.35)}} \\ &= 9.0 \times 10^{-5} \, \text{m} \end{aligned}$$

2.2 I–V Characteristics

Resistance R is affected by 2 opposing mechanisms:

1. increase in rate of collision of free electrons (charge carriers) with lattice ions – R increases
2. increase in number of free electrons due to bound electrons being freed – R decreases

2.2.1 A metallic conductor at constant temperature (linear conductor)

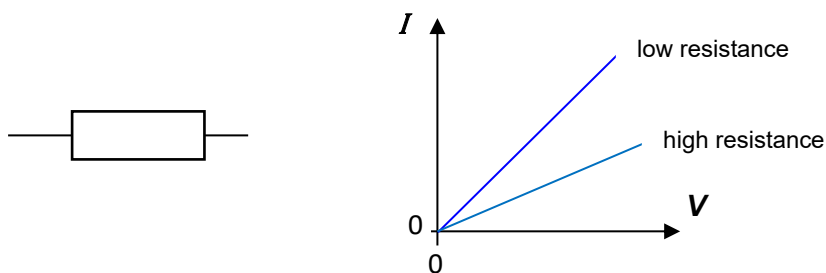


Fig. 2.1

- All metals contain lattice ions (which vibrate continuously about their equilibrium positions) and free electrons (number density is characteristic of each metal).
- The I – V characteristic graph is a straight line passing through the origin. Thus, $I \propto V$ ie resistance R is constant. Metallic conductors obey Ohm's law at constant temperature.

- When p. d. V increases,
 - acceleration of electrons increase, drift velocity increases, rate of flow of electrons increases, thus electric current I increases.
 - as free electrons move, they collide with the vibrating lattice ions,
 - which gains kinetic energy and vibrate with larger amplitudes (natural frequency remains constant), the larger the amplitude, the greater is the chance of collision and obstructing the path of the free electrons (so by right R should increase),
 - The number of bound electrons freed during collision with free electrons does not increase significantly (as metals has a large number of free electrons),
 - In this type of metals, kinetic energy gained by lattice ions is dissipated quickly to the surroundings, hence temperature and amplitude of lattice ions remain constant, hence R is constant.

2.2.2 A filament lamp (non-linear conductor)

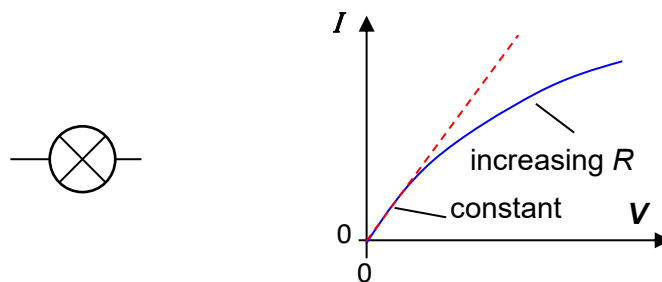


Fig. 2.2

- A filament is a thin metal wire housed inside a glass enclosure filled with inert gas.
 - At low p.d. (and current), ratio $\frac{I}{V}$ is quite constant, resistance R is constant
 - At higher p.d. (and current), resistance R increases
- The heated filament releases thermal electromagnetic radiation (light energy) by blackbody radiation.
- When p.d. V increases,
 - acceleration of electrons increase, drift velocity increases, rate of flow of electrons increases, thus electric current I increases.
 - as free electrons move, they collide with the vibrating lattice ions,
 - which gains kinetic energy and vibrate with larger amplitudes (natural frequency remains constant), the larger the amplitude, the greater is the chance of collision and obstructing the path of the free electrons, and the current increase ΔI decreases, leads to increase in R ,
 - The number of bound electrons freed during collision with free electrons does not increase significantly (as metals has a large number of free electrons) – insignificant effect on R ,
 - Overall, R increases. That is, further increase of the potential difference does not increase the current substantially.

2.2.3 A semiconductor diode

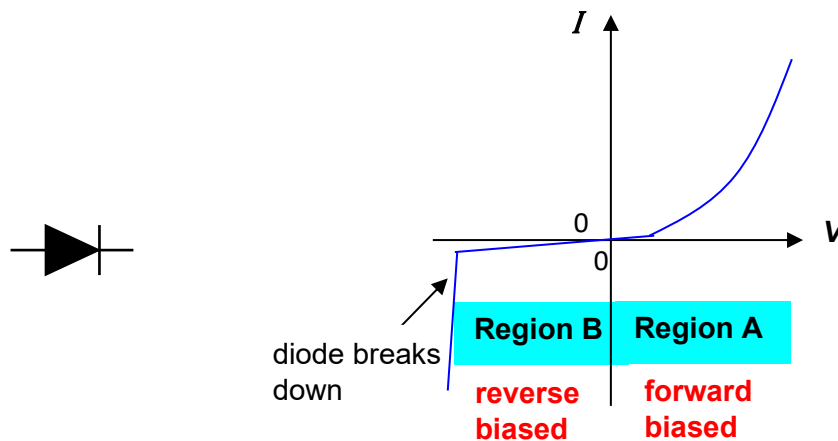
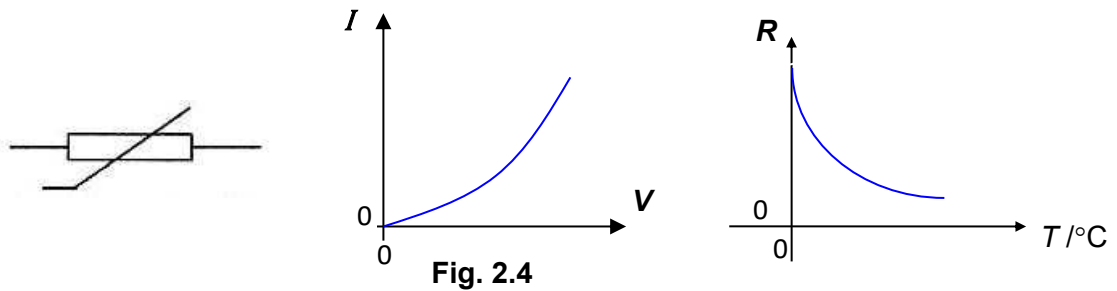


Fig. 2.3

- Region A: When diode is connected in forward biased, it behaves like a simple resistor
 - When p.d. V increases, more moving electrons collide the lattice ions, which gain kinetic energy (and temperature), the lattice ions vibrate with larger amplitudes and more collisions between the free electrons and the lattice ions, obstruct the path of electrons (to increase R)
 - But the higher kinetic energy of lattice ions releases more bound electrons as charge carriers (to decrease R)
 - Overall, the increase in no. of charge carriers (to decrease R) predominates increase in collision due to larger amplitude of lattice vibration (to increase R). Hence, the overall effect is that R decreases as V increases
- Region B: When diode is connected in a reverse biased, it behaves like a semiconductor diode
 - $V > -80\text{ V}$, diode is in normal operating condition. No current flows. $I = 0$
 - $V < -80\text{ V}$, diode breaks down and is permanently damaged, very large current flows ($R \rightarrow 0$).
(Usually not tested for 'A' Levels. You will see more of this in university.)

2.2.4 A thermistor with negative temperature coefficient



- A thermistor is made of a semiconductor material. Resistance is 1000 times that of a metallic conductor.
- I - V characteristic is the similar to that for a semiconductor diode shown in 2.2.3.
- For *negative temperature coefficient* (NTC) thermistor, when p.d. increases, the temperature increases, and resistance decreases.

Summary

	metals	NTC thermistor	filament lamp	semiconductor
graph	straight line	curve up	curve down	Forward: curve up Reverse: normal $I = 0$
passes through origin	yes	yes	yes	yes
resistance	$R = \text{gradient}$	$R \neq \text{gradient}$	$R \neq \text{gradient}$	$R \neq \text{gradient}$
Ohm's law	yes	no	no	no

Fig. 2.5

2.3 Internal Resistance

The internal resistance r is an inherent part of any e.m.f. source. Energy is dissipated as heat in the internal resistance.

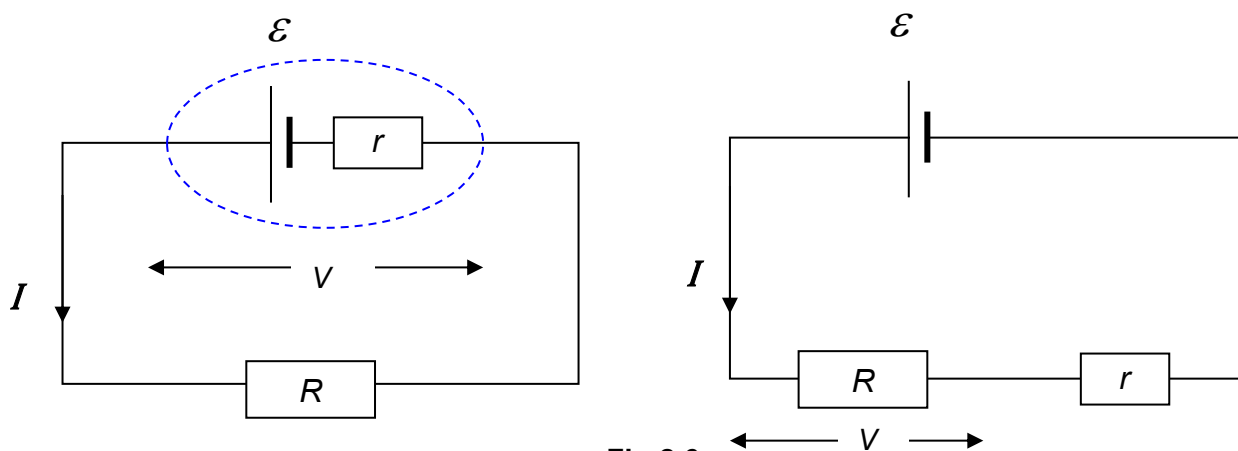


Fig 2.6

By principle of conservation of energy,

Power supplied by e.m.f. source = power dissipated in resistors R and r

$$P = P_R + P_r$$

$$I\varepsilon = I^2 R + I^2 r$$

$$\varepsilon = IR + Ir$$

$$\varepsilon = V + Ir$$

$$V = \varepsilon - Ir$$

where

ε - e.m.f. of the source

P_R - power dissipated in external resistance R

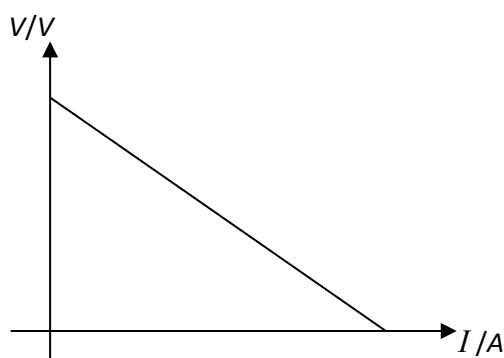
P_r - power dissipated in internal resistance r

$V = IR$ - p.d. across resistor R = terminal p.d. of source

I - current flowing

Note:

- When a current flows, potential difference across the terminals of the source is V
- $V = \varepsilon$ when:
 - $R \gg r$, i.e. negligible internal resistance
 - Circuit is open, then current I through a source is zero



Terminal potential difference (V) against current (I) graph

To increase I , the external resistance decreases while the internal resistance (r) and emf (ε) remains constant.

Gradient of the graph is $-r$.

Vertical intercept is ε .

Horizontal intercept is the maximum current when the external resistance is zero.

Example 2.3 (N04/I/23)

A battery of e.m.f. $\mathcal{E}$ and internal resistance r delivers a current I through a variable resistance R . R is set at two different values and the currents I are measured using an ammeter of negligible resistance.

R / Ω	I / A
1.0	3.0
2.0	2.0

What is the value of the e.m.f. $\mathcal{E}$?

Solution:

Using $V = \mathcal{E} - Ir$

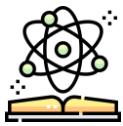
$$\mathcal{E} = I(R + r)$$

Sub values into eqn

$$\mathcal{E} = 3.0 + 3.0r \rightarrow (1)$$

$$\mathcal{E} = 4.0 + 2.0r \rightarrow (2)$$

Hence solving, $\mathcal{E} = 6.0 \text{ V}$



Nature of Science Moment – The Big Misconception About Electricity

The misconception is that electrons carry potential energy around a complete conducting loop, transferring their energy to the load. In this video, Veritasium discusses some ideas about electricity that are counterintuitive. See if you agree with them!

<https://www.youtube.com/watch?v=bHlhqxav9LY>

How are the principles of charge and energy conservation applied to analyse circuits?

3 Circuit Analysis

3.1 Combined Resistance

3.1.1 Resistors in Parallel

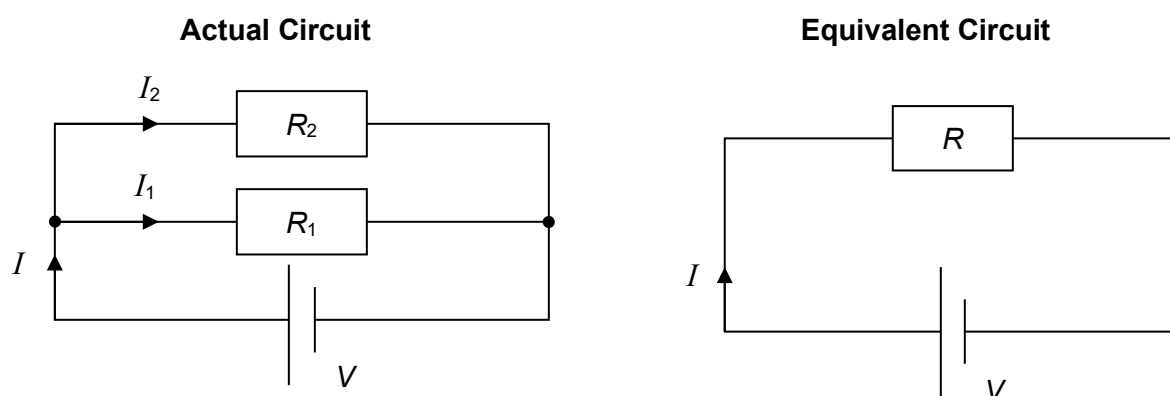


Fig. 3.1 Two resistors connected in parallel

For resistors connected in parallel, p.d. across each resistor is equal to the e.m.f. V of the cell.

$$V = I_1 R_1 \quad \text{and} \quad V = I_2 R_2$$

Total current in the circuit I is split between the two parallel branches

$$\begin{aligned} I &= I_1 + I_2 \\ \frac{V}{R} &= \frac{V}{R_1} + \frac{V}{R_2} \\ \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1 R_2}{R_1 + R_2} \end{aligned}$$

For N number of resistors in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

3.1.2 Resistors in Series

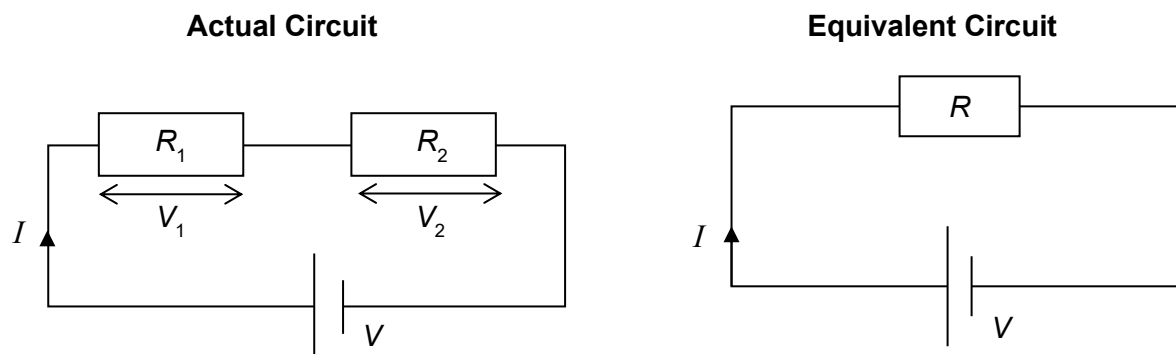


Fig. 3.2 Two resistors connected in series

For resistors connected in series, the same current I passes through each resistor.

$$V_1 = IR_1 \quad \text{and} \quad V_2 = IR_2$$

e.m.f. of the cell V is split between the two resistors

$$V = V_1 + V_2$$

$$IR = IR_1 + IR_2$$

$$R = R_1 + R_2$$

For N number of resistors in series,

$$R = R_1 + R_2 + R_3 + \dots + R_N$$



Can we try to prove the following?

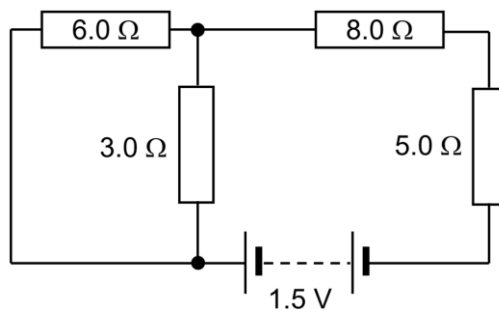
For N identical resistors each of resistance R arranged in parallel, $R_{\text{total}} = R/N$.

$$\begin{aligned} \frac{1}{R_{\text{total}}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N} \\ &= \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \dots + \frac{1}{R} \\ &= \frac{N}{R} \\ R_{\text{total}} &= \frac{R}{N} \end{aligned}$$

For N identical resistors each of resistance R arranged in series, $R_{\text{total}} = NR$.

$$\begin{aligned} R_{\text{total}} &= R_1 + R_2 + R_3 + \dots + R_N \\ &= R + R + R + \dots + R \\ &= NR \end{aligned}$$

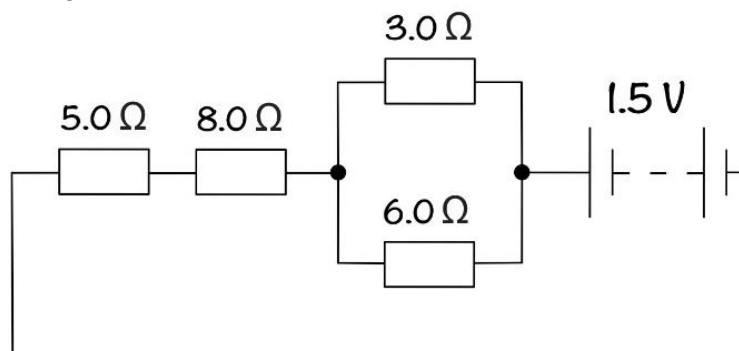
Example 3.1



Find the total resistance of the circuit shown. [$15.0\ \Omega$]

Solution

Re-drawing the circuit diagram as shown below,



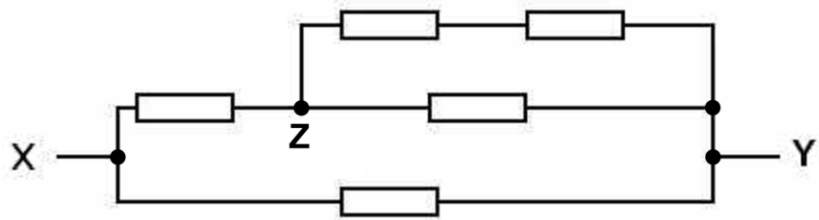
$$\begin{aligned} \text{combined resistance between the } 6.0\ \Omega \text{ and } 3.0\ \Omega \text{ resistors} &= \left(\frac{1}{6.0} + \frac{1}{3.0} \right)^{-1} \\ &= 2.0\ \Omega \\ \text{total resistance, } R_T &= 2.0 + 8.0 + 5.0 \\ &= 15.0\ \Omega \end{aligned}$$

Example 3.2

The circuit diagram shows a network of resistors each of resistance R .

Calculate the equivalent resistance between

- (a) X and Y $[0.625R]$
(b) Z and Y $[0.5R]$

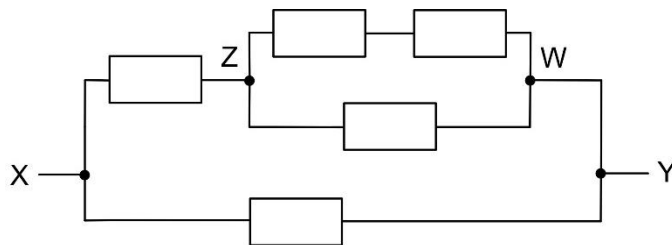


Solution

Re-draw the circuit diagram

Apply formulas to combine resistances

(a)



$$R_{ZW} = \left(\frac{1}{R+R} + \frac{1}{R} \right)^{-1}$$

$$= \frac{2}{3}R$$

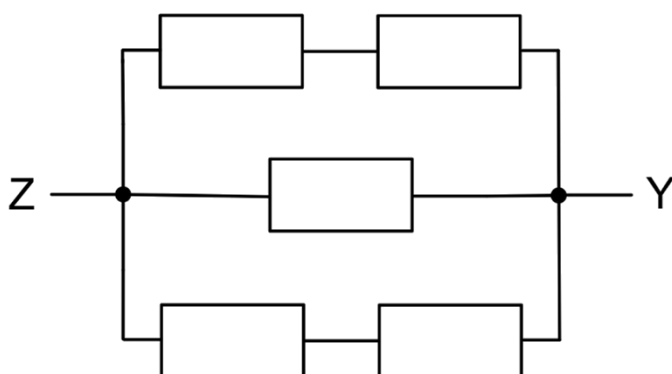
$$R_{\text{top branch between XY}} = R + \frac{2}{3}R$$

$$= \frac{5}{3}R$$

$$R_{XY} = \left(\frac{1}{\frac{5}{3}R} + \frac{1}{R} \right)^{-1}$$

$$= 0.625R$$

(b)



$$R_{\text{top branch}} = R + R$$

$$= 2R$$

$$R_{\text{middle branch}} = R$$

$$R_{\text{bottom branch}} = R + R$$

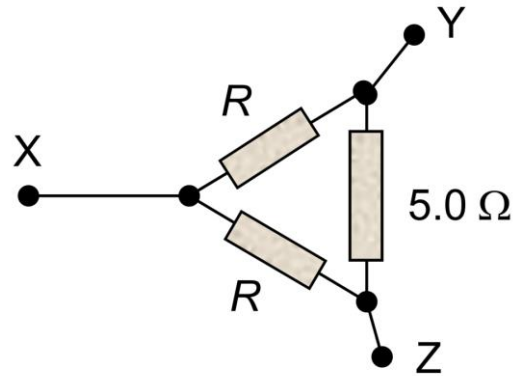
$$= 2R$$

$$R_{ZY} = \left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{2R} \right)^{-1}$$

$$= 0.5R$$

Example 3.3

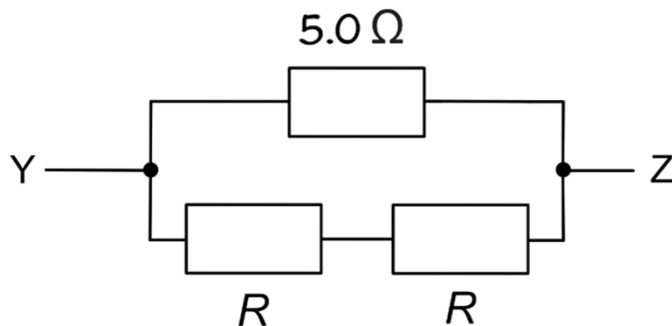
The circuit diagram shows a network of resistors. Two identical resistors are marked R . The third resistor has resistance of 5.0Ω . The resistance between Y and Z is 2.5Ω .



- (a) Write down an equation in terms of R for the effective resistance across YZ.
 (b) Calculate the resistance of R . [2.5Ω]
 (c) Determine the effective resistance between X and Y. [1.9Ω]

Solution

(a)

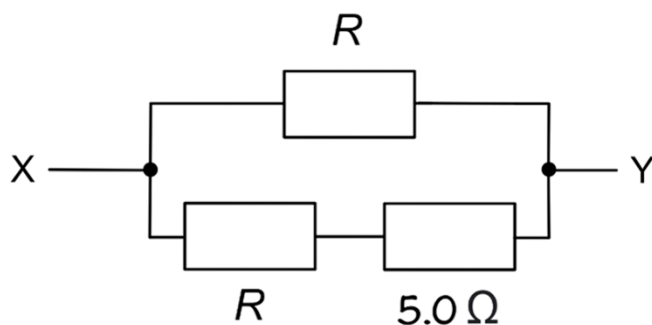


$$R_{\text{bottom branch}} = R + R \\ = 2R$$

$$R_{YZ} = \left(\frac{1}{5.0} + \frac{1}{2R} \right)^{-1} \\ = \left(\frac{2R + 5.0}{10R} \right)^{-1} \\ = \frac{10R}{2R + 5.0}$$

(b) $2.5 = \frac{10R}{2R + 5.0}$
 $R = 2.5 \Omega$

(c)



$$R_{\text{bottom branch}} = 2.5 + 5.0 \\ = 7.5 \Omega$$

$$R_{XY} = \left(\frac{1}{2.5} + \frac{1}{7.5} \right)^{-1} \\ = 1.875 \\ \approx 1.9 \Omega$$

Example 3.4

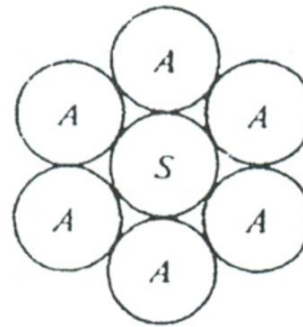
An electricity supply cable consists of a steel core of area of cross-section 50 mm^2 with six other conductors of aluminium of the same cross-sectional area arranged around it as shown.

resistivity of steel $= 9.0 \times 10^{-8} \Omega \text{ m}$

resistivity of aluminium $= 2.5 \times 10^{-8} \Omega \text{ m}$

Find the resistance of a 120 m length of the cable.

[9.6 mΩ]

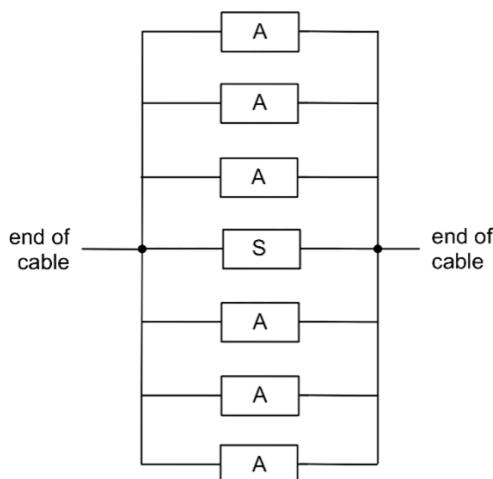


A = aluminium

S = steel

Solution

Six conductors made of aluminium and one made of steel are connected in parallel.



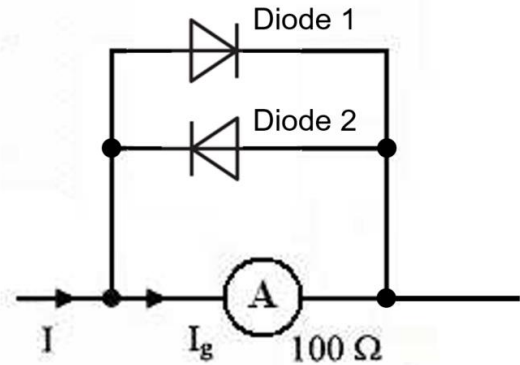
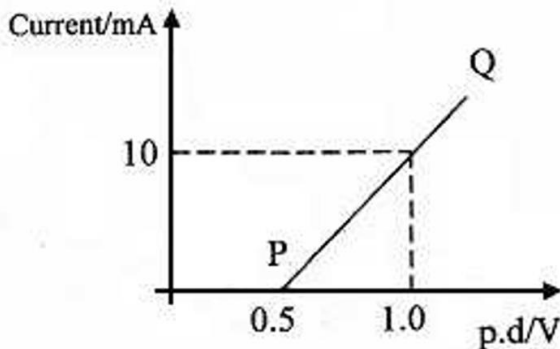
$$\begin{aligned} \text{resistance of steel core, } R_S &= \frac{\rho_S l}{A} \\ &= \frac{(9.0 \times 10^{-8})(120)}{50 \times 10^{-6}} \\ &= 0.216 \Omega \end{aligned}$$

$$\begin{aligned} \text{resistance of one aluminium cable, } R_A &= \frac{\rho_A l}{A} \\ &= \frac{(2.5 \times 10^{-8})(120)}{50 \times 10^{-6}} \\ &= 0.060 \Omega \end{aligned}$$

$$\begin{aligned} \text{effective resistance of the cable, } R_{\text{cable}} &= \left[\frac{1}{R_S} + 6 \left(\frac{1}{R_A} \right) \right]^{-1} \\ &= \left[\frac{1}{0.216} + 6 \left(\frac{1}{0.060} \right) \right]^{-1} \\ &= 9.6 \times 10^{-3} \Omega \\ &= 9.6 \text{ m}\Omega \end{aligned}$$

Example 3.5

The current that passes through a certain diode varies with the potential difference across it as shown. PQ is a straight line. Two such diodes are connected in parallel with a milliammeter of resistance $100\ \Omega$ as shown.



- Determine the potential difference across the milliammeter when the current I_g through the milliammeter is 8 mA.
- State which diode is conducting, and determine the current flowing through it. [6 mA]
- Determine the value of the direct current I . [14 mA]

Solution

$$\begin{aligned}
 \text{(a)} \quad V &= IR \\
 &= (8 \times 10^{-3})(100) \\
 &= 0.8\ \text{V}
 \end{aligned}$$

- Diode 2 is in a reverse bias arrangement, hence it is not conducting. Current through it is zero.

Diode 1 is in a forward bias arrangement, hence it is conducting.

$$\begin{aligned}
 \text{p.d. across Diode 1} &= \text{p.d. across milliammeter} \\
 &= 0.8\ \text{V}
 \end{aligned}$$

$$\begin{aligned}
 \text{gradient of } I\text{-}V \text{ graph} &= \frac{10 - 0}{1.0 - 0.5} \\
 &= 20\ \text{mA V}^{-1}
 \end{aligned}$$

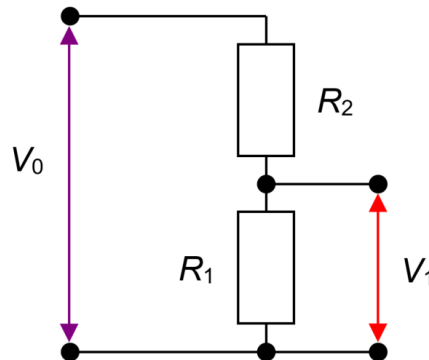
$$\begin{aligned}
 \text{Equation of line in } I\text{-}V \\
 \text{graph:} \quad I - 0 &= 20(V - 0.5) \\
 I &= 20V - 10
 \end{aligned}$$

$$\begin{aligned}
 \text{when } V &= 0.8\ \text{V}, \\
 I_1 &= 20V - 10 \\
 &= 20(0.8) - 10 \\
 &= 6\ \text{mA}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad I &= I_1 + I_g \\
 &= 6 + 8 \\
 &= 14\ \text{mA}
 \end{aligned}$$

3.2 Potential Divider

The arrangement of two or more resistances in series is known as a *potential divider* where V_0 is the supply voltage and V_1 is the output voltage.



Total current I flowing through the circuit is given by

$$I = \frac{V_0}{R_1 + R_2}$$

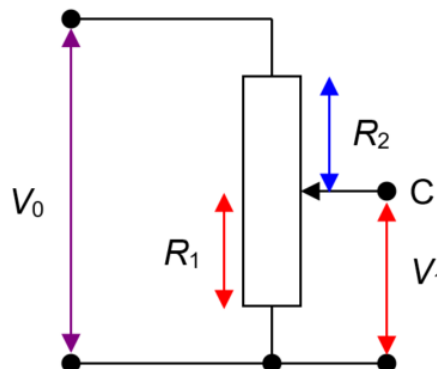
Output voltage is equal to the potential difference across R_1

$$\begin{aligned} V_1 &= IR_1 \\ &= \left(\frac{V_0}{R_1 + R_2} \right) R_1 \\ &= \frac{R_1}{R_1 + R_2} V_0 \end{aligned}$$

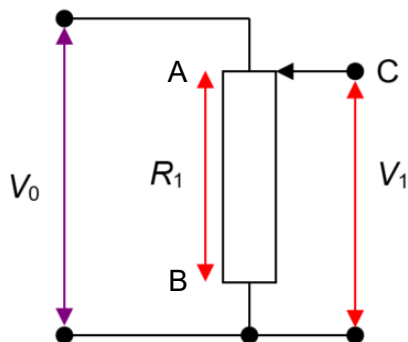
$$\boxed{V_1 = \frac{R_1}{R_{\text{total}}} V_0}$$

The potential difference across two points depends on the ratio of the resistance between the two points to the total resistance.

Besides using two resistors, a rheostat which is a variable resistor can be used to vary the output voltage.

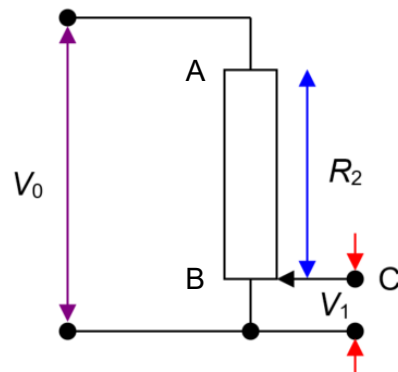


The resistances R_1 and R_2 can be varied by sliding a contact C on the rheostat. This allows V_1 to be varied from 0 V to V_0 .



When C is at end A of the rheostat,

$$\begin{aligned} R_1 &= R_{\text{rheostat}} \\ R_2 &= 0 \, \Omega \\ V_1 &= \frac{R_{\text{rheostat}}}{R_{\text{rheostat}}} V_0 \\ &= V_0 \end{aligned}$$



When C is at end B of the rheostat,

$$\begin{aligned} R_1 &= 0 \, \Omega \\ R_2 &= R_{\text{rheostat}} \\ V_1 &= \frac{0}{R_{\text{rheostat}}} V_0 \\ &= 0 \, \text{V} \end{aligned}$$



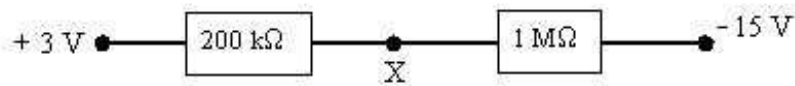
What is the difference between potential and potential difference?

Potential (unit: Volt) is a characteristic of a point in the circuit. '*potential at a point*'

Potential difference (unit: Volt) is the difference in electric potentials between two points in a circuit. '*potential difference across a component / between two points*'

Example 3.6

Two resistors of resistance $200\text{ k}\Omega$ and $1\text{ M}\Omega$ respectively, form a potential divider with outer junctions maintained at potentials $+3\text{ V}$ and -15 V .



(a) Calculate

(i) the potential difference across the $200\text{ k}\Omega$ resistor, [3 V]

(ii) the potential difference across the $1\text{ M}\Omega$ resistor. [15 V]

(b) Determine the potential at the junction X. [0 V]

Solution

(a) (i) $V_{200\text{ k}\Omega} = \left(\frac{200 \times 10^3}{200 \times 10^3 + 1 \times 10^6} \right) [3 - (-15)]$ $= 3\text{ V}$	(ii) $V_{1\text{ M}\Omega} = \left(\frac{1 \times 10^6}{200 \times 10^3 + 1 \times 10^6} \right) [3 - (-15)]$ $= 15\text{ V}$
---	---

(b) 3 V is a higher potential than -15 V . Hence, potential decreases from the left to the right.

Method #1

X is at a lower potential than 3 V .

p.d. across $200\text{ k}\Omega$ resistor = $3 - \text{potential at X}$

$$\begin{aligned} \text{potential at X, } V_X &= 3 - \text{p.d. across } 200\text{ k}\Omega \text{ resistor} \\ &= 3 - 3 \\ &= 0\text{ V} \end{aligned}$$

Method #2

X is at a higher potential than -15 V .

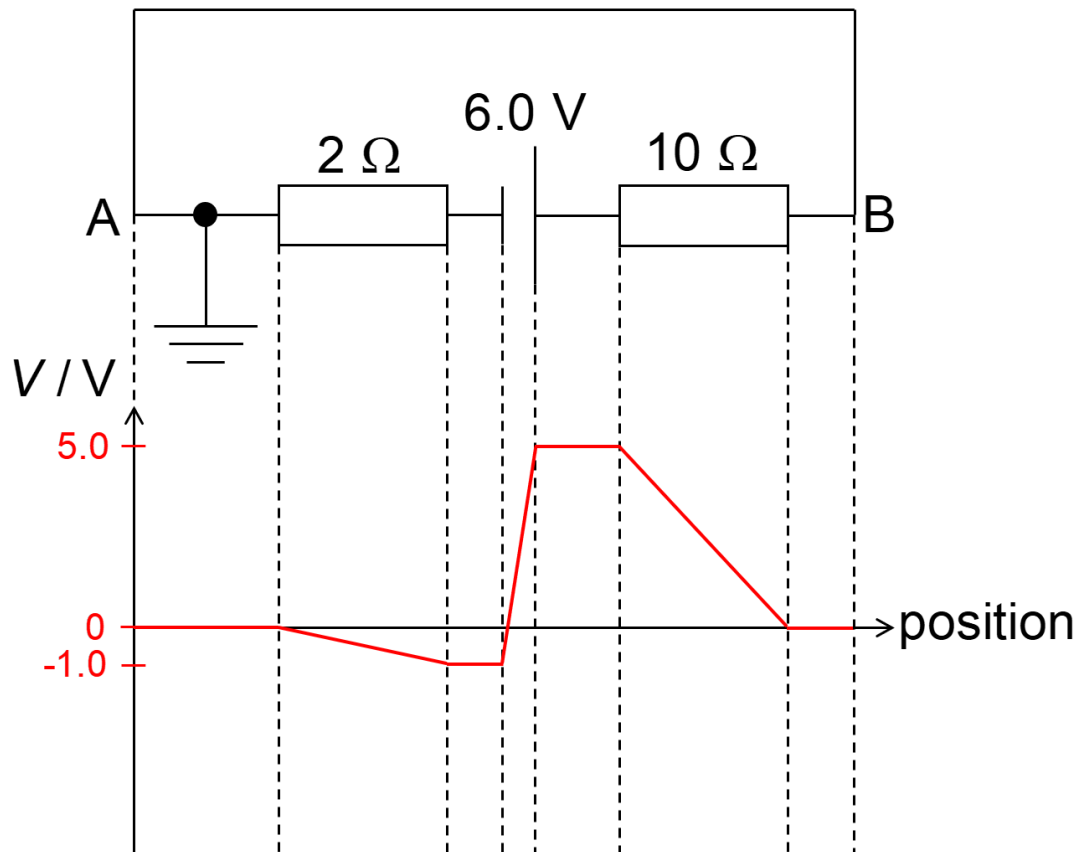
p.d. across $1\text{ M}\Omega$ resistor = potential at X $- (-15)$

$$\begin{aligned} \text{potential at X, } V_X &= -15 + \text{p.d. across } 1\text{ M}\Omega \text{ resistor} \\ &= -15 + 15 \\ &= 0\text{ V} \end{aligned}$$

Double check that potential at X is lower than 3 V and higher than -15 V .

Example 3.7

Sketch the potential V to indicate the change in potential along the circuit from A to B. Label the potential values accordingly.



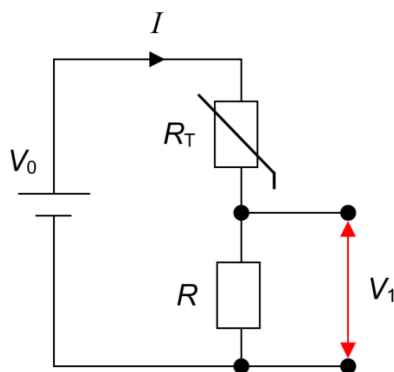
3.3 Thermistors and Light-Dependent Resistors (LDR)

3.3.1 Using thermistors to vary potential difference

Thermistors are made from semiconductors. Thermistors can be heated externally (by external heat source) or internally (by the current flowing through them). When its temperature changes, its resistance and hence its p.d. varies. There are 2 types of thermistors:

- (a) Positive temperature coefficient (PTC) thermistor (as temperature increases, resistance increases)
- (b) Negative temperature coefficient (NTC) thermistor (as temperature increases, resistance decreases)

In the syllabus, unless stated otherwise, assume that the word ‘thermistors’ refer to NTC thermistors instead of PTC thermistors. Thermistors can be connected in series with a fixed resistor R and a source of e.m.f. to form a potential divider circuit.



$$V_1 = \frac{R}{R + R_T} V_0$$

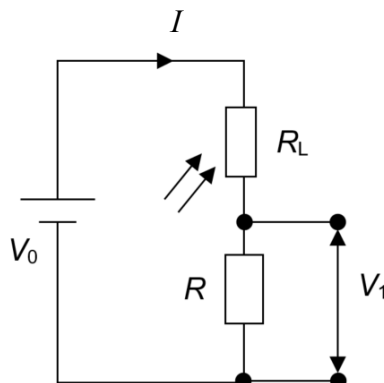
When temperature increases, R_T decreases. This decreases the total resistance and hence increases the total current. Since R remains constant, V_1 increases.

At low temperatures, R_T increases and V_1 decreases.

At high temperatures, R_T decreases and V_1 increases.

3.3.2 Using light-dependent resistors to vary potential difference

LDRs are made of semiconductors. When intensity of light on LDR increases, its resistance decreases. LDRs can be connected in series with a fixed resistor R and a source of e.m.f. to form a potential divider.



$$V_1 = \frac{R}{R + R_L} V_0$$

When light intensity increases, R_L decreases. This decreases the total resistance and hence increases the total current. Since R remains constant, V_1 increases.

At low illumination, R_L increases and V_1 decreases.

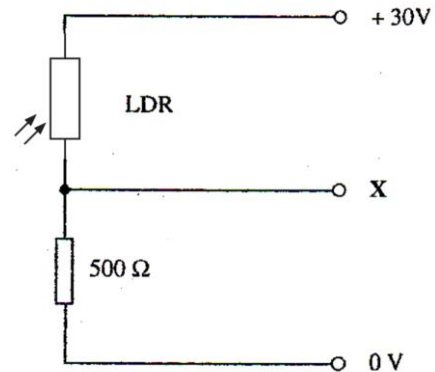
At high illumination, R_L decreases and V_1 increases.

Example 3.8

The light-dependent resistor (LDR) and a $500\ \Omega$ resistor form a potential divider between voltage lines held at $+30\text{ V}$ and 0 V as shown. The resistance of the LDR is $1000\ \Omega$ in the dark but drops to $100\ \Omega$ in bright light.

Determine the corresponding change in the potential at X.

[a rise of 15 V]



Solution

In the **dark**, p.d. across XO,

$$V_{xo} = \left(\frac{500}{500 + 1000} \right) (30)$$

$$= 10\text{ V}$$

$$V_{xo} = V_x - V_o$$

$$V_x = V_{xo} + V_o$$

$$= 10 + 0$$

$$= 10\text{ V}$$

In **bright light**, p.d. across XO,

$$V_{xo} = \left(\frac{500}{500 + 100} \right) (30)$$

$$= 25\text{ V}$$

$$V_{xo} = V_x - V_o$$

$$V_x = V_{xo} + V_o$$

$$= 25 + 0$$

$$= 25\text{ V}$$

change in potential at X = $25 - 10$

$$= 15\text{ V}$$

3.4 Combined Capacitance

3.4.1 Introduction to Capacitance

Recall from the topic Electric Fields,

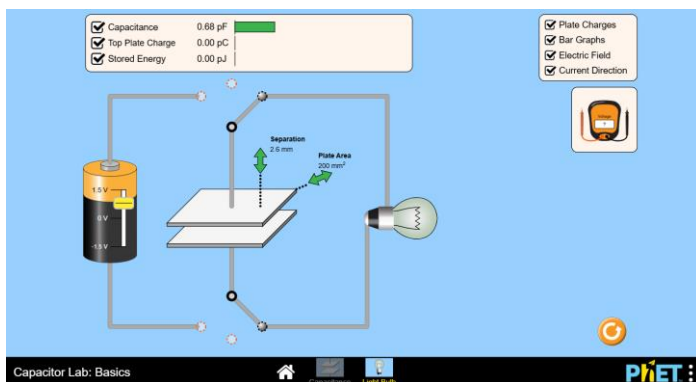


Capacitance is the ratio of the charge stored to the potential difference of a capacitor.

$$C = \frac{Q}{V}$$

The amount of electric potential energy U stored in the capacitor when it is charged to a voltage of V is given by

$$U = \frac{1}{2} QV = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2$$



This simulation demonstrates the charging and discharging of a capacitor.



Nature of Science Moment – What Are Capacitors? The Silent Heroes of Modern Electronics

Capacitors are one of the most important components in electronics, but how do they work? In this video, you will get a summary about capacitors — how they store and release energy, their different types, and where they are used in everyday devices.

https://www.youtube.com/watch?v=kJDNgMweX_A

3.4.2 Capacitors in Parallel

When two capacitors are connected in parallel as shown in Fig. 3.3, the potential difference across each of the capacitors is the same. That is,

$$\Delta V_1 = \Delta V_2 = \Delta V$$

where ΔV is the battery terminal voltage.

After the battery is attached to the circuit, the capacitors quickly reach their maximum charge. The maximum charge for the two capacitors is Q_1 and Q_2 respectively. Hence the total charge stored by the two capacitors, Q_{tot} , is the sum of the charges of the individual capacitors, $Q_1 + Q_2$.

$$\text{Since } Q_{tot} = Q_1 + Q_2$$

$$C_{eq}\Delta V = C_1\Delta V + C_2\Delta V$$

Hence

$$C_{eq} = C_1 + C_2$$

Therefore, the equivalent capacitance of a parallel combination of capacitors is (1) the algebraic sum of the individual capacitances and (2) greater than any of the individual capacitances.

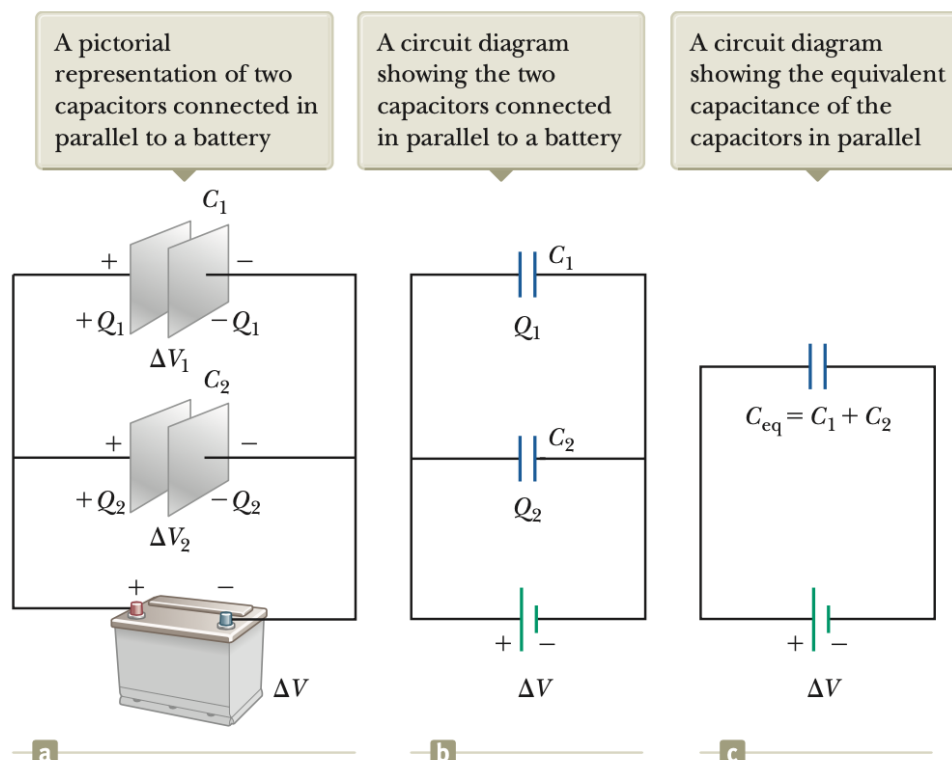


Fig. 3.3 Two capacitors connected in parallel

3.4.3 Capacitors in Series

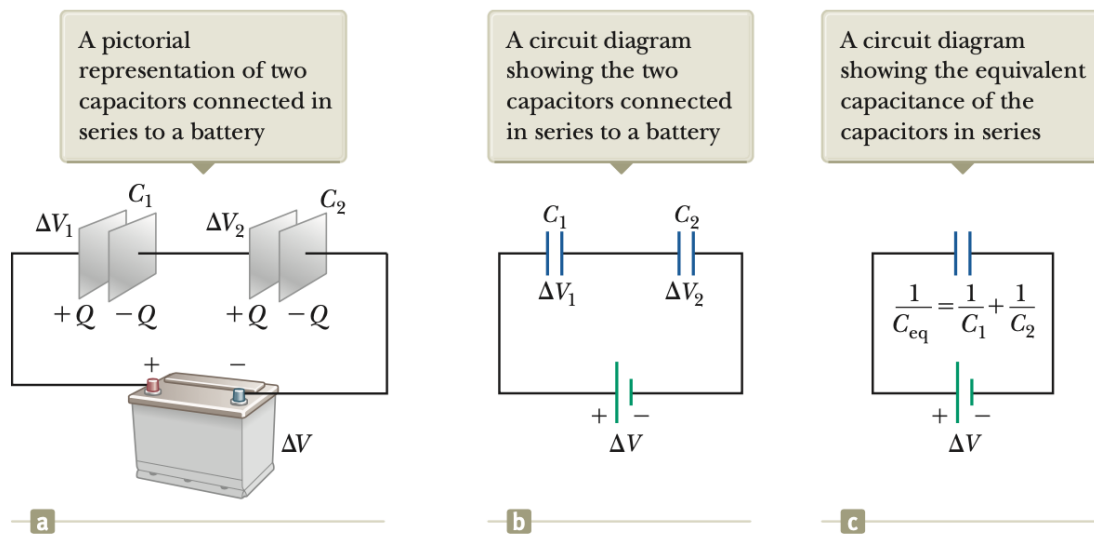


Fig. 3.4 Two capacitors connected in series

When two capacitors are connected in series as shown in Fig. 3.4, the charge across each of the capacitors is the same. That is,

$$Q_1 = Q_2 = Q$$

where Q is the charge that moved between a wire and the connected outside plate of one of the capacitors.

The individual voltages ΔV_1 and ΔV_2 across the capacitors add to give the total voltage ΔV_{tot} across the combination.

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Hence

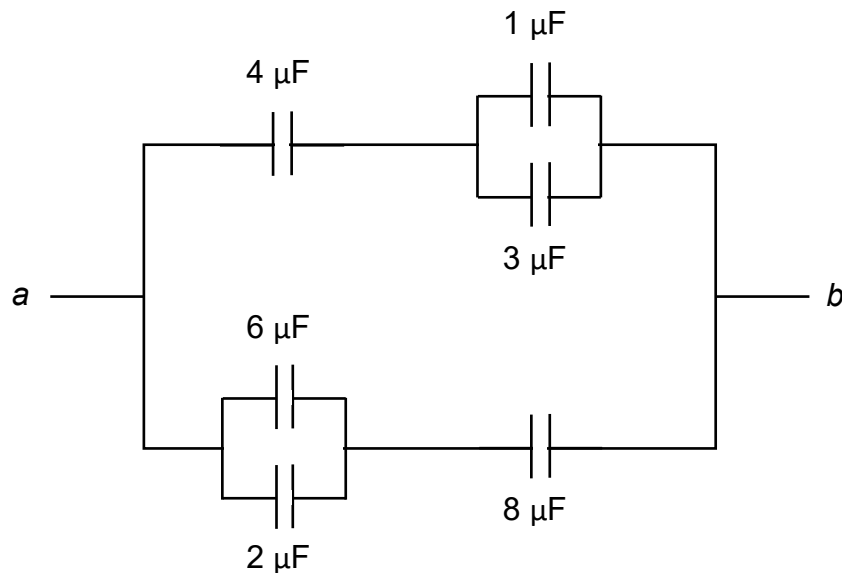
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

This expression shows that (1) the inverse of the equivalent capacitance is the algebraic sum of the inverses of the individual capacitances and (2) the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.

Example 3.9

Find the equivalent capacitance between a and b for the combination of capacitors shown below.

[6 μF]



Solution

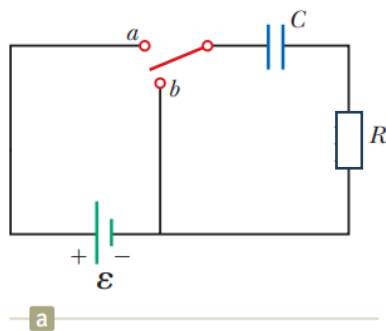
$$\text{Capacitance of the top branch} = \left(\frac{1}{4} + \frac{1}{1+3} \right)^{-1} = 2\ \mu\text{F}$$

$$\text{Capacitance of the bottom branch} = \left(\frac{1}{6+2} + \frac{1}{8} \right)^{-1} = 4\ \mu\text{F}$$

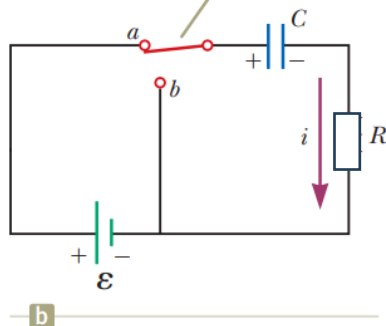
$$\text{Total capacitance} = 2 + 4 = 6\ \mu\text{F}$$

3.5 RC Circuits

3.5.1 Charging a Capacitor



When the switch is thrown to position *a*, the capacitor begins to charge up.



When the switch is thrown to position *b*, the capacitor discharges.

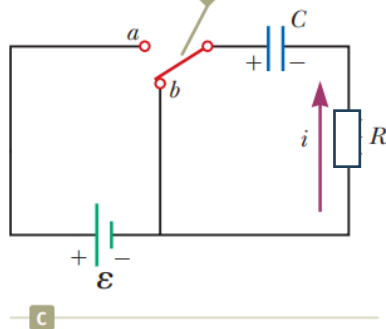


Fig. 3.5

Fig. 3.5 shows a simple series RC circuit. The capacitor in this circuit is initially uncharged. There is no current while the switch is open (Fig. 3.5(a)). The switch is moved to position *a* at $t = 0$ (Fig. 3.6(b)), however, charge begins to flow, setting up a current in the circuit and the capacitor begins to charge. Charge does not jump across the capacitor plates since the gap between the plates represents an open circuit. Charge is transferred between the plates and its connecting wires due to the electric field established in the wires by the battery until the capacitor is fully charged. As the plates are being charged, the potential difference across the capacitor increases. The value of the maximum charge on the plates depends on the voltage of the battery. Once the maximum charge is reached, the current in the circuit is zero since the potential difference across the capacitor matches that supplied by the battery.

Consider the algebraic sum of the e.m.f. sources must be equal to the algebraic sum of the potential difference across the components, the loop in Fig. 3.5(b) in the clockwise direction gives

$$\varepsilon - \frac{q}{C} - iR = 0$$

where $\frac{q}{C}$ is the potential difference across the capacitor and iR is the potential difference across the resistor. Note that q and i are instantaneous values that depend on time as the capacitor is being charged.

At the instant the switch is thrown to position *a* ($t = 0$), the charge on the capacitor is zero. Hence the initial current I_i in the circuit is maximum and given by

$$I_i = \frac{\varepsilon}{R}$$

At this time, the potential difference from the battery terminals appears entirely across the resistor. Later, when the capacitor is charged to its maximum value $Q_{\max}$, charges cease to flow, the current in the circuit is zero and the potential difference from the battery terminals appears entirely across the capacitor.

Hence $\varepsilon - \frac{Q_{\max}}{C} = 0$. So $Q_{\max} = C\varepsilon$.

Derivation of the expressions is not in syllabus

To obtain the expressions for time dependence of the charge and current, consider the equations:

$$i = \frac{dq}{dt} \text{ and } \mathcal{E} - \frac{q}{C} - iR = 0.$$

$$\text{This leads to } \mathcal{E} - \frac{q}{C} - \left(\frac{dq}{dt}\right)R = 0.$$

$$\text{Make } \frac{dq}{dt} \text{ the subject: } \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = \frac{C\mathcal{E} - q}{RC}$$

$$\text{Hence } \frac{dq}{q - C\mathcal{E}} = -\frac{dt}{RC}$$

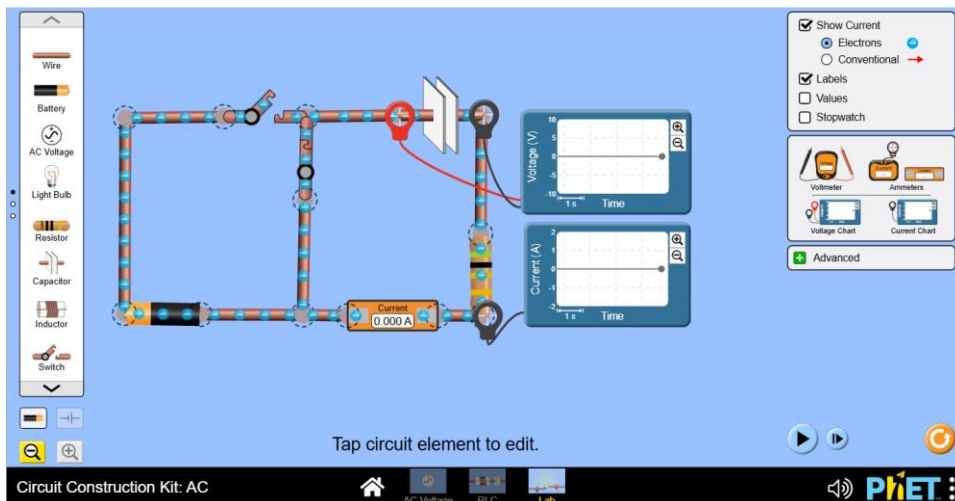
Integrating both sides

$$\int_0^q \frac{dq}{q - C\mathcal{E}} = -\int_0^t \frac{dt}{RC} \text{ leads to } \ln[q - C\mathcal{E}]_0^q = \left[-\frac{t}{RC}\right]_0^t$$

$$\text{This results in } \ln[q - C\mathcal{E}] - \ln[-C\mathcal{E}] = -\frac{t}{RC} \text{ which can be simplified to } \ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

$$\text{Inserting the exponential on both sides: } \frac{q - C\mathcal{E}}{-C\mathcal{E}} = e^{-\frac{t}{RC}}$$

$$\text{Recall } Q_{\max} = C\mathcal{E}, \text{ so } q = Q_{\max} (1 - e^{-\frac{t}{RC}}).$$



You can use this simulation to create the above circuit to obtain the variation with time of potential difference and current when a capacitor is charged and discharged.

You can also analyze circuits involving parallel and series configurations of resistors and capacitors.

To determine the current i , it is simply the differential of the instantaneous charge q .

$$q = Q_{\max} - Q_{\max} e^{-\frac{t}{RC}}$$

$$q = C\varepsilon - C\varepsilon e^{-\frac{t}{RC}}$$

$$i = \frac{dq}{dt} = \frac{-C\varepsilon}{-RC} e^{-\frac{t}{RC}} = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

$$i = I_i e^{-\frac{t}{RC}}$$

Fig. 3.6 shows the plots of capacitor charge and circuit current versus time. Charge, q , is zero at $t = 0$ and approaches the maximum value $C\varepsilon$ as t tends to infinity. The quantity RC which appears in the equations is called the time constant τ of the circuit. It represents the time interval that the current decreases to $\frac{1}{e}$ of its initial value ($0.368I_i$) or for the charge to increase from zero to $(1 - \frac{1}{e})Q_{\max}$ or $0.632Q_{\max}$.

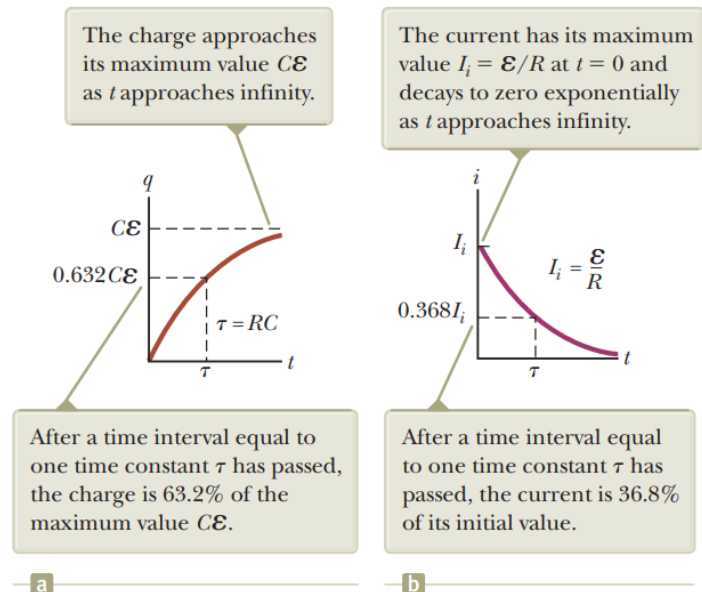


Fig. 3.6

3.5.2 Discharging a Capacitor

When the capacitor in Fig. 3.5(b) is completely charged, the initial potential difference that exists across the capacitor is $\frac{Q_i}{C}$ while there is zero potential difference across the resistor since $i = 0$.

When the switch is thrown on position b at $t = 0$ as shown in Fig. 3.5(c) the capacitor begins to discharge through the resistor. The instantaneous current in the circuit is i and the instantaneous charge on the capacitor is q . Hence the closed loop is now without the battery, the emf source is now the capacitor while the component is now only the resistor:

$$\frac{q}{C} + iR = 0$$

$$-\frac{q}{RC} = \frac{dq}{dt}$$

$$\int_0^t -\frac{1}{RC} dt = \int_{Q_i}^q \frac{1}{q} dq$$

$$-\frac{t}{RC} = \ln\left(\frac{q}{Q_i}\right)$$

$$e^{-\frac{t}{RC}} = \frac{q}{Q_i}$$

$$q = Q_i e^{-\frac{t}{RC}}$$

To determine the instantaneous current, $i = \frac{dq}{dt} = -\frac{Q_i}{RC} e^{-\frac{t}{RC}} = -\frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$

$$i = -I_i e^{-\frac{t}{RC}}$$

The negative sign indicates that as the capacitor discharges, the current direction is opposite to its direction when the capacitor was being charged. Now both the charge on the capacitor and the current decay exponentially at a rate characterized by the time constant $\tau = RC$.

Example 3.10

A $3.55 \mu\text{F}$ capacitor C_1 is fully charged using a 6.30 V battery.

(a) Determine the charge stored in the capacitor. [$2.24 \times 10^{-5} \text{ C}$]

The battery is then removed, and the capacitor is connected to an uncharged $8.95 \mu\text{F}$ capacitor C_2 . Charges move from C_1 to C_2 until equilibrium is reached.

(b)(i) Determine the final potential difference across each capacitor. [1.79 V]

(b)(ii) Calculate the final charge on capacitor C_2 . [$1.60 \times 10^{-5} \text{ C}$]

Capacitor C_2 is then connected to a 50Ω resistor separately.

(c) Determine the time taken for capacitor C_2 to discharge 90% of its initial charge. [$1.03 \times 10^{-3} \text{ s}$]

Solution

(a) $Q_0 = C_1 V_0 = (3.55 \times 10^{-6})(6.30) = 2.24 \times 10^{-5} \text{ C}$

(b)(i) By conservation of charge,

$$\begin{aligned} Q_0 &= Q_1 + Q_2 \\ C_1 V_0 &= C_1 V + C_2 V \\ V &= \frac{C_1 V_0}{C_1 + C_2} = \frac{(3.55 \times 10^{-6})(6.30)}{(3.55 \times 10^{-6} + 8.95 \times 10^{-6})} = 1.79 \text{ V} \end{aligned}$$

(b)(ii) $Q_2 = C_2 V = (8.95 \times 10^{-6})(1.79) = 1.60 \times 10^{-5} \text{ C}$

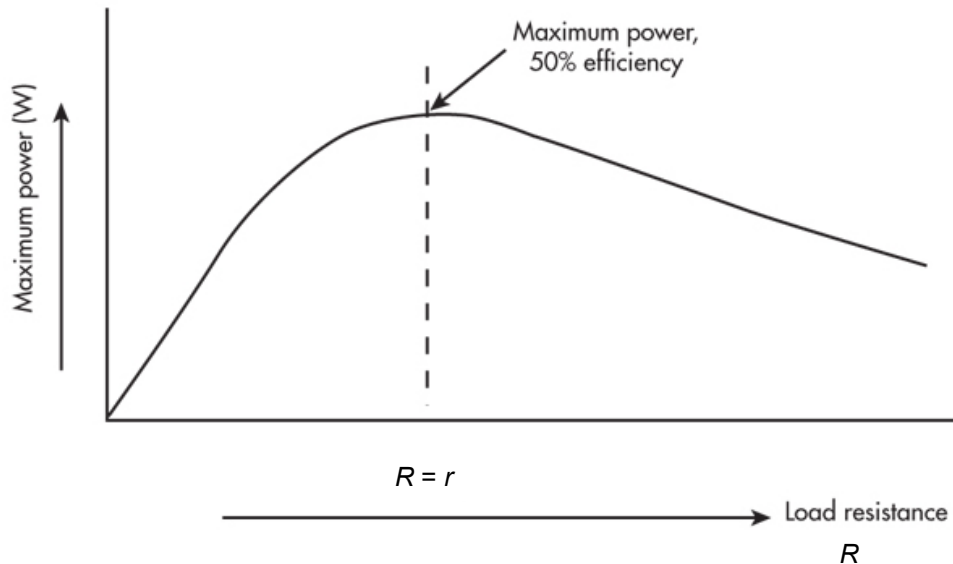
(c) $Q = Q_0 e^{-\frac{t}{RC}}$

$$\begin{aligned} 0.10 &= e^{-\frac{t}{RC}} \\ \ln 0.10 &= -\frac{t}{(50)(8.95 \times 10^{-6})} \\ t &= 1.03 \times 10^{-3} \text{ s} \end{aligned}$$

Annex: Output Efficiency and Maximum Power Output

Maximum Power Theorem:

Maximum power supplied by the e.m.f. source to the external circuit is when the resistance of the external resistor R is *equal* to the internal resistance r of the e.m.f. source.



Output power:

$$P_{out} = I^2 R \rightarrow (1)$$

Also,

$$P_{gen} = P_R + P_r$$

$$I\varepsilon = I^2 R + I^2 r$$

$$\varepsilon = I(R + r)$$

$$I = \frac{\varepsilon}{(R + r)} \rightarrow (2)$$

Sub (2) into (1)

$$P_{out} = \left[\frac{\varepsilon}{(R + r)} \right]^2 (R) \quad \text{where } \varepsilon, R, r \text{ are constants}$$

Power generated:

$$P_{gen} = I \varepsilon$$

$$= \frac{\varepsilon^2}{R + r}$$

Output efficiency:

$$\eta = \frac{P_{out}}{P_{gen}} = \frac{R}{R + r}$$

Derivation for Max Power Theorem (Not in syllabus)

When P_{out} is maximum,

$$\frac{dP_{out}}{dR} = 0$$

$$\varepsilon^2 \left[\frac{(R+r) - 2R}{(R+r)^3} \right] = 0$$

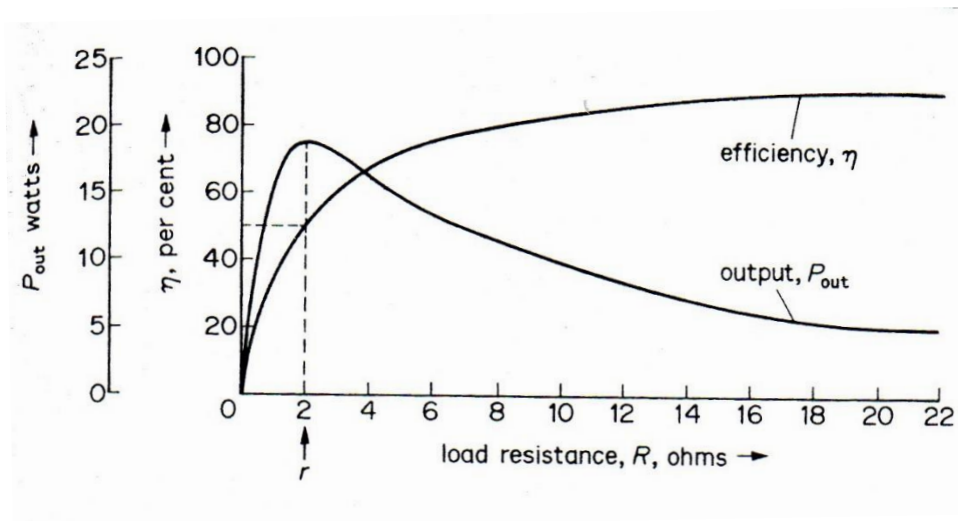
$$\boxed{R = r}$$

maximum output power, $(P_{out})_{max} = \frac{\varepsilon^2 R}{(R+r)^2}$

$$= \frac{\varepsilon^2 r}{(2r)^2} = \frac{\varepsilon^2}{4r}$$

external resistance R	$R = r$	$R \gg r$
maximum output power $(P_{out})_{max}$	$\frac{\varepsilon^2}{4r}$	$\frac{\varepsilon^2}{R}$
efficiency η	0.50	$\rightarrow 1$

Graph below is based on an internal resistance, $r = 2 \Omega$



Review Questions

- 1 One element of an electric cooker has an $I - V$ characteristic as shown Fig. 1.1.

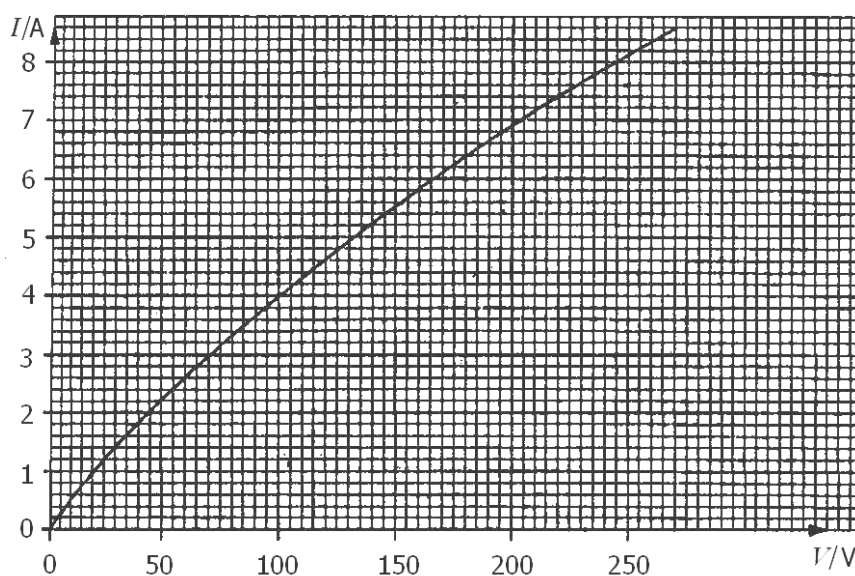
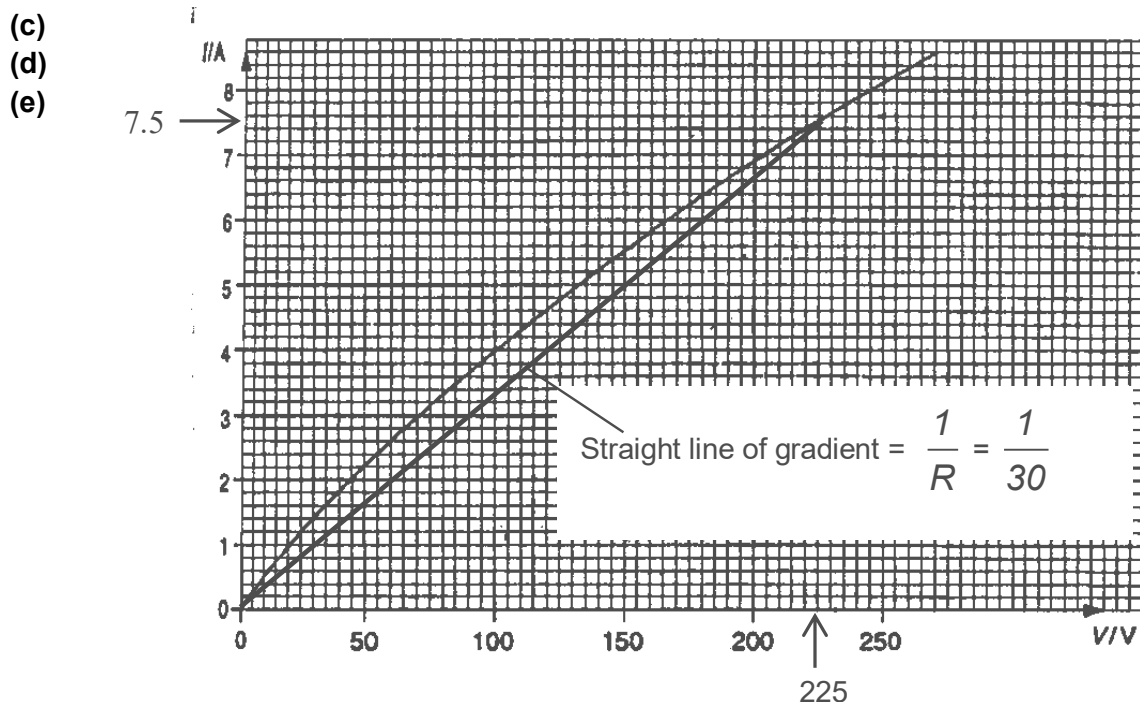


Fig. 1.1

- (a) Explain how the characteristic shows that the resistance of the element increases with potential difference.
- (b) Explain in terms of the movement of charged particles why the resistance increases with potential difference.
- (c) Use the graph to estimate the potential difference which should be applied to the element if it is to have a resistance of $30\ \Omega$.
- (d) Determine the current in the element when it has a resistance of $30\ \Omega$.
- (e) Determine the power of the element when it has a resistance of $30\ \Omega$.

Solution:

- 1 (a)
- Resistance is defined as the ratio of potential difference across the conductor to the current flowing through it.
 - If the resistance is constant, the graph is a straight line passing through the origin.
 - Here, the graph shows that the ratio $\frac{I}{V}$ decreases as V increases. This would imply that the resistance of the element increases as V increases.
- It is difficult to quote a value for the resistance of a filament lamp because the resistance changes as temperature changes.
- (b) As V increases, the rate of lattice vibration increases.
- Electric current is the rate of flow of free electrons through the metallic conductor. As the free electrons move, they collide with the vibrating lattice, so their movement is impeded.
 - The more the lattice vibrates, the greater the chance of collision. This means the greater the resistance.
 - As the increase in number of free electrons is insignificant, the no. of free electrons remains unchanged. Insignificant effect on resistance.
 - The overall effect is that resistance R increases. That is, further increase of the potential difference does not increase the current substantially.



Draw a straight line with gradient $= \frac{1}{30}$ from the origin (as shown in the graph).

From the graph:

when $R = 30 \, \Omega$, $V \approx 225 \, \text{V}$ and $I \approx 7.5 \, \text{A}$.

Power of the element at resistance of $30 \, \Omega$,

$$P = I^2 R = 7.5^2 (30) = \underline{1.69 \times 10^3 \, \text{W}}$$

- 2 Archaeologists use a variety of techniques to locate ancient buildings. One technique requires the measurement of electrical resistance. From this, measurements of resistivity may be determined.

- (a) State the relationship between resistance and resistivity. State the meaning of any symbols that you use.
- (b) Some resistors used in electronic circuits contain a thin film of carbon. Fig. 2.1 shows a square film of carbon of resistivity ρ . Current flows between the shaded faces.

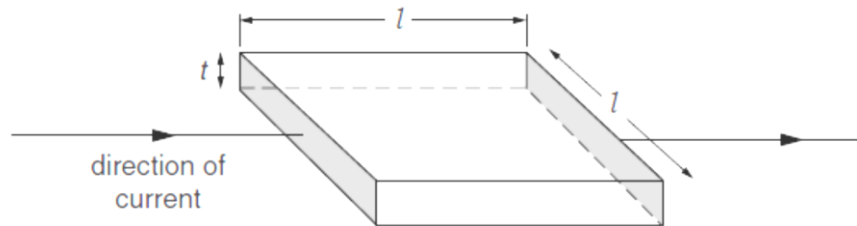


Fig. 2.1

Show that the resistance R of the square carbon film in Fig. 2.1 is independent of the length l of the sides of the square when the film has uniform thickness.

- (c) An archaeologist places pairs of probes at regular intervals in the ground and measures the resistance of the soil. Different resistance values at different locations may indicate the presence of a prehistoric ditch. Fig. 2.2 illustrates this method.

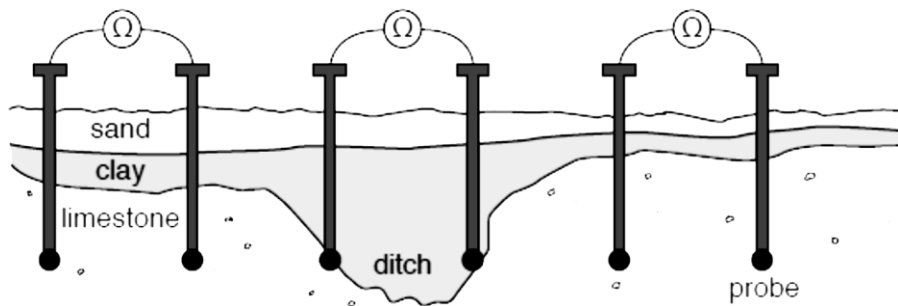


Fig. 2.2

Fig. 2.3 shows a block of clay. Fig. 2.4 shows a sample of clay and limestone incorporating the clay block shown in Fig. 2.3.

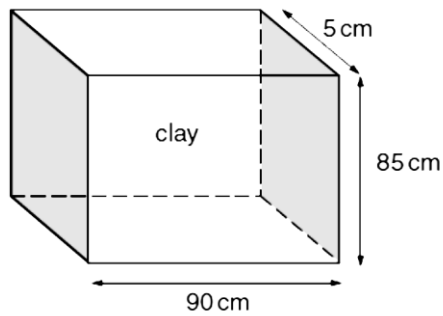


Fig. 2.3

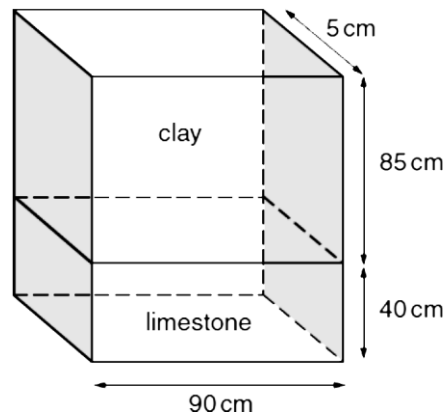


Fig. 2.4

- (i) The resistance between the shaded faces of the block of clay in Fig. 2.3 is $1650\ \Omega$. The resistance between the shaded faces of the clay-limestone sample in Fig. 2.4 is $1600\ \Omega$.

Calculate the resistance between the shaded faces of the limestone block.

- (ii) Determine the resistivity of this limestone.

Solution:

2 (a) $R = \frac{\rho L}{A}$ where ρ – resistivity, L – length of wire, A – cross-sectional area

(b) $R = \frac{\rho \ell}{t \ell} = \frac{\rho}{t}$

(c) (i) $\frac{1}{R} = \frac{1}{R_c} + \frac{1}{R_L}$
 $\frac{1}{1600} = \frac{1}{1650} + \frac{1}{R_L}$
 $R_L = 52\ 800\ \Omega$

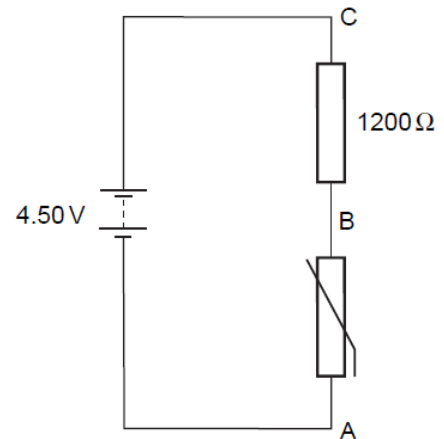
(ii) $R = \frac{\rho L}{A}$
 $52\ 800 = \frac{\rho (0.9)}{0.4 (0.05)}$
 $\rho = 1170\ \Omega\ \text{m}$

3 AJC 2015 Prelim P3 Q2

A battery of e.m.f. 4.50 V and negligible internal resistance is connected in series with a fixed resistor of resistance 1200 Ω and a thermistor, as shown.

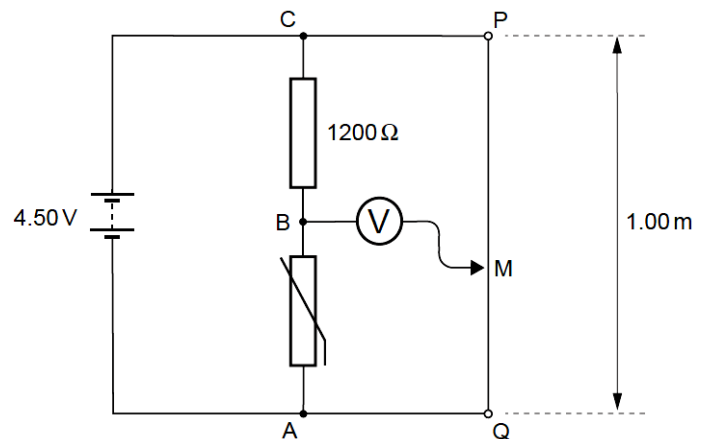
- (a) At room temperature, the thermistor has a resistance of 1800 Ω .

Deduce that the potential difference across the thermistor (across AB) is 2.70 V.



- (b) A uniform resistance wire PQ of length 1.00 m is now connected in parallel with the resistor and the thermistor, as shown.

A sensitive voltmeter is connected between point B and a moveable contact M on the wire.



- (i) Explain why, for constant current in the resistance wire, the potential difference between any two points on the wire is proportional to the distance between the points.
- (ii) The contact M is moved along PQ until the voltmeter shows zero reading.
 1. State the potential difference between the contact at M and the point Q.
 2. Calculate the length of wire between M and Q.
- (ii) The thermistor is warmed slightly. State and explain the effect on the length of wire between M and Q for the voltmeter to remain at zero deflection.

Solution:

$$\begin{aligned}
 3 \quad (a) \quad V &= \frac{ER_1}{R_1 + R_2} \\
 &= \frac{1800}{3000} \times 4.50 \\
 &= 2.70 \text{ V}
 \end{aligned}$$

- (b) (i) for a wire, $V = IR = I(\rho L/A)$
 ρ and A are constant
so $V \propto L$
- (ii) 1. 2.70 V
2. $\frac{L}{100} = \frac{2.70}{4.50}$
 $L = 60.0 \text{ cm}$
- (iii) thermistor resistance decreases as temperature rises
so QM is shorter