

Name: _____ () Class: _____



華僑中學

HWA CHONG INSTITUTION
PAPER G

INTEGRATED MATHEMATICS

Level : Secondary Three

Duration : 2 hours 30 minutes

Do not open this booklet until you are told to do so.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

The use of a graphing calculator is not permitted.

You are reminded of the need for clear presentation in your answers. Omission of essential working will result in loss of marks.

This question paper consists 7 printed pages, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Mensuration

Arc Length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

2. TRIGONOMETRY

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

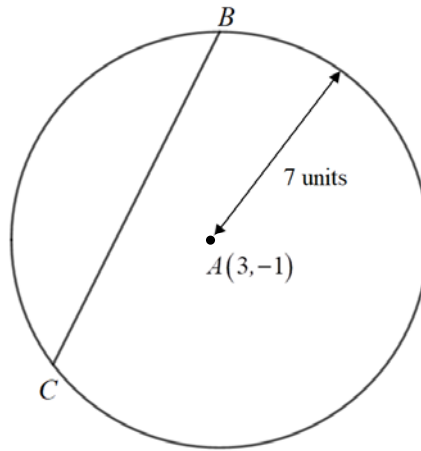
- 1 (i) Express $\frac{3-2x}{x^2-4} - \frac{2}{2-x}$ as a single fraction in its simplest form. [3]
- (ii) Hence, solve $\frac{3-2x}{x^2-4} - \frac{2}{2-x} = \frac{x^2-2x+4}{x^3+8}$. [3]
- 2 (a) Without using a calculator, given the equation $2^{x+1} = 5^{x-1}$,
- (i) find the value of $\left(\frac{2}{5}\right)^x$. [2]
- (ii) Hence, solve for x , leaving your answer in exact form. [2]
- (b) Solve the equation $x + \sqrt{x+1} = 5$. [4]
- (c) A rectangle has length $(5 - \sqrt{12})$ cm and the area is $(8 + 2\sqrt{3})$ cm². Find the breadth of the rectangle in the form $(a + b\sqrt{3})$ cm, where a and b are integers. [4]
- 3 (a) The equation $x^2 - 3kx + k(1-k) = 0$ has two non-zero roots. Find the range of values of k . [3]
- (b) The function g is defined by $g: x \rightarrow 3 + \frac{4}{x+2}$, $x \neq -2$.
- (i) Find g^{-1} . [3]
- (ii) Hence, state the domain and range of g^{-1} . [2]
- 4 The polynomial $P(y) = 3y^3 + 2y^2 - 7y + 2$.
- (i) Show that $y = 1$ is a root of $P(y) = 0$. [1]
- (ii) Factorise $3y^3 + 2y^2 - 7y + 2$ completely. [3]
- (iii) Hence, solve $24y^6 + 8y^4 = 14y^2 - 2$, leaving your answers in exact form. [2]
- 5 (i) Show that $\log_x(2x-1) \times \log_6 x = \log_6(2x-1)$. [1]
- (ii) Hence, solve $\log_6 x + \log_x(2x-1) \times \log_6 x = 1$. [3]

[Turn Over

- 6 A curve has the equation $y = \frac{1}{2}x^2 - x - 4$.
- (i) Using completing the square method, show that the minimum point on the curve has coordinates $\left(1, -4\frac{1}{2}\right)$. [3]
- (ii) Find the coordinates of the points at which the curve intersects the x -axis. [2]
- (iii) Hence, sketch $y = \left|\frac{1}{2}x^2 - x - 4\right|$ for $-2 \leq x \leq 6$, indicating clearly the x -intercepts, y -intercept and turning point. [3]
- (iv) Write down the range of values of k such that $\left|\frac{1}{2}x^2 - x - 4\right| = k$ has 3 solutions. [1]

- 7 (i) State precisely 2 successive geometrical transformations which would transform the graph of $y = e^{-x}$ into the graph of $y = -2e^{-x}$. [2]
- (ii) Hence, sketch the graph of $y = -2e^{-x}$, showing clearly the x -intercept, y -intercept and asymptote if any. [3]

- 8 The diagram shows a circle with centre $A(3, -1)$ and radius 7 units.



- (i) Given that the equation of the circle is $x^2 + y^2 + 2ax + 2by + c = 0$, find the value of a , of b and of c . [4]

The points B and C lie on the circle such that AB is parallel to the y -axis and the line BC passes through the origin.

- (ii) Write down the coordinates of B . [1]
- (iii) Find the coordinates of C . [4]

- 9 (a) Given that $A = \begin{pmatrix} 8 & 2 \\ 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix}$, find matrix C such that
- $$2C = 2B - 3A .$$
- [3]

- (b) Tea D'light sells beverages at Orchard and Ang Mo Kio outlets. The number of cups of milk tea, white coffee and green tea sold at the two outlets on the first day of reopening in Phase 2 on 19th June was shown in the table below.

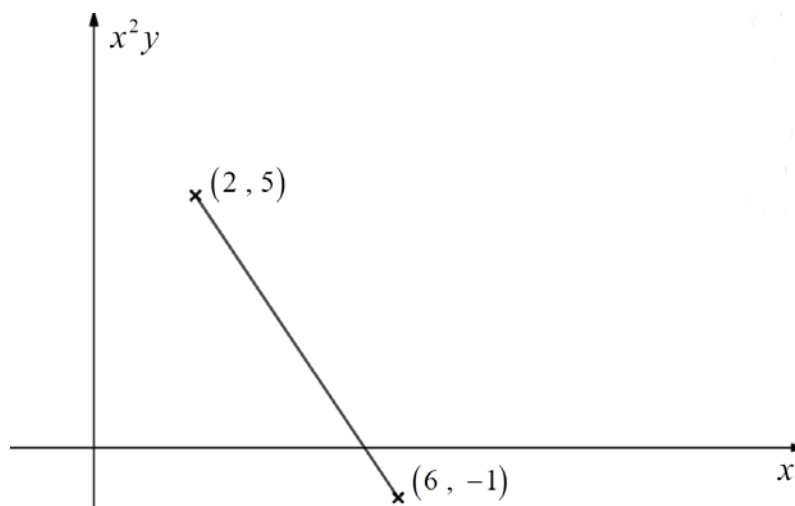
	Orchard	Ang Mo Kio
Milk Tea	400	450
White Coffee	250	200
Green Tea	50	140

- (i) The information in the table can be represented by a 2×3 matrix Q .
Write down the matrix Q .
- [1]

The profit made from selling milk tea, white coffee and green tea are \$3.00, \$1.80 and \$2.00 respectively.

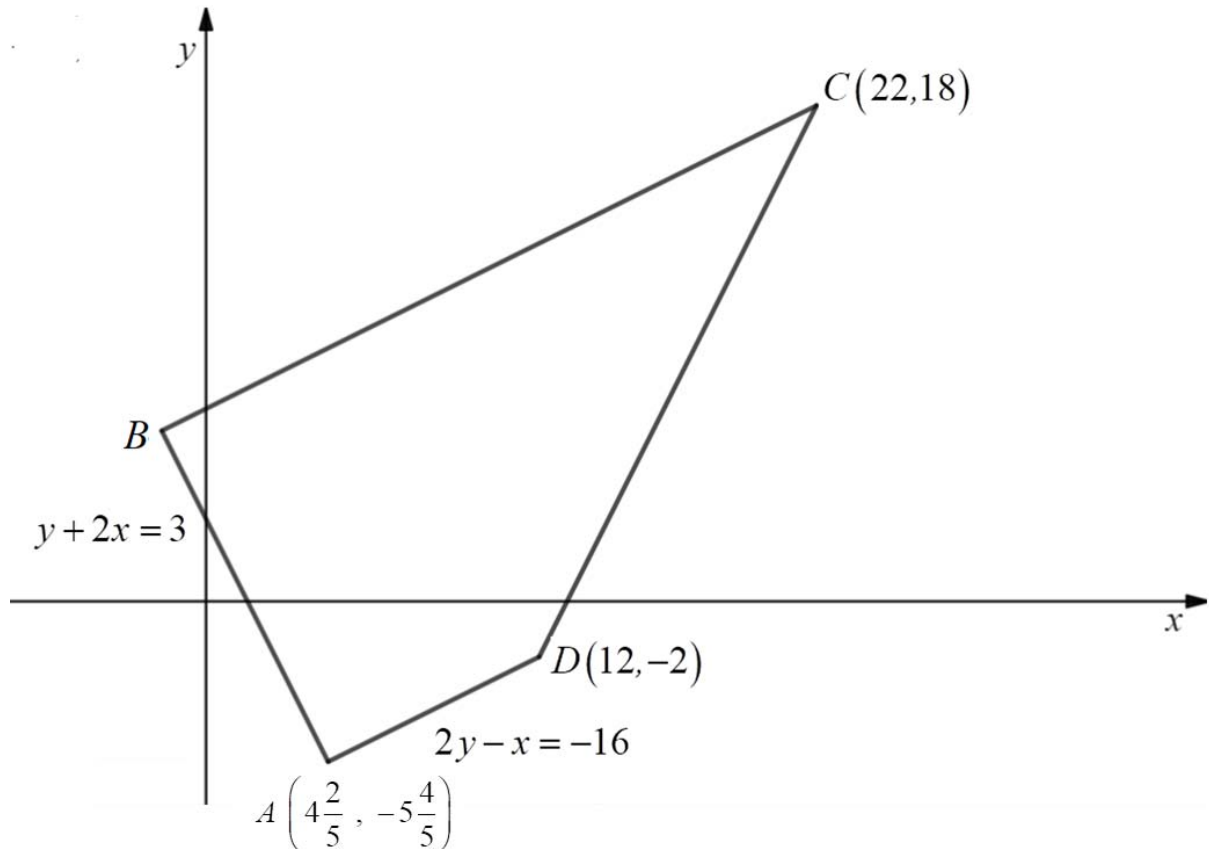
- (ii) Write down a column matrix P to represent the above information. [1]
 (iii) Given that $V = QP$, find matrix V . [2]
 (iv) Using matrix multiplication, find the total amount of profit made by Tea D'light on the first day of reopening in Phase 2. [2]

- 10 The figure shows part of a straight line graph obtained by plotting x^2y against x .
Express y in terms of x .
- [3]



[Turn Over]

11 Solution to this part of the question by accurate drawing will not be accepted.



The diagram shows a trapezium $ABCD$ with $A\left(4\frac{2}{5}, -5\frac{4}{5}\right)$, $C(22, 18)$ and $D(12, -2)$.

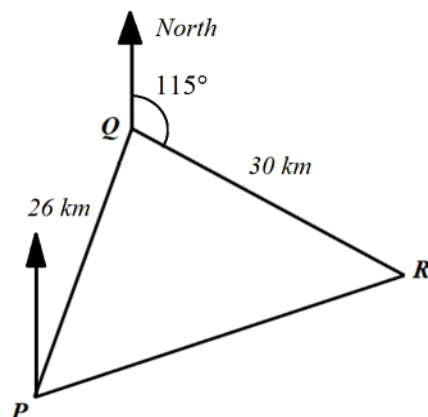
The equation of AB is $y + 2x = 3$ and the equation of AD is $2y - x = -16$.

- (i) Find the coordinates of B . [4]
- (ii) Show that $\angle ABC = 90^\circ$. [2]
- (iii) Given that $ABHD$ is a parallelogram, find the coordinates of H . [3]

Point $E(-16, 35)$ lies on AB produced.

- (iv) Find the value of $\left(\frac{\text{area of triangle } AEC}{\text{area of triangle } ABC}\right)$. [3]

12



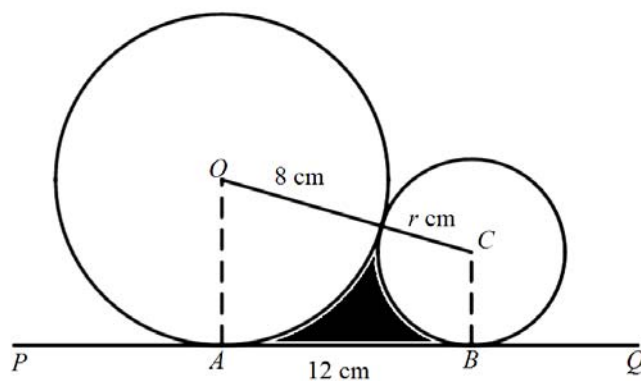
In the diagram P , Q and R represent three small islands.

Island P is 26 km from island Q on a bearing of 200° .

Island R is 30 km from island Q and on a bearing of 115° .

- (i) Calculate the distance of island R from island P . [2]
- (ii) Calculate the bearing of island R from island P . [3]
- (iii) A boat leaves island P and travels directly to island Q .
Find the shortest distance of the boat from island R . [2]

13



The diagram shows two circles, centres O and C with radii 8 cm and r cm respectively, which touch each other externally. $PABQ$ is a common tangent to both circles. Given that $AB = 12$ cm,

- (i) show that $r = 4.5$, [2]
- (ii) find $\angle AOC$ and $\angle OCB$ in radians, [4]
- (iii) find area of the shaded region. [4]

End of Paper