RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Tutorial S5: Hypothesis Testing

1 Observations over a long period of time have shown that the mid-day temperature at a particular place during the month of June has a mean value of 23.9°C. An ecologist sets up an experiment to collect data for a hypothesis test of whether the climate is getting hotter. She selects at random 20 June days over a five-year period and records the mid-day temperature. Her results (in °C) are as follows :

20.1	26.2	23.3	28.9	30.4	28.4	17.3	22.7	25.1	24.2
15.4	26.3	19.3	24.0	19.9	30.3	32.1	26.7	27.6	23.1

(i) State the null and alternative hypotheses that the ecologist should use.

(ii) Given that the standard deviation is 2.3°C, carry out an appropriate test at the 10% significance level, stating any assumptions made. State your conclusion with regard to the mid-day temperature.

[(ii) p-value $= 0.0980 \le 0.1$, reject H_0**]**

2 'Brilliant' fireworks are intended to burn for 40 seconds. A random sample of 50 'Brilliant' fireworks is taken. Each firework in the sample is ignited and the burning time, x seconds, is measured. The results are summarised by

$$\sum(x-40)=-27\,,\ \sum(x-40)^2=167\,.$$

Test, at the 5% level of significance, whether the mean burning time of 'Brilliant' fireworks differs from 40 seconds. **[7]**

State what you understand by the expression 'at the 5% significance level' in the context of this question. **[1]**

State with a reason, whether, in using the above test, it is necessary to assume that the burning times of 'Brilliant' fireworks have a normal distribution. **[1]**

[p-value $= 0.0304 \le 0.05$, reject H_0**]**

3 The mass, x kg, of the contents of each packet in a random sample of 80 cereal packets is measured, and the results are summarized by $\sum x = 79.53$, $\sum x^2 = 100.4621$.

Test, at the 4% significance level, whether the population mean mass of the contents is less than 1.10 kg. **[7]**

In another test, using the same data and also at the 4% significance level, the hypotheses are as follows.

Null hypothesis: the population mean mass of the contents is equal to μ_0 kg

Alternative hypothesis: the population mean mass of the contents is not equal to μ_0 kg

Given that the null hypothesis is rejected in favour of the alternative hypothesis, find the set of possible values of μ_0. **[3]**

[p-value $= 0.0344 \le 0.04$, reject H_0 ; $(0, 0.875] \cup [1.11, \infty)$ **]**

4 An office worker, Natalie, has diabetes and has to monitor her blood glucose levels, which vary throughout the day. The results from a sample of 75 readings, x (in mmol/L), taken at random times over a week, are summarised by $\sum x = 511.5$ and $\sum x^2 = 4027.89$.

(i) Calculate unbiased estimates of the population mean and variance for the blood glucose levels. **[2]**

(ii) Test at 5% significance level whether Natalie's mean blood glucose level, μ (in mmol/L), is greater than 6.0. You should state your hypotheses and give your conclusion in context. **[4]**

(iii) State, giving a reason, whether the conclusion of the test in part **(ii)** would be valid if the 75 readings were all taken at weekends. **[1]**

Following a change in her diet, Natalie claims that her mean blood glucose level μ is now less than 6.0. She takes another random sample of 75 readings and notes that the total blood glucose levels is now 420. Using this sample, Natalie concludes that there is no reason to reject her claim at 6% level of significance.

(iv) Find the range of possible values of the variance used in calculating the test statistic. **[4]**

Explain why there is no need for Natalie to know anything about the population distribution of the glucose blood levels when carrying out the tests in **(ii)** and **(iv)**. **[1]**

[**(i)** $\bar{x} = 6.82; s^2 = 7.29$ **(ii)** p-value = 0.00427, reject H_0 **(iv)** $s^2 \leq 4.96$]

5 At an early stage in analyzing the marks scored by the large number of candidates in an examination paper, the Examining Board takes a random sample of 250 candidates and find that the marks, x, of these candidates give $\sum x = 11\,872$ and $\sum x^2 = 646\,193$. Using the figures obtained in this sample, the null hypothesis $\mu = 49.5$ is tested against the alternative hypothesis $\mu < 49.5$ at the α% significance level. Determine the set of values of α for which the null hypothesis is rejected in favour of the alternative hypothesis.

[$\{\alpha \in \mathbb{R} : 4.02 \leq \alpha \leq 100\}$]

6 A random sample of 90 batteries, used in a particular model of mobile phone, is tested and the 'standby time' is measured. The results are summarized by $\sum x = 3033$, and $\sum (x - 33.7)^2 = 13034.8$.

Test, at 1% significance level, whether the mean standby time is less than 36.0h.

In a test, at the 5% significance level, it is found that there is sufficient evidence that the population mean talk-time is less than 5 hours. Using only this information and giving a reason in each case, state whether each of the following statements is

(i) necessarily true, **(ii)** necessarily false, or **(iii)** neither necessarily true nor necessarily false.

(a) There is significant evidence at the 10% significance level that the population mean talk-time is less than 5 hours.

(b) There is significant evidence at the 5% significance level that the population mean talk-time is not 5 hours.

[p-value = 0.0357 > 0.01, do not reject H_0]

7 In a factory, the time in minutes for an employee to install an electronic component is a normally distributed continuous random variable T. The standard deviation of T is 5.0 and under ordinary conditions the expected value of T is 38.0. After background music is introduced into the factory, a sample of n components is taken and the mean time for randomly chosen employees to install them is found to be $\overline{t}$ minutes. A test is carried out, at the 5% significance level, to determine whether the mean time taken to install a component has been reduced.

(i) State appropriate hypotheses for the test, defining any symbols you use. **[2]**

(ii) Given that $n = 50$, state the set of values of $\overline{t}$ for which the result of the test would be to reject the null hypothesis. **[3]**

(iii) It is given instead that $\overline{t} = 37.1$ and the result of the test is that the null hypothesis is not rejected. Obtain an inequality involving n, and hence find the set of values that n can take. **[4]**

[(ii) $\{\overline{t} \in \mathbb{R}: 0 < \overline{t} \le 36.8\}$, **(iii)** $\{n \in \mathbb{Z}: 1 \le n \le 83\}$**]**

8 The manufacturer of a certain type of fan used for cooling electronic devices claims that the mean time to failure (MTTF) is 65 000 hours. The quality control manager suspects that the MTTF is actually less than 65 000 hours and decides to carry out a hypothesis test on a sample of these fans. (An accelerated testing procedure is used to find the MTTF.)

(i) Explain why the manager should take a sample of at least 30 fans, and state how these fans should be chosen. **[2]**

(ii) State suitable hypotheses for the test, defining any symbols that you use. **[2]**

The quality control manager takes a suitable sample of 43 fans, and finds that they have an MTTF of 64 230 hours.

(iii) Given that the manager concludes that there is no reason to reject the null hypothesis at the 5% level of significance, find the range of possible values of the variance used in calculating the test statistic. **[3]**

[(iii) $s^2 \ge 9420000$ **]**

9 On a remote island a zoologist measures the tail lengths of a random sample of 20 squirrels. In a species of squirrel known to her, the tail lengths have mean 14.0 cm. She carries out a test, at the 5% significance level, of whether the squirrels on the island have the same mean tail length as the species known to her. She assumes that the tail lengths of squirrels on the island are normally distributed with standard deviation 3.8 cm.

(i) State appropriate hypotheses for the test. **[1]**

The sample mean tail length is denoted by $\overline{x}$ cm.

(ii) Use an algebraic method to calculate the set of values of $\overline{x}$ for which the null hypothesis would not be rejected. (Answers obtained by trial and improvement from a calculator will obtain no marks.) **[3]**

(iii) State the conclusion of the test in the case where $\overline{x} = 15.8$. **[2]**

[(ii) $\{\overline{x} \in \mathbb{R}:\ 12.3 < \overline{x} < 15.7\}$, **(iii)** Reject H_0**]**

10 Carbon steel is made by adding carbon to iron; this makes the iron stronger, though less flexible. A steel manufacturing company makes carbon steel in the form of round bars. The process is designed to manufacture bars in which the amount of carbon in each bar, by weight, is 1.5%. It is known that the percentage of carbon in the steel bars is distributed normally, and that the standard deviation is 0.09%.

After comments from customers, the production manager wishes to test, at the 5% level of significance, if the percentage of carbon in the steel bars is, in fact, 1.5%. He examines a random sample of 15 bars to determine the percentage of carbon in each bar.

(i) Find the critical region for this test. **[4]**

The company recently launched a new line of flat bars made from mild steel. In mild steel the amount of carbon in each bar, by weight, is 0.25%.

Comments from customers suggest that these bars are not sufficiently flexible, which makes the production manager suspect that too much carbon has been added. He decides to perform a hypothesis test on a random sample of 40 of the new flat bars to find out if this is the case.

(ii) Explain why the production manager takes a sample of 40 flat bars for this test when he only took a sample of 15 round bars in his earlier test. **[2]**

The amounts of carbon, x%, in a random sample of 40 flat bars are summarized as follows.

$$n = 40 \qquad \sum x = 10.16 \qquad \text{sample variance} = 0.00014255$$

(iii) Calculate unbiased estimates of the population mean and variance for the percentage amount of carbon in the flat bars. **[2]**

(iv) Test, at the $2\frac{1}{2}\%$ level of significance, whether the mean amount of carbon in the flat bars is more than 0.25%. You should state your hypotheses and define any symbols that you use. **[5]**

[(i) $\left\{\overline{r} \in \mathbb{R} : \overline{r} \le 1.45 \text{ or } \overline{r} \ge 1.55\right\}$, **(iii)** 0.254, 0.000146, **(iv)** 0.0182**]**