1 It is given that $f(x) = 2x^3 + ax^2 + x + b$, where a and b are constants.

(a) Find the value of a and of b for which $2x^2 + 3x - 2$ is a factor of $f(x)$. [4]

(b) Find the remainder when $f(x)$ is divided by $x^2 - 2$. [2]

2 Do not use a calculator in answering this question.

It is given that $4+x=\sqrt{3}(x-1)$.

Find x in the form $\frac{p+q\sqrt{3}}{2}$, where p and q are integers. [4]

3 Express $\frac{2x^2-3x-1}{(x-3)(x+2)}$ in partial fractions. [5]

4 A curve has equation $y = x^{-2}(2x-3)^5$, where $x \neq 0$.

(a) Express $\frac{dy}{dx}$ in the form $x^{-2}(2x-3)^4(ax+b)$, where a and b are integers to be *found*. [4]

(b) State the range of values of x for which y is a decreasing function. [2]

5 At all points on a curve, $\frac{dy}{dx} = 4x - \frac{5}{x^2} - 2$.

The curve passes through the point $P(1, 4)$. Find the equation of the curve. [4]

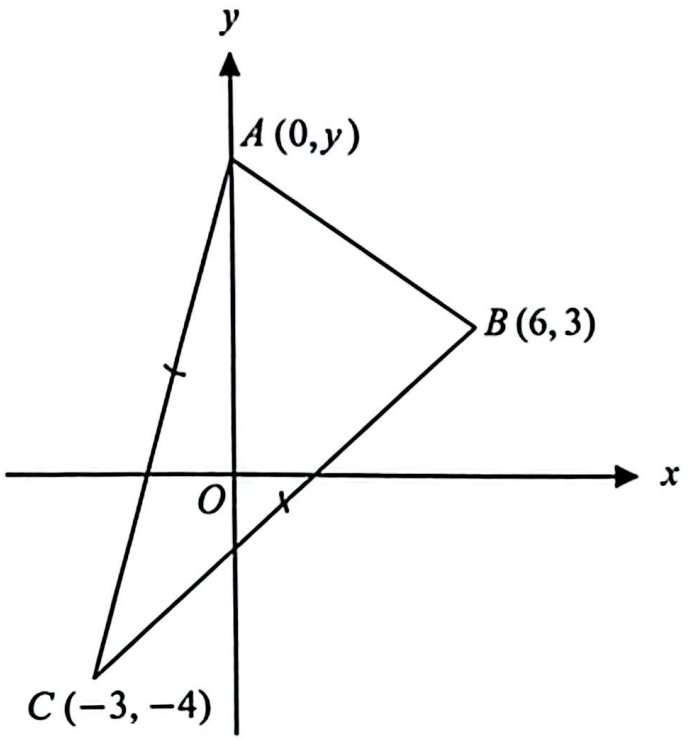
6 (a) Show that $\dfrac{\sec^2 A}{2-\sec^2 A} = \sec 2A$. [3]

(b) Hence, find the exact value of $\dfrac{\sec^2 15°}{2-\sec^2 15°}$. [2]

7 (a) Differentiate $\sqrt{2x^2-1}$ with respect to x. [2]

(b) Hence, evaluate $\int_1^2 \frac{3x}{\sqrt{2x^2-1}}\,dx$. [4]

8 Solutions to this question by accurate drawing will not be accepted.



In the diagram, ABC is an isosceles triangle in which $AC = BC$ and the coordinates of B and C are (6, 3) and (−3, −4) respectively. A is a point on the y-axis.

(a) Find the coordinates of A. **[2]**

(b) Find the equation of the perpendicular bisector of AB. [3]

(c) Find the area of triangle ABC. [2]

10 Do not use a calculator in answering this question.

(a) Given that $a^2 - b^2 = (a-b)(a^2 + ab + b^2)$, factorise $8x^3 - c^3$. **[2]**

(b) Hence, prove that $x^3 - \frac{c^3}{8}$ has only one real root. **[3]**

11 Find the range of values of k for which $kx^2 + 8x + k \leq 6$ for all real values of x. [5]

9 The volume of a sphere is given by $\frac{4}{3}\pi r^3$,

A spherical balloon has a radius of r cm.

It is being inflated at a rate of 35 cm^3/s.

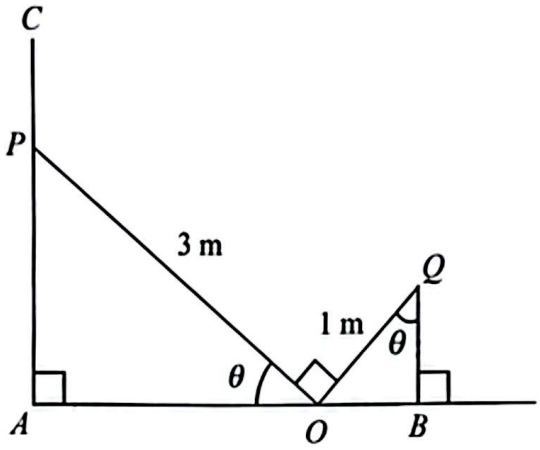
Find the rate at which the radius is increasing at the instant when the volume of the balloon is 100 cm^3. [5]

12 (a) Express, in terms of π, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$. [1]

(b) Express, in terms of π, the principal value of $\tan^{-1}(-1)$. [1]

(c) State the value of x for which $\sin^{-1}\left(-\frac{1}{4}\right)$ is defined. [1]

13



An L-shaped bar POQ with $OP = 3\,\text{m}$, $OQ = 1\,\text{m}$ and angle $POQ = 90°$, is placed against a vertical wall AC as shown in the diagram.

O is the point of the bar in contact with the horizontal ground. B is the point on the ground which is vertically below Q.

OP is inclined at an angle θ with OA.

(a) Find the length AB in terms of θ. [2]

(b) Express AB in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(c) Find the maximum distance between A and B, and the corresponding value of θ. [2]

(d) Find the value of θ when $AB = 2$ m. [2]

End of Paper