Chapter 6: Circular Motion

<u>Detailed Solutions</u>

1 **A** If resultant force zero, astronaut will move off at a tangent in a straight line (N1L).

B,**C** The seat exerts zero normal force on the astronaut as both the seat and astronaut are 'falling' with the same acceleration.

D The gravitational force on the astronaut provides the centripetal force for him to move in a circular path. **(Answer)**

Worked Example 1

B Resultant acceleration must have a component opposite to linear velocity for it to slow down. Hence the resultant acceleration is the vector sum of centripetal acceleration and a component towards the left.

2 **D** Since gravitational force provides centripetal force, hence

$$G\frac{Mm}{r^2} = \frac{mv^2}{r} \quad \rightarrow \quad \therefore v^2 = G\frac{M}{r} \quad \rightarrow \quad \therefore v^2 \propto \frac{1}{r}$$

Note: Students tend to apply $v = r\omega$ wrongly and choose B as the answer.

3 **C** At the bottom:

$$T_{\max} - mg = \frac{mv^2}{r}$$

$$10 - 2.5 = \frac{mv^2}{r} \text{ -------(1)}$$

At the top:

$$T_{\min} + mg = \frac{mv^2}{r}$$

$$T_{\min} + 2.5 = \frac{mv^2}{r} \text{ -------(2)}$$

Solving: $T_{\min} = 7.5 - 2.5 = 5.0$ N

4 **B** Inner and outer rings experience the same ω. Since normal force (which provides the sense of artificial weight mg) on the objects provides their centripetal forces,

$mg = mr\omega^2 \Rightarrow g \propto r$

Hence,

$$\frac{r_1}{r_0} = \frac{g_1}{g} \quad \Rightarrow \quad \frac{r_1}{2150} = \frac{3.72}{9.81}$$

$$\therefore r_1 = 815 \approx 800 \text{ m}$$

Worked Example 2

(a)

$$T\sin 30^o = \frac{mv^2}{r}$$ B1

$$T\cos 30^o - mg = 0$$ B1

$$\tan 30^o = \frac{mv^2}{mrg} = \frac{v^2}{rg}$$

$$v = \sqrt{(0.50)(9.81)(\tan 30^o)}$$ B1

$$= 1.68 \text{ m s}^{-1}$$

(b) $T\cos 30^o = mg$

$$T = \frac{(0.20)(9.81)}{\cos 30^o} = 2.27 \text{ N}$$ A1

5 **(a)** The aircraft must be tilted so that there is a horizontal component for the lift force as there is other horizontal force acting in the radial direction. B1
This horizontal component of the lift force provides the centripetal force for the aircraft to move in a circular motion. B1

(b)
$$L\sin\theta = \frac{mv^2}{r}$$ M1
$$L\cos\theta - mg = 0$$ M1
$$\tan\theta = \frac{mv^2}{mrg} = \frac{v^2}{rg} = \frac{(150)^2}{(5000)(9.81)}$$ M1
$$\theta = 25^o$$ A1

(c) $L\cos\theta - mg = 0$
$$L = \frac{(650000)}{\cos 24.6} = 7.1\times10^5\text{N}$$ A1

6 **(a)**
$$F_c = \frac{mv^2}{r} = \frac{(1350)(24)^2}{65} = 1.2\times10^4\text{N}$$ M1 A1

(b) The friction is required to provide the centripetal force to do circular motion. B1
The will results in a centripetal acceleration which results in a change in the direction of the velocity. B1

(c) max. frictional force $= (0.70)(1350)(9.81) = 9.2\times10^4$ N B1
Since the frictional force is less then the centripetal force needed, the car cannot turn at 24 m s^{-1}. B1

7 **(a)** **(i)** The centripetal force will decrease as it moves from X to Y. B1
As the car moves up, there is a increase in GPE . resulting in a decrease in the kinetic energy of the car. B1
The speed will hence decrease. B1

(ii) If the car is in contact with the track at Y, there net force in the radial direction must be more than the weight of the car since B1
the normal contact force is also acting vertically downward. B1
Hence by Newton's 2nd law, the acceleration will also be more than 9.81 m s^{-2}.

(b) **(i)**
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_Y^2 + mgh$$
$$(0.5)(3.8)^2 = (0.5)v^2 + (9.81)(0.62)$$ M1
$$v_Y = 1.5 \text{ m s}^{-1}$$
$$a_c = \frac{v^2}{r} = \frac{(1.5)^2}{0.31} = 7.3 \text{ m s}^{-2}$$ A1
Since the centripetal acceleration is less than 9.81 m s^{-2}, cannot remains in contact at point Y. B1

(ii)
$$N - mg = \frac{mv^2}{r}$$ B1
$$N = \frac{(0.230)(3.8)^2}{0.31} + (0.230)(9.81) = 13 \text{ N}$$ A1