**NANYANG JUNIOR COLLEGE**

**DEPARTMENT OF MATHEMATICS**

**2026 J2 H2 FM**

**FM Statistics 1 - Discrete and Continuous Random Variable**

**Instruction:** *Attempt the questions marked with *.*

**1*** The piano shop opens at 9 am every day. The waiting time (in hours) between successive arrivals of customers at the shop has an exponential distribution with mean $\frac{1}{\lambda}$. Records show that on average, the shop sees no customers in the first 5 minutes on 20% of the days.

**(a)** Show that the mean number of customers in one hour is $12\ln 5$. [2]

**(b)** Given that the first customer has not arrived by 9.05 am, find the probability that the shop sees its first customer by 9.15 am. [2]

**(c)** Find the probability that the 15th customer arrives after 10.30 am. [2]

Records show that, on average, 1 in 10 of its customers bought a piano.
Let $W$ be the random variable denoting the number of customers served **before** the customer who bought the second piano.

**(d)** Find $\mathrm{P}(W = w)$. [2]

**(e)** By considering the mean and variance of an appropriate geometric distribution, find $\mathrm{E}(W)$. [3]

**2*** The continuous random variable $X$ has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{2}{25}(x+1), & -1 \le x \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

The random variable $Y$ is defined by $Y = X^2$.

**(a)** Show that $\mathrm{P}(Y \le y) = \dfrac{4}{25}\sqrt{y}$ for $0 \le y \le 1$. [3]

**(b)** Find the cumulative distribution function of $Y$. [4]

**(c)** Find the probability density function of $Y$. [2]

**3** The number of distinct uranium deposits in a given area is a Poisson random variable with parameter 10. Independently, the number of distinct plutonium deposits in this area is a Poisson random variable with parameter 5. In a fixed period of time, each deposit is discovered independently with probability $\frac{1}{50}$.

**(a)** Find the probability that no less than 2 deposits are discovered during this period. [2]

**(b)** Given that no more than 3 deposits are found in this period of time, find the probability that there are at least 1 uranium deposit. [4]

**4** A particle moves along a horizontal line, relative to a fixed origin, so that its position at a given time $t$ is $x = x_0 \sin t$, where $x_0$ is a positive constant.

Let $X$ be the continuous random variable denoting the particle's position.

**(a)** Show that the cumulative density function of $X$ is given by

$$\mathrm{F}(x) = \begin{cases} 1 & x > x_0, \\ \dfrac{1}{2} + \dfrac{1}{\pi}\sin^{-1}\left(\dfrac{x}{x_0}\right) & -x_0 \le x \le x_0, \\ 0 & x < -x_0, \end{cases}$$

and hence, find the probability density function of $X$. [5]

**(b)** Find the exact probability that the magnitude of $X$ is at most $\frac{1}{2}x_0$. [2]

**(c)** Show that the expectation of $X$ is 0, and find the exact variance of $X$ using the substitution $x = x_0 \sin\theta$. [7]

**[H2 Math DRV]**

**5** Three fair six-sided die are thrown. The random variable $X$ represents the highest of the three scores on the dice.

**(a)** Show that $\mathrm{P}(X = 6) = \dfrac{91}{216}$. [2]

**(b)** By finding the probability distribution of $X$, show that $\mathrm{E}(X) = \dfrac{119}{24}$ and find the exact value of Var ($X$). [5]