JURONG PIONEER JUNIOR COLLEGE
9478 H2 PHYSICS/8867 H1 PHYSICS

MOTION AND FORCES TUTORIAL SOLUTIONS

Part 1: Motion (Kinematics)

Self-Check Questions

S1 Displacement: The change in position of an object in a particular direction.
Velocity: The rate of change of displacement.
Acceleration: The rate of change of velocity. It can be an increase or decrease in speed, or a change in direction.

S2 Information that we can obtain from

- Displacement-time graph:
    - Position of an object at a particular instance
    - Average velocity of an object over a time interval
    - Instantaneous velocity of an object via the gradient at that instance
    - How the velocity is changing by analysing the change in gradient for the graph trend
- Velocity-time graph, we can obtain:
    - Velocity of an object at a particular instance
    - Average Acceleration of an object over a time interval
    - Acceleration of an object via the gradient at that instance
    - Change in displacement of an object over a time interval via the area under graph for that time interval
    - How the acceleration is changing by analysing the change in gradient for the graph trend
- Acceleration-time graph, we can obtain:
    - Acceleration of an object at a particular instance
    - Change in velocity of an object over a time interval via the area under graph for that time interval

S3 For kinematics graphs, (1) gradient of velocity-time graph = acceleration *a* and
(2) area under velocity-time graph = displacement *s*

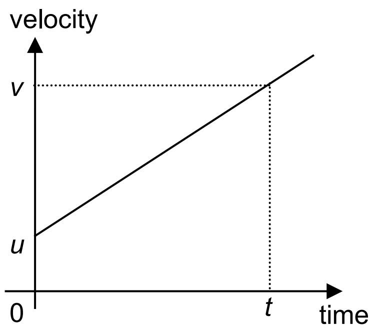
For the velocity-time graph shown,

gradient = $a = \frac{v-u}{t-0}$

area under graph = $s = \frac{1}{2}(v+u)t$

Hence, $a = \frac{v-u}{t} \Rightarrow v = u + at$ - - - - (1)

$s = \frac{1}{2}(v+u)t$ - - - - (2)



A velocity-time graph illustrating uniform acceleration

Substituing (1) into (2) by replacing *v*, $s = ut + \frac{1}{2}at^2$ - - - - (3)

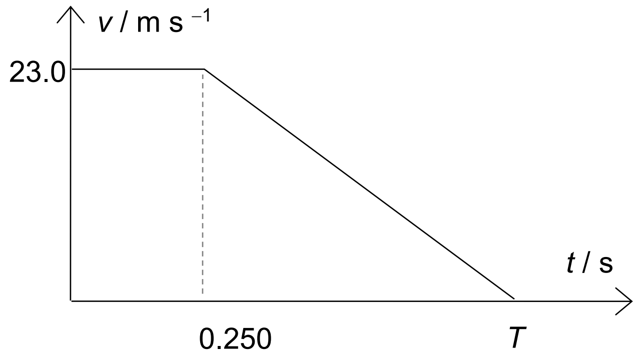
Substituting (1) into (2) by replacing *t*, $v^2 = u^2 + 2as$ - - - - (4)

Self-Practice Questions

1 Velocity is determined by gradient of s-t graph. Gradient in first half of s-t graph is positive and decreasing in magnitude, second half is negative and increasing in magnitude. Gradient is zero in the middle. [Ans: **C**]

2 For the portion of the graph between 0 and point B, acceleration is positive, implying that acceleration is in the same direction as direction of motion of object. Hence, object's speed keeps increasing until it reaches its maximum speed at point B. After point B, acceleration becomes negative. This implies that the acceleration is now directed opposite to the direction of motion. Hence, speed of the object will start to decrease after point B. [Ans: **B**]

3



<u>Find T:</u>

$-35.0 = (0.0 - 23.0) / (T - 0.250)$

$T = 0.9071$ s

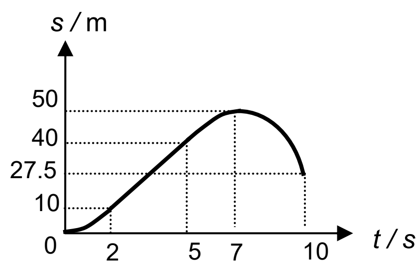
Minimum stopping distance = area under graph

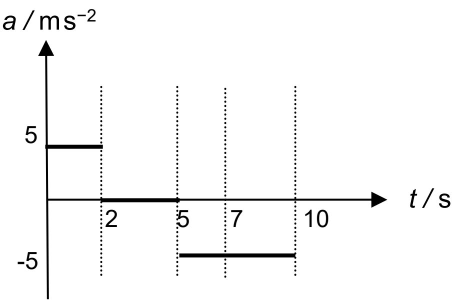
$= 23.0 \times 0.250 + ½(0.9071 - 0.250) \times 23.0$

$= 13.3$ m

[Ans: **13.3 m**]

4





5 (a) (i) Take downward as positive, given $u = 0$ m s^{-1}, $s = 125$ m, $a = g = 9.81$ m s^{-2}.

Using $s = ut + \frac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2(125)}{9.81}} = 5.05$ s

(ii) Using $v = u + at$, final velocity is $v = 0 + (9.81)(5.05) = 49.5$ m s^{-1}

(b) Take downward as positive, given s = 125 m, $a = g$ = 9.81 m s^{-2}, t = 4.0 s.

Using $s = ut + \frac{1}{2}at^2$, we have $125 = u(4.0) + \frac{1}{2}(9.81)(4.0)^2$

→ the initial downward velocity, u = 11.6 m s^{-1}

(c) (i) Take downward as positive, given u = −40.0 m s^{-1} , s = 125 m, $a = g$ = 9.81 m s^{-2}

Using $s = ut + \frac{1}{2}at^2$, we have $125 = (-40.0)t + \frac{1}{2}(9.81)t^2$

→ $4.905t^2 - 40t - 125 = 0$

→ t = 10.6 s or t = −2.41 s (rejected since t > 0)

(ii) Take upward as positive, given u = 40.0 m s^{-1} , a = –9.81 m s^{-2}

Using $v^2 = u^2 + 2as$, let h be the maximum height from point of projection.

$0 = 40.0^2 + 2(-9.81)h$ → h = 81.5 m

Maximum height reached from bottom of cliff = 125 + 81.5 = 207 m

6 **(a)** $s_A = (40)(20) = 800$ m

(b) $a_B = \dfrac{50-25}{20}$ = 1.25 m s^{-2}

(c) $s_B = \frac{1}{2}(20)(25 + 50) = 750$ m

(d) For B to catch up with A, they must have the same displacement from t = 0.

$s_A = s_B$

$800 + 40t = 750 + 50t$

t = 5.0 s

(e) $s_A = s_B = 800 + 40(5.0) = 1000$ m

(f) $s_A = 40t$

$s_B = 25t + \frac{1}{2}(1.25)t^2 = 25t + 0.625t^2$

$s_{A-B} = 40t - 25t - 0.625t^2$

$= 15t - 0.625t^2$

For maximum s_{A-B} , let $\dfrac{ds_{A-B}}{dt} = 0$

→ $15 - 2(0.625)t = 0$ → t = 12 s

Maximum $s_{A-B} = 15(12) - 0.625(12)^2$

$= 90$ m

OR

Let t_1 be the time when the two graphs intersect, when speed of B is 40 m s^{-1}.

So, $a_B = \dfrac{40-25}{t_1} = 1.25$ (from **(b)**)

→ $t_1 = \dfrac{40-25}{1.25}$ = 12 s

Before t_1, B is slower than A. After t_1, B is faster than A. So the maximum distance between cars occurs at t_1. Refer left column for answer.

Discussion Questions

Rectilinear Motion

1 **(a)** Yes it is possible. An object is moving with constant velocity on a frictionless surface or an object falling with terminal velocity in air.

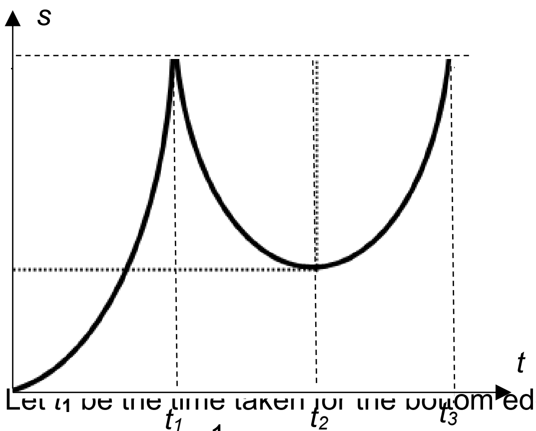
(b) Yes it is possible. An object thrown upwards and resting momentarily at its maximum height experiences acceleration of free fall.

(c) Yes it is possible. (1) An object being thrown upwards experiences acceleration of free fall. Taking upward to be positive, its velocity is positive while its acceleration is negative. (2) A car (moving to the right) that is slowing down. Taking right to be positive, the velocity is positive (in the direction of motion) while the acceleration is negative (in the opposite direction to that of motion). Refer notes p7: graph (e).

2 **(a)** **(i)** The height may be determined by the area under the graph prior to hitting the ground (i.e. height of drop = $\frac{1}{2}(v_1)(t_1)$).

(ii) t_4

(b)



3 **(a)** **(i)** Let t_1 be the time taken for the bottom edge to reach the light beam.

Using $s = ut + \frac{1}{2}at^2$, where u = 0 m s^{-1}, s = 1.00 m, g = 9.79 m s^{-2}

$$t_1 = \sqrt{\frac{2s_1}{g}} = \sqrt{\frac{2(1)}{9.79}} = 0.452\text{ s}$$

(ii) Let t_2 be the time taken for the top edge to reach the light beam.
Since the time taken for plate to pass through the beam is 0.052 s,
$t_2 = t_1 + 0.052 = 0.504$ s
Using $s_2 = ut_2 + \frac{1}{2}a(t_2)^2$, where u = 0 m s^{-1}, $a = g$ = 9.79 m s^{-2}

$$s_2 = 0 + \tfrac{1}{2}(9.79)(0.504)^2 \rightarrow s_2 = 1.24\text{ m}$$

Length of metal plate = 1.24 − 1.00 = 0.24 m

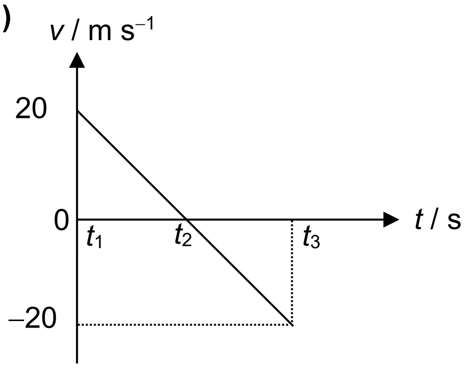
(b) **1.** The effect of air resistance may cause the time taken to be of a higher value.
2. The metal plate does not fall vertically downwards with the base remaining horizontal throughout. It may tilt as it falls and cause the time to be different.

4 **(a)** **(i)** Taking upward to be positive,
$v^2 = u^2 + 2as$
$0 = 20.0^2 + 2(-9.81)s$
Therefore, maximum height s = 20.4 m.

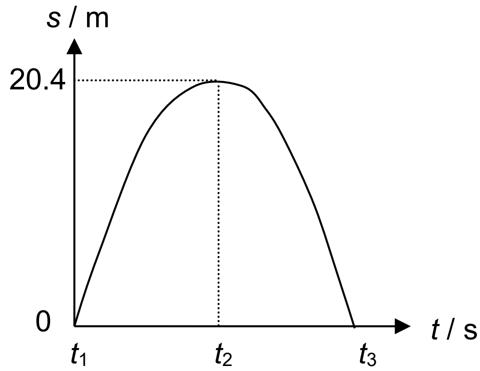
(ii) $s = ut + \frac{1}{2}at^2$
$0 = 20.0\,t + \frac{1}{2}(-9.81)t^2$
$4.905\,t^2 - 20\,t = 0$
t = 0 or 4.08 s

Therefore, the time required is 4.08 s.

(b) (i)



(ii)



Part 2: Forces (Dynamics)

Self-Check Questions

S1 1st law: a body at rest will stay at rest, and a body in motion will continue to move at constant velocity, unless acted on by a resultant external force;

2nd law: the rate of change of momentum of a body is (directly) proportional to the resultant force acting on the body and is in the same direction as the resultant force;

3rd law: the force exerted by one body on a second body is equal in magnitude and opposite in direction to the force simultaneously exerted by the second body on the first body.

S2 The mass of an object is a measure of its inertia. The weight of an object refers to the gravitational force exerted on it by the Earth. The mass of an object remains the same regardless of its location but the weight of the object depends on the strength of the gravitational field it is placed in.

S3 The two forces must be equal in magnitude and opposite in direction, be of the same type and they must act on different bodies.

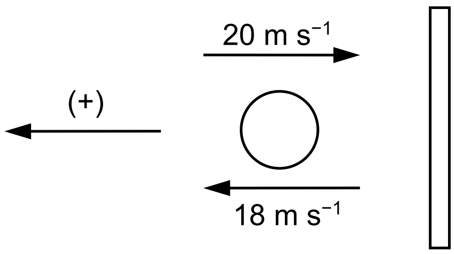
S4 Linear momentum is the product of mass and velocity. It is a vector quantity and its unit can be expressed as either N s or kg m s^{-1}.

S5 The rate of change of momentum of a body is directly proportional to the resultant force acting on it. When there is no net external force acting on a system, the total momentum remains constant. However, it should be noted that within the system, each body can exert an internal force on each other, the momentum of the individual bodies can thus change.

S6 Impulse of a force is the product of the force and the time interval for which the force acts on the body. It is mathematically equal to the area under the force-time graph.

Self-Practice Questions

Qn	Ans	Solution
P1	**D**	By Newton's 2nd law, no external resultant force acts on the aircraft moving with constant velocity relative to the ground. Hence, the resultant force acting on the aircraft is equal to zero.

Qn	Ans	Solution
P2	B	When a trolley runs down a slope with negligible friction and air resistance, the resultant force acting on the trolley parallel to the slope is $mg\sin\theta$, where m is the mass of trolley and θ is the angle the slope makes with the horizontal. By Newton's 2nd law, $mg\sin\theta = ma$ $a = g\sin\theta$ Since acceleration is independent of mass m, along the same slope, the acceleration for the second experiment is a.
P3	B	Consider the system of 2 blocks, letting acceleration be a, By Newton's 2nd law, $54 - 6.0 = (6.0 + 2.0)a$ $a = 6.0 \text{ m s}^{-2}$ Resultant force on 6.0 kg mass $= ma$ $= 6.0 \times 6.0$ $= 36 \text{ N}$
P4	B	 Impulse provided by the wall $=$ change in momentum experiened by the ball $= p_f - p_i$ $= mv_f - mv_i$ $= 0.070(18 - (-23))$ $= 2.87$ ≈ 2.9 N s (away from the wall)
P5	A	Let acceleration of box and mass be a, and tension in the string be T. Applying Newton's 2nd law on 2.0 kg mass, $(+\downarrow)\ 2.0g - T = 2.0a$ --- (1) Applying Newton's 2nd law on 8.0 kg box, $\left(\overset{+}{\rightarrow}\right)\ T - 6.0 = 8.0a$ --- (2) Solving for a, (1) + (2), $(2.0)(9.81) - 6.0 = 10.0a$ $a = 1.36$ $\approx 1.4 \text{ m s}^{-2}$

Discussion Questions

1 Applying Newton's first law of motion, when the bus comes to a sudden stop, the backpacks will continue in their original state of uniform motion in a straight line, maintaining their velocity initially. Hence they will slide forward. The friction between the backpacks and the surface will slow them down.

2 By Newton's second law:

$2F = (10)(60)$ ------ (1)

$5F = M(50)$ ------ (2)

Solving for M gives 30 kg.

3 **(a)** $s = ut + \frac{1}{2}at^2$

$= \frac{1}{2}(1.5)2^2$

$= 3.0$ m

vertical height moved

$h = s\sin 40^\circ$

$= 3.0\sin 40^\circ$

$= 1.928$

≈ 1.9 m

(b) (i) $\sum F = ma$

$N - mg = ma$

$N - (95)(9.81) = (95)(0)$

$N = 931.95$

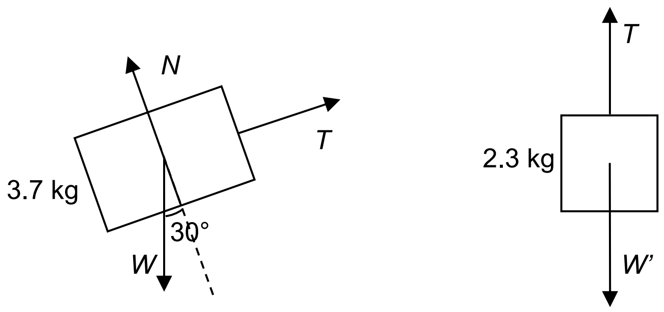
≈ 930N

(b) (ii) According to Newton's 2nd law of motion, his rate of change of momentum is directly proportional to the resultant force acting on him. Since he is moving at a constant speed in the same direction, his rate of change of momentum is zero and so must the resultant force acting on him. Therefore, the normal reaction force N must exactly balance his weight mg.

(c) (i) An upward normal reaction force greater in magnitude than his weight, a sideways frictional towards the right side on the soles of his shoes, and a downward gravitational force.

(c) (ii) Vertically, since the upward normal reaction force is greater than his weight, there will be a resultant upward force F_y. Horizontally, the resultant force will be the rightward frictional force F_x. The resultant force $F = \sqrt{F_x^2 + F_y^2}$ where $\tan 40^\circ = \frac{F_y}{F_x}$, results in an acceleration along the direction of cable.

4



(a) By Newton's second law and taking direction down slope as positive for 3.70 kg block:

$3.70\ g \sin 30^\circ - T = 3.70\ a$ ---- (1)

Taking upward as positive for 2.30 kg block:

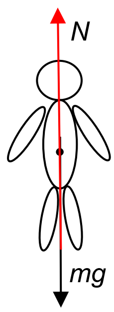
$T - 2.30\ g = 2.30\ a$ ---- (2)

Solving for T and a yields 20.9 N and – 0.736 m s^{-2}.
Hence magnitude of the acceleration = 0.736 ms^{-2}.

(b) Since the answer for a is negative, this indicates that the 2.30 kg block is accelerating downwards.

(c) 20.9 N

5 By Newton's second law and taking upwards as +ve (for (a) and (b))



$N - mg = ma$
$N = m(a+g)$

(a) Constant speed $\Rightarrow$ $a = 0$

$$N = (75)(0 + 9.81)$$
$$= 736\ \text{N}$$

(b) Upward acceleration a = 2.00 m s^{-2}

$$N = (75)(2.00 + 9.81)$$
$$= 886\ \text{N}$$

(c) Taking downwards as +ve:

$mg - N = ma$
$N = m(g - a)$

Downward acceleration a = 4.00 ms^{-2}

$$N = (75)(9.81 - 4.00)$$
$$= 436\ \text{N}$$

(d) Falls freely, $a = g$

$$N = (75)(9.81 - 9.81)$$
$$= 0\ \text{N}$$

6 (a) (i) Volume of air displaced = Area $\times$ vertical distance moved in 1 s
$= 80v \text{ m}^3$

(ii) Mass of air displaced = Volume $\times$ density
$= 80v \times 1.3$
$= 104v$ kg

(iii) Momentum of air displaced = mv
$= 104v \times v$
$= 104v^2 \text{ kg m s}^{-1}$

(iv) Initial momentum of air displaced = 0
Final momentum of air displaced = $104v^2$
Change in momentum of air displaced = final momentum – initial momentum
$= 104v^2 \text{ kg m s}^{-1}$

(v) Force exerted on air = rate of change of momentum of air

$$= \frac{\text{change in momentum}}{\text{time taken}}$$

$$= \frac{104v^2}{1}$$

$= 104v^2$ N

(vi) The rotor pushes the air downwards, so by N3L:
The direction of force exerted on the rotor blades by the air is upwards.
The magnitude of the force = $104v^2$ N

(b) Since the helicopter hovers, it is in equilibrium => resultant force = 0
Hence upward force on rotor = weight of helicopter

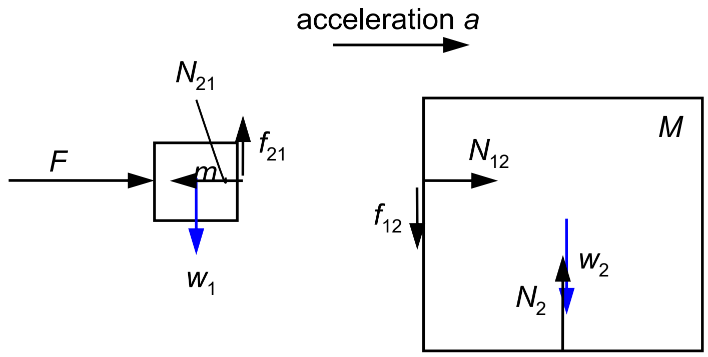
$104v^2 = 1000 \times 9.81$

$v = 9.71 \text{ m s}^{-1}$

7(a)	Change in momentum $= 18(0 - 7.2)$ $= -130 \text{ kg m s}^{-1}$
(b)	Force on water by wall = – 130 N Force by the water on the wall = – Force on water by wall = 130 N
(c)	If the water rebound with a non-zero speed after striking the wall, the <u>change in momentum of the water is greater</u>. If the water still maintains the same mass flow rate, <u>the wall must exert a larger force on the water to cause this larger rate of change of momentum and hence, water will exert a larger force on the wall</u>.

8 (a) Free-body diagrams of forces on m and M.

Legend of forces:
w_1 : weight of m
w_2 : weight of M
N_{21} : normal contact force exerted on m by M
N_{12} : normal contact force exerted on M by m $(|N_{21}| = |N_{12}| = N)$
N_2 : normal contact force exerted on M by the floor
f_{21} : friction force exerted on m by M
f_{12} : friction force exerted on M by m $(|f_{21}| = |f_{12}| = f_r)$



(b) & (c) Consider block m, using Newton's second law,
$F - N = ma$ --- (1)

Consider block M, using Newton's second law,
$N = Ma$ --- (2)

Since $f_r = \mu_s N$, to keep block m from sliding down block M, $f_r = w_1 = mg$,

$mg = \mu_s N$

$$N = \frac{(1.6)(9.81)}{0.38} = 41.31 \text{ N}$$

From (2),

$$a = \frac{N}{M} = \frac{41.31}{8.8} = 4.694$$

$$\approx 4.7 \text{ m s}^{-2}$$

From (1),

$$F = N + ma$$
$$= (1.6)(4.694) + 41.31$$
$$= 48.82$$
$$\approx 49 \text{ N}$$