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DUNMAN SECONDARY SCHOOL

CANDIDATE NAME			
CLASS		**INDEX NUMBER**	

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS **4052/01**

Paper 1

20 August 2025

2 hours 15 minutes

Solutions

(Updated 1 Dec 2022 by HOD/Math aligned to Marking Scheme for Specimen Papers 4049)

Question	Answer
1	$x-4=\dfrac{2}{x-3}$
	$(x-4)(x-3)=2$
	$x^2-7x+12=2$
	$x^2-7x+10=0$
	$(x-2)(x-5)=0$
	$x=2$ or 5
	When $x = 2$, $y = -2$
	When $x = 5$, $y = 1$
	Length = $\sqrt{(2-5)^2+(-2-1)^2}$ = 4.24 unit
2a	$-90° < \tan^{-1} p < 90°$ or $-\dfrac{\pi}{2} < \tan^{-1} p < \dfrac{\pi}{2}$
2b	$\pi - x$
3	$3x^2+15x+20 = 3\left(x^2+5x\right)+20$
	$= 3\left[(x+2.5)^2-(2.5)^2\right]+20$
	$= 3(x+2.5)^2+1.25$
	Since $3(x+2.5)^2+1.25 \ge 1.25$, $3x^2+15x+20$ cannot be smaller than 1.

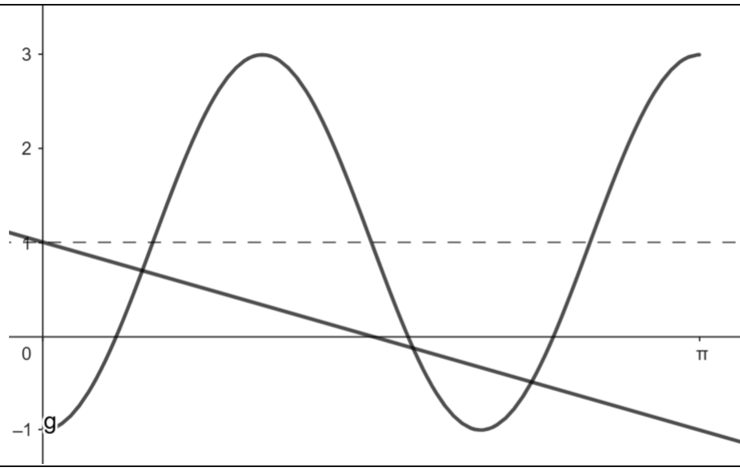
Question	Answer
4a	$\dfrac{x^2+3x+2}{x^2(2x+1)} = \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{2x+1}$
	$x^2+3x+2 = Ax(2x+1)+B(2x+1)+Cx^2$
	When $x=-\dfrac{1}{2}$, $\dfrac{1}{4}-\dfrac{3}{2}+2=C\left(\dfrac{1}{4}\right)$ $C=3$
	When $x = 0$, $B = 2$
	Comparing coefficients of x^2, $A = -1$
	$\dfrac{x^2+3x+2}{x^2(2x+1)} = -\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{2x+1}$
4b	$\int \dfrac{x^2+3x+2}{x^2(2x+1)}\ \mathrm{d}x = \int -\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{2x+1}\ \mathrm{d}x$ $= -\ln x-\dfrac{2}{x}+\dfrac{3}{2}\ln(2x+1)+c$
5	$3\log_5 y-\log_y 5=2$
	$3\log_5 y-\dfrac{\log_5 5}{\log_5 y}=2$
	$3\log_5 y-\dfrac{1}{\log_5 y}=2$
	$3(\log_5 y)^2-2\log_5 y-1=0$
	$(3\log_5 y+1)(\log_5 y-1)=0$
	$\log_5 y=1$ or $-\dfrac{1}{3}$
	$y=5$ or $y=5^{-\frac{1}{3}}=0.585$ (3s.f.)
Question	**Answer**
6a	$\sin(105^\circ)=\sin(60^\circ+45^\circ)$
	$= \sin 60^\circ \cos 45^\circ+\cos 60^\circ \sin 45^\circ$
	$= \dfrac{\sqrt{3}}{2}\cdot\dfrac{1}{\sqrt{2}}+\dfrac{1}{2}\cdot\dfrac{1}{\sqrt{2}}$
	$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}$
	$= \dfrac{\sqrt{6}+\sqrt{2}}{4}$
6b	$\dfrac{1}{2}(4)(BC)\sin 105^\circ=10+2\sqrt{3}$

	$\frac{1}{2}(4)(BC)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)=10+2\sqrt{3}$
	$BC=\frac{2\left(10+2\sqrt{3}\right)}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$
	$=\frac{4\left(5+\sqrt{3}\right)\left(\sqrt{6}-\sqrt{2}\right)}{6-2}$
	$=5\sqrt{6}+\sqrt{18}-5\sqrt{2}-\sqrt{6}$
	$=4\sqrt{6}+3\sqrt{2}-5\sqrt{2}$
	$=4\sqrt{6}-2\sqrt{2}$
	$=\sqrt{2}\left(4\sqrt{3}-2\right)$

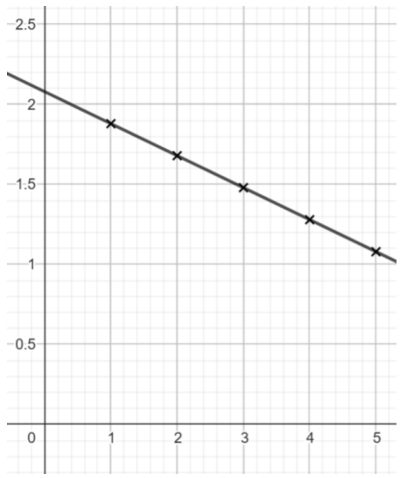
Question	Answer
7a	$9^x + 6^x = 6\left(4^x\right)$
	$\dfrac{9^x}{4^x} + \dfrac{6^x}{4^x} = 6$
	$\dfrac{3^{2x}}{2^{2x}} + \left(\dfrac{6}{4}\right)^x - 6 = 0$
	$\left(\dfrac{3}{2}\right)^{2x} + \left(\dfrac{3}{2}\right)^x - 6 = 0$
	When $u = \left(\dfrac{3}{2}\right)^x$, $u^2 + u - 6 = 0$
7b	$(u+3)(u-2) = 0$
	$u = 2$ or -3 (n.a.)
	$\left(\dfrac{3}{2}\right)^x = 2$
	$x = \dfrac{\ln 2}{\ln \frac{3}{2}}$ or $\dfrac{\lg 2}{\lg \frac{3}{2}}$ $= 1.71$
8	When $x = -2$, $2(-2)^3 + a(-2)^2 + b(-2) + 10 = 0$
	$4a - 2b = 6$
	$2a - b = 3$ - eqn 1
	When $x = 3$, $2(3)^3 + a(3)^2 + b(3) + 10 = 10$
	$9a + 3b = -54$
	$3a + b = -18$ - eqn 2
	eqn 1 + eqn 2: $5a = -15$
	$a = -3$
	$\therefore b = -9$
Question	**Answer**
9a	$12t - \dfrac{3}{2}t^2 = 0$
	$t\left(12 - \dfrac{3}{2}t\right) = 0$
	$t = 0$ or 8
	$\therefore 8$ s
9b	distance $= \int_0^8 12t - \dfrac{3}{2}t^2 \ dx$
	$= \left[6t^2 - \dfrac{1}{2}t^3\right]_0^8$

	$=[128]-[0]$
	= 128 m
9c	$a=12-3t$
	When t = 5, a = -3 m/s^2

Question	**Answer**
10	$y = xe^{-3x}$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = e^{-3x}(1) + x\left(-3e^{-3x}\right)$
	$\quad = e^{-3x}(1-3x)$
	$e^{-3x}(1-3x) = 0$
	$x = \dfrac{1}{3}$
	$y = \dfrac{1}{3e}$ or 0.123
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = (1-3x)\left(-3e^{-3x}\right) + e^{-3x}(-3)$
	When $x = \dfrac{1}{3}$, $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$
	$\left(\dfrac{1}{3}, -\dfrac{1}{3e}\right)$ is a maximum point.

Question	**Answer**
11a	
11b	The $2\cos 3x = \frac{2}{\pi}x$ is equivalent to $-2\cos 3x + 1 = -\frac{2}{\pi}x + 1$ and so its solutions can be found from the interception points of the two graphs.

Question	**Answer**
12a	Midpoint of $PQ = \left(\frac{1+9}{2},\frac{-2+10}{2}\right) = (5,4)$
	Gradient of $PQ = \frac{-2-10}{1-9} = \frac{3}{2}$
	Gradient of perpendicular bisector $= -\frac{2}{3}$
	$y-4=-\frac{2}{3}(x-5)$
	$y=-\frac{2}{3}x+\frac{22}{3}$
12b	$-\frac{2}{3}x+\frac{22}{3}=8x-10$
	$-\frac{26}{3}x=-\frac{52}{3}$
	$x=2$
	$R(2,6)$
12c	Area $= \frac{1}{2}\begin{vmatrix} 9 & 2 & 1 & 9 \\ 10 & 6 & -2 & 10 \end{vmatrix}$
	$= \frac{1}{2}\left[(54-4+10)-(20+6-18)\right]$
	$= 26$ unit^2

Question	Answer
13a	<table><tr><td>t</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>m</td><td>6.55</td><td>5.36</td><td>4.40</td><td>3.60</td><td>2.94</td></tr><tr><td>ln m</td><td>1.88</td><td>1.68</td><td>1.48</td><td>1.28</td><td>1.08</td></tr></table> 
13b	$\ln M = 2.08$
	$M = 8.00$
13c	Gradient = -0.2
	$k = 0.2$

Question	Answer
14a	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -4\sin(2x+\pi)$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\cos(2x+\pi) + c_1$
	$3 = 2\cos\left[2\left(\dfrac{\pi}{2}\right)+\pi\right] + c_1$
	$3 = 2(1) + c_1$
	$c_1 = 1$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\cos(2x+\pi) + 1$
	$y = \sin(2x+\pi) + x + c_2$
	$\pi = \sin\left[2\left(\dfrac{\pi}{2}\right)+\pi\right] + \dfrac{\pi}{2} + c_2$
	$c_2 = \dfrac{\pi}{2}$
	$y = \sin(2x+\pi) + x + \dfrac{\pi}{2}$
14b	When $x = 3$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\cos\left[2(3)+\pi\right]+1 = -0.920$
	y is decreasing when $x = 3$.

Question	Answer
15	$y=\dfrac{16}{(x+1)^2}-1=16(x+1)^{-2}-1$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-32(x+1)^{-3}$
	$-4=-32(x+1)^{-3}$
	$(x+1)^3=8$
	$x=1$
	At L, $y=\dfrac{16}{(1+1)^2}-1=3$
	$y-3=-4(x-1)$
	$y=-4x+7$
	At N, $0=-4x+7$
	$x=1.75$
	At M, $0=\dfrac{16}{(x+1)^2}-1$
	$x=3$ or -5 (n.a.)
	Area $=\left[\int_1^3 16(x+1)^{-2}-1\ \mathrm{d}x\right]-\int_1^{1.75}-4x+7\ \mathrm{d}x$
	$=\left[-16(x+1)^{-1}-x\right]_1^3-\left[-2x^2+7x\right]_1^{1.75}$
	$=2-1.125$
	$=0.875$ unit^2