

2025 NJC H3 Math Prelim Solutions

Qn	Solution
1(a)	Consider the points $P(b, a)$ and $Q(d, c)$ in the x-y plane. Since $\frac{a}{b} < \frac{c}{d}$, OQ has a more positive gradient than OP. Also, the point $R(b+d, a+c)$ is such that $\overrightarrow{OR}$ is the vector sum of $\overrightarrow{OP}$ and $\overrightarrow{OQ}$. From the above diagram, we see that the gradient of OR is in between the gradients of OP and OQ. Thus, $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}.$
1(b)	For $0 < x < \frac{\pi}{2}$, $0 < \cos x < 1$ and $\sin x > 0$ $\Rightarrow 1 < \frac{1}{\cos x}$ and $\sin x > 0$ $\Rightarrow \sin x < \frac{\sin x}{\cos x}$ $\Rightarrow \sin x < \tan x$ By applying the result from part (a), $\frac{\sin x}{1} < \frac{2 \sin x}{1 + \cos x} < \frac{\sin x}{\cos x}$ $\frac{\sin x}{1} < \frac{4 \sin \frac{x}{2} \cos \frac{x}{2}}{1 + 2 \cos^2 \frac{x}{2} - 1} < \frac{\sin x}{\cos x}$ $\frac{\sin x}{1} < \frac{4 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} < \frac{\sin x}{\cos x}$ $\sin x < 2 \tan \frac{x}{2} < \tan x \text{ (shown)}$

Qn	Solution
1(c)	By AM-GM Inequality, $ab < \frac{a^2 + b^2}{2}$ $\frac{2ab}{2} < \frac{a^2 + b^2}{2}$ $\frac{2ab}{2} < \frac{2ab + a^2 + b^2}{2+2} < \frac{a^2 + b^2}{2}$ $\frac{2ab}{2} < \frac{a^2 + 2ab + b^2}{4} < \frac{a^2 + b^2}{2}$ $4ab < (a+b)^2 < 2(a^2 + b^2)$ $4 < \frac{(a+b)^2}{ab} < \frac{2(a^2 + b^2)}{ab}$ $4 < \left(\frac{a+b}{\sqrt{ab}} \right)^2 < 2 \left(\frac{a^2 + b^2}{ab} \right)$ $4 < \left(\frac{a}{\sqrt{ab}} + \frac{b}{\sqrt{ab}} \right)^2 < 2 \left(\frac{a^2}{ab} + \frac{b^2}{ab} \right)$ $4 < \left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} \right)^2 < 2 \left(\frac{a}{b} + \frac{b}{a} \right) \text{ (shown)}$

Qn	Solution
2(a)	Substituting $m = 0$ into the equation, $f(0) + 2f(n) = f^2(0 + n)$ $f(0) + 2f(n) = f^2(n) \dots\dots (1)$ Substituting $m = 1$ into the equation, $f(2) + 2f(n) = f^2(1 + n)$ $f(2) + 2f(n) = f^2(n + 1) \dots\dots (2)$ Since n is arbitrary, from equation (1), $f^2(n + 1) = f(0) + 2f(n + 1) \dots\dots (3)$ Substituting equation (3) into equation (2), $f(2) + 2f(n) = f(0) + 2f(n + 1)$ $f(n + 1) - f(n) = \frac{f(2) - f(0)}{2}$, which is a constant. This implies that f is an arithmetic progression. Thus $f(x) = ax + b$ for some constants a and b. Hence, $f(2m) + 2f(n) = f^2(m + n)$ $a(2m) + b + 2(an + b) = a(a(m + n) + b) + b$ $2ma + b + 2an + 2b = a^2m + a^2n + ab + b$ $2ma - ma^2 + 2na - na^2 + 2b - ab = 0$ $(ma + na + b)(2 - a) = 0$ $(m + n)a + b = 0$ or $2 - a = 0$ For $(m + n)a + b = 0$ for all integers m and n, $a = b = 0$. Thus, f is the zero function in this case. For $2 - a = 0$, $a = 2$ and $b \in \mathbb{R}$. But since f is an integer-valued function with domain $\mathbb{Z}$, $b \in \mathbb{Z}$. Thus either $f(x) = 0$ or $f(x) = 2x + b$, where $b \in \mathbb{Z}$.

Qn	Solution
2(b)	$x + y = 4 \Rightarrow (x + y)^2 = 16$ $\Rightarrow x^2 + 2xy + y^2 = 16$ $\Rightarrow x^2 + y^2 = 16 - 2xy \dots\dots (1)$ $x + y = 4 \Rightarrow (x + y)^3 = 64$ $\Rightarrow x^3 + 3x^2y + 3xy^2 + y^3 = 64$ $\Rightarrow x^3 + y^3 + 3x^2y + 3xy^2 = 64$ $\Rightarrow x^3 + y^3 + 3xy(x + y) = 64$ $\Rightarrow x^3 + y^3 + 3xy(4) = 64$ $\Rightarrow x^3 + y^3 = 64 - 12xy \dots\dots (2)$ From equations (1) and (2), $(x^2 + y^2)(x^3 + y^3) = (16 - 2xy)(64 - 12xy)$ $x^5 + x^3y^2 + x^2y^3 + y^5 = 2(8 - xy) \times 4(16 - 3xy)$ $x^5 + y^5 + x^2y^2(x + y) = 8(8 - xy)(16 - 3xy)$ $464 + x^2y^2(4) = 8(128 - 16xy - 24xy + 3x^2y^2)$ $4(116 + x^2y^2) = 8(128 - 40xy + 3x^2y^2)$ $116 + x^2y^2 = 256 - 80xy + 6x^2y^2$ $5x^2y^2 - 80xy + 140 = 0$ $x^2y^2 - 16xy + 28 = 0$ $(xy - 14)(xy - 2) = 0$ $xy = 14 \text{ or } xy = 2$ If $xy = 14$, then $x(4 - x) = 14$ $4x - x^2 = 14$ $x^2 - 4x + 14 = 0$ $D = (-4)^2 - 4(14) = -40 < 0$ This equation has no real solution for x (and thus y). So it must be that $xy = 2$. Hence $x(4 - x) = 2$ $4x - x^2 = 2$ $x^2 - 4x + 2 = 0$ $x = \frac{4 \pm \sqrt{(-4)^2 - 4(2)}}{2} = 2 + \sqrt{2} \text{ or } 2 - \sqrt{2}$ If $x = 2 - \sqrt{2}$, $y = 4 - (2 - \sqrt{2}) = 2 + \sqrt{2}$ If $x = 2 + \sqrt{2}$, $y = 4 - (2 + \sqrt{2}) = 2 - \sqrt{2}$ Since $x > y$, $x = 2 + \sqrt{2}$ and $y = 2 - \sqrt{2}$.

Qn	Solution
3(a)	Let S_i be the total training hours up till the ith day, where $1 \leq i \leq 140$. Then $1 \leq S_1 < S_2 < \dots < S_{140} \leq 20(13) = 260$. Let $T_i = S_i + 19$, where $1 \leq i \leq 140$. Then $20 \leq T_1 < T_2 < \dots < T_{140} \leq 279$. Each S_i and T_i is a value between 1 and 279 (inclusive), and the list $\{S_1, S_2, \dots, S_{140}, T_1, T_2, \dots, T_{140}\}$ has 280 elements. Consider slotting the 280 elements into boxes labelled 1 to 279. By Pigeonhole Principle, there exists S_i and T_j ($i > j$) such that $S_i = T_j$, since all S_i's are distinct and all T_j's are distinct. Thus, $S_i = S_j + 19 \Rightarrow S_i - S_j = 19$. Hence, the player will have trained for exactly 19 hours successively from day $j+1$ to day i. <u>Alternative:</u> let S_i be the total training hours up till the i^{th} day, where $1 \leq i \leq 140$ and $1 \leq S_i \leq 20 \cdot 13 = 260$. $S_i \equiv r \pmod{19}$, where $0 \leq r \leq 18$, $r \in \mathbf{Z}$. Consider slotting the 140 elements S_i into 19 boxes labelled 0 to 18, depending on their corresponding value of r when divided by 19. Notice that $140 = 19 \cdot 7 + 7$. By Pigeonhole Principle, there exists $S_1^*, S_2^*, \dots, S_8^*$ such that each of them is congruent to r^* modulo 19, where $0 \leq r^* \leq 18$, $r^* \in \mathbf{Z}$. We can therefore re-write each term in the following form: $S_1^* = 19q_1 + r^*$ $S_2^* = 19q_2 + r^*$... $S_8^* = 19q_8 + r^*$ where $q_1, \dots, q_8 \in \mathbf{Z}$ Notice that since $1 \leq S_i \leq 260$ and $\left\lfloor \frac{260}{19} \right\rfloor = 13$, we have $0 \leq q_1, q_2, \dots, q_8 \leq 13$.

	Thus, there must be at least two q_i 's which are consecutive values. Let this two q_i 's be q_j and q_k where $q_k > q_j$. $19q_k + r^* - 19q_j + r^* = 19$ $\Rightarrow S_k^* - S_j^* = 19$ Hence the player would have trained for exactly 19 hours from day $j+1$ to day k.
3(b)(i)	Let $y_i = x_i$ for $i = 1, 2, 3, 4$ and $y_5 \geq 0$, where $y_5 \in \mathbb{Z}$.
3(b)(ii)	Let $y_1 = x_1 - 3$, $y_2 = x_2$, $y_3 = x_3 + 8$, $y_4 = x_4 - 6$ and $y_5 \geq 0$. Then $x_1 + x_2 + x_3 + x_4 \leq 50$ becomes $(y_1 + 3) + y_2 + (y_3 - 8) + (y_4 + 6) + y_5 = 50$ i.e. $y_1 + y_2 + y_3 + y_4 + y_5 = 49$, where $0 \leq y_1 \leq 9$, $0 \leq y_2 \leq 7$, $0 \leq y_3 \leq 20$, $0 \leq y_4 \leq 14$ and $y_5 \geq 0$. Let A_1, A_2, A_3, A_4 be the set of integer solutions such that $y_1 \geq 10$, $y_2 \geq 8$, $y_3 \geq 21$, $y_4 \geq 15$ respectively. $ A_1 = \binom{49-10+4}{4} = 123410$ $ A_2 = \binom{49-8+4}{4} = 148995$ $ A_3 = \binom{49-21+4}{4} = 35960$ $ A_4 = \binom{49-15+4}{4} = 73815$ $ A_1 \cap A_2 = \binom{49-18+4}{4} = 52360$ $ A_1 \cap A_3 = \binom{49-31+4}{4} = 7315$ $ A_1 \cap A_4 = \binom{49-25+4}{4} = 20475$ $ A_2 \cap A_3 = \binom{49-29+4}{4} = 10626$ $ A_2 \cap A_4 = \binom{49-23+4}{4} = 27405$ $ A_3 \cap A_4 = \binom{49-36+4}{4} = 2380$

$$|A_1 \cap A_2 \cap A_3| = \binom{49-39+4}{4} = 1001$$

$$|A_1 \cap A_2 \cap A_4| = \binom{49-33+4}{4} = 4845$$

$$|A_1 \cap A_3 \cap A_4| = \binom{49-46+4}{4} = 35$$

$$|A_2 \cap A_3 \cap A_4| = \binom{49-44+4}{4} = 126$$

$$|A_1 \cap A_2 \cap A_3 \cap A_4| = 0$$

By PIE,
required number of integer solutions

$$\begin{aligned} &= \binom{49+4}{4} \\ &\quad - \left(\sum_{i=1}^4 A_i - \sum_{i \neq j} (A_i \cap A_j) \right. \\ &\quad \left. + \sum_{i \neq j \neq k} (A_i \cap A_j \cap A_k) - |A_1 \cap A_2 \cap A_3 \cap A_4| \right) \\ &= 25199 \end{aligned}$$

Qn	Solution
4(a)	The line passing through point (x_0, y_0) with gradient m, has an equation $y - y_0 = m(x - x_0)$, where $m \neq 0$. For its axial intercepts: When $x = 0$, $y = y_0 - mx_0$... y-intercept When $y = 0$, $x = x_0 - \frac{y_0}{m}$... x-intercept For the y-intercept $(0, y_0 - mx_0)$ to be the midpoint of (x_0, y_0) and the x-intercept $\left(x_0 - \frac{y_0}{m}, 0\right)$: $\frac{x_0 + \left(x_0 - \frac{y_0}{m}\right)}{2} = 0 \text{ and } \frac{y_0 + 0}{2} = y_0 - mx_0$ $x_0 = \frac{y_0}{2m} \text{ and } y_0 = 2mx_0$ $\frac{y_0}{2x_0} = m$ For $Q(x, y)$ on C, $-\frac{dx}{dy} = \frac{y}{2x}$ $-2x \frac{dx}{dy} = y$ Integrating both sides with respect to y: $\int -2x \, dx = \int y \, dy$ $-x^2 = \frac{y^2}{2} + c$ $2x^2 + y^2 = D, D = -2c > 0$
4(b)	$\frac{d}{dx} \left(e^{3\sin x} v \right) = \left(\frac{d}{dx} e^{3\sin x} \right) \left(\frac{dv}{dx} \right)$ $e^{3\sin x} \frac{dv}{dx} + (3e^{3\sin x} \cos x) v = 3e^{3\sin x} \cos x \frac{dv}{dx}$ $(3e^{3\sin x} \cos x - e^{3\sin x}) \frac{dv}{dx} = (3e^{3\sin x} \cos x) v$ $e^{3\sin x} (3 \cos x - 1) \frac{dv}{dx} = (3e^{\sin x} \cos x) v$ $\frac{1}{v} \frac{dv}{dx} = \frac{3 \cos x}{3 \cos x - 1}$

Integrating both sides with respect to x :

$$\frac{1}{v} \frac{dv}{dx} = 1 + \frac{1}{3 \cos x - 1}$$

$$\int \frac{1}{v} dv = \int 1 + \frac{1}{3 \cos x - 1} dx$$

Let $\cos x = \frac{1-t^2}{1+t^2}$. Then

$$-\sin x \frac{dx}{dt} = \frac{(1+t^2)(-2t) - (1-t^2)2t}{(1+t^2)^2}$$

$$-\frac{2t}{1+t^2} \frac{dx}{dt} = \frac{-4t}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{2}{1+t^2}$$

$$\int \frac{1}{v} dv = x + \int \frac{1}{3 \cos x - 1} dx$$

$$= x + \int \frac{1}{3 \left(\frac{1-t^2}{1+t^2} \right) - 1} \frac{2}{1+t^2} dt$$

$$= x + \int \frac{1}{\frac{3-3t^2-1-t^2}{1+t^2}} \frac{2}{1+t^2} dt$$

$$= x + \int \frac{1}{1-2t^2} dt$$

$$= x + \frac{1}{2\sqrt{2}} \ln \left| \frac{1+\sqrt{2}t}{1-\sqrt{2}t} \right| + c$$

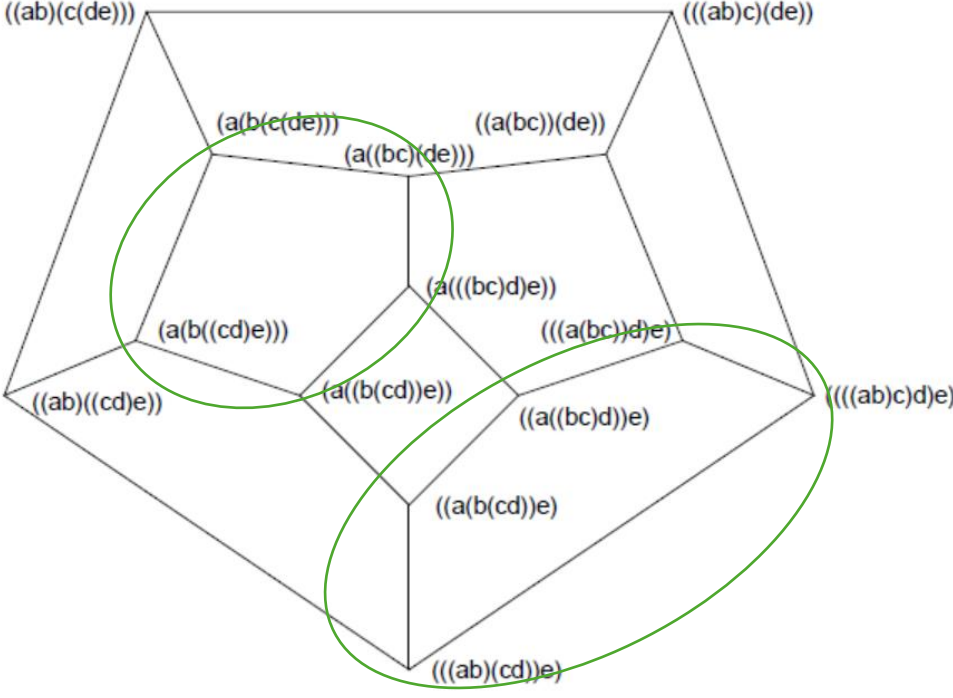
$$\ln |v| = x + \frac{1}{2\sqrt{2}} \ln \left| \frac{1+\sqrt{2} \tan^{-1} \frac{x}{2}}{1-\sqrt{2} \tan^{-1} \frac{x}{2}} \right| + c$$

$$= x + \frac{1}{2\sqrt{2}} \ln \left| \frac{1+\sqrt{2} \tan^{-1} \frac{x}{2}}{1-\sqrt{2} \tan^{-1} \frac{x}{2}} \right| + c$$

$$v = Ae^x \left| \frac{1+\sqrt{2} \tan^{-1} \frac{x}{2}}{1-\sqrt{2} \tan^{-1} \frac{x}{2}} \right|^{\frac{1}{2\sqrt{2}}}, \quad A = \pm e^c$$

Qn	Solution												
5(i)	$F_0 = 2^1 + 1 = 3$ $F_1 = 2^2 + 1 = 5$ $F_2 = 2^4 + 1 = 17$ $F_3 = 2^8 + 1 = 257$												
5(ii)	<table border="1"><tr><td>k</td><td>$F_0 F_1 F_2 \dots F_{k-1}$</td><td>$F_k$</td></tr><tr><td>1</td><td>$F_0 = 3$</td><td>5</td></tr><tr><td>2</td><td>$F_0 F_1 = 15$</td><td>17</td></tr><tr><td>3</td><td>$F_0 F_1 F_2 = 255$</td><td>257</td></tr></table> Conjecture: $F_0 F_1 F_2 \dots F_{k-1} = F_k - 2, \quad k \in \mathbb{Z}^+$	k	$F_0 F_1 F_2 \dots F_{k-1}$	F_k	1	$F_0 = 3$	5	2	$F_0 F_1 = 15$	17	3	$F_0 F_1 F_2 = 255$	257
k	$F_0 F_1 F_2 \dots F_{k-1}$	F_k											
1	$F_0 = 3$	5											
2	$F_0 F_1 = 15$	17											
3	$F_0 F_1 F_2 = 255$	257											
5(iii)	Let $P(k)$ be the statement: $F_0 F_1 F_2 \dots F_{k-1} = F_k - 2$ for $k \in \mathbb{Z}^+$. When $k = 1$, LHS = $F_0 = 3$ RHS = $F_1 - 2 = 5 - 2 = 3$. Since LHS = RHS, $P(1)$ is true. Assume that $P(m)$ is true for some $m \in \mathbb{Z}^+$ i.e. $F_0 F_1 F_2 \dots F_{m-1} = F_m - 2$. (Need to prove that $P(m+1)$ is true i.e. $F_0 F_1 F_2 \dots F_m = F_{m+1} - 2$.) $\begin{aligned} &F_0 F_1 F_2 \dots F_{m-1} F_m \\ &= (F_m - 2) F_m \\ &= (2^{2^m} - 1)(2^{2^m} + 1) \\ &= (2^{2^m})^2 - 1 \\ &= (2^{2^{m+1}} + 1) - 2 \\ &= F_{m+1} - 2 \end{aligned}$ $\therefore P(m)$ is true $\Rightarrow P(m+1)$ is true. Since $P(1)$ is true, and $P(m)$ is true $\Rightarrow P(m+1)$ is true, by mathematical induction, $P(k)$ is true for all $k \in \mathbb{Z}^+$.												

Qn	Solution
5(iv)	Suppose $p > 1$ is a common divisor of two distinct Fermat numbers F_i and F_j, where $i < j$. Then $p \mid F_i$ and $p \mid F_j$. Since $F_0 F_1 \dots F_i \dots F_{j-1} = F_j - 2$, $p \mid F_0 F_1 \dots F_i \dots F_{j-1}$ and hence, $p \mid F_j - 2$. Given $p \mid F_j$, we have $p \mid F_j - (F_j - 2) \Rightarrow p \mid 2$. Thus, $p = 2$ (since $p > 1$) which is a contradiction because all Fermat numbers $2^{2^n} + 1$ are odd and hence, not divisible by $p = 2$. $\therefore$ the only common divisor of any two distinct Fermat numbers is 1 and hence, they are coprime.
5(v)(a)	$5 \times 2^7 + 1 \equiv 0 \pmod{641}$ $5 \times 2^7 \equiv -1 \pmod{641}$ $5 \times 2^{7^4} \equiv -1^4 \pmod{641}$ $5^4 \times 2^{28} \equiv 1 \pmod{641}$ Remainder when $5^4 \times 2^{28}$ is divided by 641 is 1.
5(v)(b)	From part (v)(a), $5^4 \times 2^{28} = 641q_1 + 1, q_1 \in \mathbb{Z} \text{ -----(1)}$ $2^4 + 5^4 \equiv 0 \pmod{641}$ $\Rightarrow 5^4 \equiv -2^4 \pmod{641}$ $\Rightarrow 5^4 = 641q_2 - 2^4, q_2 \in \mathbb{Z} \text{ -----(2)}$ Substitute (2) into (1), $(641q_2 - 2^4) \times 2^{28} = 641q_1 + 1$ $\Rightarrow 2^{32} + 1 = 641(q_2 - q_1), q_2 - q_1 \in \mathbb{Z}$ $\Rightarrow F_5 \equiv 0 \pmod{641} \text{ (Shown)}$

Qn	Solution
6(i)	For a five-fold product: P_5 $= P_1P_4 + P_2P_3 + P_3P_2 + P_4P_1$ $= 1(5) + 1(2) + 2(1) + 5(1) = 14$ $C_4 = C_4 = \frac{1}{4+1} \binom{2(4)}{4} = \frac{1}{5} \binom{8}{4} = 14$
6(ii)	Let $k = 2i + 1$ where $i \in \mathbb{Z}^+$. P_k $= P_1P_{k-1} + \dots + P_{\lfloor \frac{k}{2} \rfloor} P_{\lfloor \frac{k}{2} \rfloor + 1} + P_{\lfloor \frac{k}{2} \rfloor + 1} P_{\lfloor \frac{k}{2} \rfloor} + \dots + P_{k-1}P_1$ $= 2 \left(P_1P_{k-1} + \dots + P_{\lfloor \frac{k}{2} \rfloor} P_{\lfloor \frac{k}{2} \rfloor + 1} \right)$ Since $P_k = P_{2i+1}$ is even, $P_{2i+1} \equiv 0 \pmod{2}$ for $i \in \mathbb{Z}^+$.
6(iii) (a)	$g_1 = (a(b(c(de))))$ $g_2 = (a((bc)(de)))$
6(iii) (b)	 B1: I will look for groupings in the associahedron where a is the last number to be associated or where e is the last number to be associated. Note: The circled sub-associahedron counts p_4. Not needed as an answer. But students can circle to illustrate their point.

Qn	Solution
6(iv)	We're looking for a way to pair each non-good sequence of n 1's and n (-1)'s with a unique sequence of $(n+1)$ 1's and $(n-1)$ (-1)'s. $A = \left\{ \begin{array}{l} s: \text{sequence of } n \text{ 1's and } n \text{ } (-1)\text{'s where there exists an} \\ \text{index } m \text{ such that } x_1 + x_2 + \dots + x_m < 0, \text{ for } 1 \leq m \leq 2n. \end{array} \right\}.$ $B = \{s^*: \text{sequence of } (n+1) \text{ 1's and } (n-1) \text{ } (-1)\text{'s}\}$ Identify the First “Negative” from s: For a non-good sequence of n 1's and n (-1)'s i.e. $\{x_1, x_2, \dots, x_{2n}\}$, find the smallest index i such that $x_1 + x_2 + \dots + x_i < 0$. Note that the sequence $\{x_1, x_2, \dots, x_{i-1}\}$ must have an equal number of 1's and (-1)'s. So $i = 2j + 1$ must be odd. Transform the sequence s: Let $s^* = \{y_1, y_2, \dots, y_{2n}\}$ where $y_k = -x_k$ for $k \leq 2j + 1$ and $y_k = x_k$ otherwise. s^* is a sequence of $(n+1)$ 1's and $(n-1)$ (-1)'s.
	1 mark: Definition of A. 1 mark: Definition of B. 1 mark: Identify the First Negative. 1 mark: Description of the transformation.
6(v)	C_n counts the number of ways p_{n+1} to associate the product $x_1 x_2 \dots x_{n+1}$ into products of two expressions using parenthesis, which is exactly the number of good sequences of n 1's and n (-1)'s. By the bijection in part (iv), the set of non-good sequences of n 1's and n (-1)'s and the set of all sequences of $(n+1)$ 1's and $(n-1)$ (-1)'s has the same size. Since every sequence of n 1's and n (-1)'s is either good or non-good, and no such sequence is both, $\begin{aligned} & \text{set of sequences of } n \text{ 1's and } n \text{ } (-1)\text{'s} \\ &= \text{set of good sequences of } n \text{ 1's and } n \text{ } (-1)\text{'s} \\ &+ \text{set of non-good sequences of } n \text{ 1's and } n \text{ } (-1)\text{'s} \end{aligned}$ $\binom{2n}{n} = C_n + \binom{2n}{n+1}$

	C_n $= \binom{2n}{n} - \binom{2n}{n+1}$ $= \frac{(2n)!}{n!n!} - \frac{(2n)!}{(n+1)!(n-1)!}$ $= \frac{(2n)!}{n!n!} - \frac{(2n)!}{(n+1)n!(n-1)!}$ $= \frac{(2n)!}{n!n!} \left(1 - \frac{n}{(n+1)} \right)$ $= \frac{(2n)!}{n!n!} \left(1 - \frac{n}{(n+1)} \right)$ $= \left(\frac{1}{n+1} \right) \binom{2n}{n}$
6(vi)	$(\Leftarrow)$ For $n = 1, 2, 3$, $C_1 = 1, n+2 = 3$ $C_2 = 2, n+2 = 4$ $C_3 = 5, n+2 = 5$ Clearly $C_n \mid n+2$ for $n = 1, 2, 3$. $(\Rightarrow)$ Using contrapositive: Let $n > 3$ $C_n = \frac{(n+2)(n+3)\dots(2n)}{n!}$ $\frac{n!}{(n+3)(n+4)\dots(2n)} = \frac{n+2}{C_n}$ $\frac{1(2)\dots(n-1)(n)}{(n+3)(n+4)\dots(2n-1)(2n)} = \frac{n+2}{C_n}$ $\frac{(3)(4)\dots(n-1)}{(n+3)(n+4)\dots(n+(n-1))} = \frac{n+2}{C_n}$ $\frac{3}{n+3} \frac{4}{n+4} \frac{n-1}{n+(n-1)} = \frac{n+2}{C_n}$

	LHS is a product of rational numbers smaller than 1. So $\frac{n+2}{C_n}$ is not an integer, that is $C_n \nmid n+2$. Thus if $n > 3$, $C_n \nmid n+2$.
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