Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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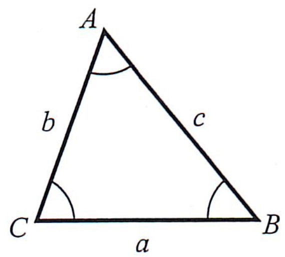
MA304.3.2 – Sine Rule

At the end of this unit, you should be able to

- **use the sine rule to solve triangles**
- **simplify trigonometric ratios of supplementary angles**
- **solve ambiguous triangles**

E1. The area of a triangle can be written as $\frac{1}{2}ab\sin C$. Write down the other two similar forms by interchanging the sides and angles.

$$\frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A.$$



(a) Show that the above relationship can be reduced to

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\frac{1}{2}\cancel{ab}\sin C}{\cancel{ab}c} = \frac{\frac{1}{2}\cancel{ac}\sin B}{\cancel{a}b\cancel{c}} = \frac{\frac{1}{2}\cancel{bc}\sin A}{a\cancel{bc}}$$

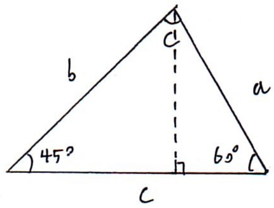
$$\frac{\sin C}{c} = \frac{\sin B}{b} = \frac{\sin A}{a}.$$

Given two sides of a triangle a and b and the angles opposite each of the sides are A and B, the sine rule states that

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

When told to solve a triangle, it means to find all the values of the angles and sides that are not given.

E2. Solve the triangle where $\measuredangle A = 45^\circ$, $\measuredangle B = 60^\circ$ and $c = 7$ cm.



$\measuredangle C = 180^\circ - 45^\circ - 60^\circ$

$= 75^\circ$

$$\frac{\sin(60^\circ)}{b} = \frac{\sin(75^\circ)}{7}$$

$b = 6.28$ cm (3 s.f.)

$$\frac{\sin(45^\circ)}{a} = \frac{\sin(75^\circ)}{7}$$

$a = 5.12$ cm (3 s.f.)

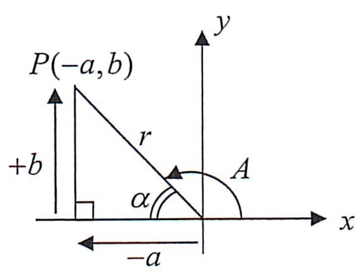
Obtuse angles

By ASTC, when an angle A is obtuse, i.e. it lies in the 2nd quadrant, only the sine ratio is positive, while both the cosine and tangent ratios are negative. We also saw that we can determine the value of $\sin A$, $\cos A$ and $\tan A$ by looking at the corresponding *basic angle* α.

$\sin A = \dfrac{b}{r}$ (since opposite side is positive)

$\cos A = \dfrac{-a}{r} = -\dfrac{a}{r}$ (since adjacent side is negative)

and $\tan A = \dfrac{b}{-a} = -\dfrac{b}{a}$

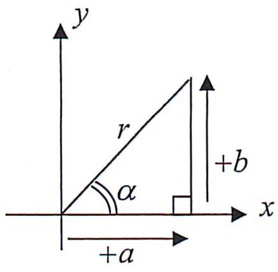


Since α is acute, we can also easily write that

$$\sin A = \frac{b}{r} = \sin\alpha$$

$$\cos A = \frac{-a}{r} = -\frac{a}{r} = -\cos\alpha$$

and $$\tan A = \frac{b}{-a} = -\frac{b}{a} = -\tan\alpha$$



But if we replace α by $180^\circ - A$, we have the following results:

$$\boxed{\begin{aligned} \sin A &= \sin(180^\circ - A) \\ \cos A &= -\cos(180^\circ - A) \\ \tan A &= -\tan(180^\circ - A) \end{aligned}}$$

P1. Evaluate the following trigonometric ratios:

(a) $\sin\frac{2\pi}{3}$

$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2}$

(b) $\tan(-135^\circ)$

$\tan(-135^\circ) = \tan(45^\circ)$

$= 1$

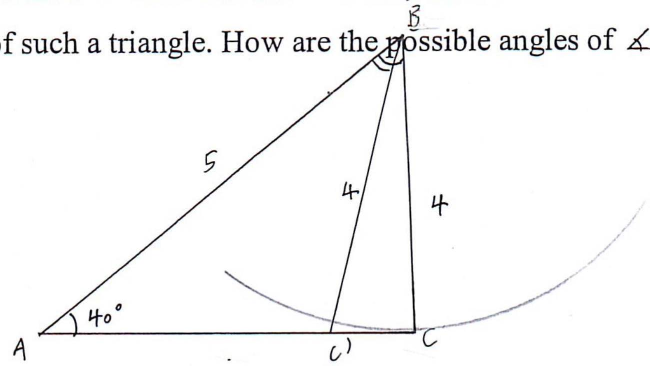
(c) $\cos(-\frac{5\pi}{6})$

$\cos\left(-\frac{5\pi}{6}\right) = -\cos(30^\circ)$

$= -\frac{\sqrt{3}}{2}$

The result $\sin A = \sin(180^\circ - A)$ is particularly important in solving some types of triangles when the information provided is not enough to determine the exact shape of the triangle, resulting in what we term as **ambiguous triangles**.

E3. A triangle ABC is such that AB = 5 cm, $\measuredangle BAC = 40^\circ$ and BC = 4 cm. Sketch possible illustrations of such a triangle. How are the possible angles of $\measuredangle ACB$ related?



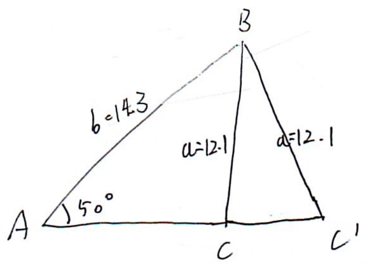
$\frac{\sin(BAC)}{BC} = \frac{\sin(ACB)}{AB}$

$\frac{\sin(40^\circ)}{4} = \frac{\sin(ACB)}{5}$

$\sin(ACB) = 1.25\sin(40^\circ)$

$\measuredangle ACB = \sin^{-1}(1.25\sin(40^\circ))$

P2. Solve the triangle where $\measuredangle A = 50^\circ$, $a = 12.1$ cm and $b = 14.3$ cm.



For ABC'

$\frac{\sin(50^\circ)}{12.1} = \frac{14.3\sin(50^\circ)}{14.3}$

$\sin(B) = \frac{14.3\sin(50^\circ)}{12.1}$

$B = \sin^{-1}\left(\frac{14.3\sin(50^\circ)}{12.1}\right)$

$= 64.5^\circ$ (1d.p.)

$\measuredangle C = 180^\circ - 64.5^\circ - 50^\circ$

$= 65.1^\circ$

$\frac{\sin(50^\circ)}{12.1} = \frac{\sin(65.1^\circ)}{c}$

$c = 14.3$ cm (3s.f.)

For ABC

$\sin(180 - \measuredangle B) = 64.9^\circ$ (1d.p.)

$\measuredangle B = 115.1^\circ$

$\measuredangle C = 180^\circ - 115.1^\circ - 50^\circ$

$= 14.9^\circ$

$\frac{\sin(50^\circ)}{12.1} = \frac{\sin(14.9^\circ)}{c}$

$c = 4.06$ cm (3s.f.)