RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial 6B: Applications of Differentiation

Section A (Basic Questions)

1 **9740/2009/02/Q1 (ii), (iii)**

A curve C has parametric equations $x = t^2 + 4t$, $y = t^3 + t^2$.

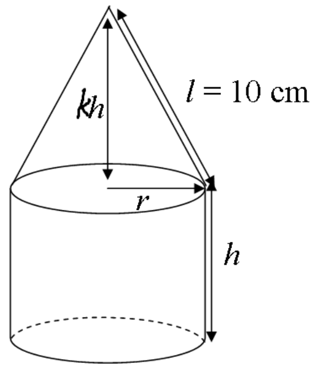
The tangent to the curve at the point P where $t = 2$ is denoted by l.

(i) Find the cartesian equation of l. **[3]**

(ii) The tangent l meets C again at the point Q. Use a non-calculator method to find the coordinates of Q. **[4]**

(i)	$x = t^2 + 4t$, $\quad y = t^3 + t^2$ $\dfrac{dx}{dt} = 2t + 4 \quad \dfrac{dy}{dt} = 3t^2 + 2t$ $\dfrac{dy}{dx} = \dfrac{3t^2 + 2t}{2t + 4}$ At $t = 2$, $\dfrac{dy}{dx} = \dfrac{3(2)^2 + 2(2)}{2(2) + 4} = \dfrac{16}{8} = 2$ $x = 2^2 + 4(2) = 12$, $y = 2^3 + 2^2 = 12$ Equation of tangent to the curve at $t = 2$, is $y - 12 = 2(x - 12)$ $y = 2x - 12$
(ii)	When the tangent l meets C again at the point Q, we have $y = 2x - 12$ ------------------------(1) and $x = t^2 + 4t$, $y = t^3 + t^2$ ----------(2) Substitute (2) into (1) : $t^3 + t^2 = 2(t^2 + 4t) - 12$ $t^3 - t^2 - 8t + 12 = 0$ $(t-2)(t^2 + t - 6) = 0$ $(t-2)(t-2)(t+3) = 0$ $t = 2$ (n.a.) or $t = -3$ When $t = -3$, $x = (-3)^2 + 4(-3) = -3$ and $y = (-3)^3 + (-3)^2 = -18$ i.e. coordinates of Q are (–3, –18).

2 JPJC Promo 9758/2021/Q8(a)



A closed container is constructed using a sheet of metal with area 100π cm^2. The container comprises 2 shapes, a cone and a cylinder. The slant height, l, of the cone is 10 cm. Given that the cylinder has height h, and radius r, and the height of the cone is kh, where k is a positive constant.

(i) show that $h = \dfrac{100 - r^2 - 10r}{2r}$, **[1]**

(ii) use differentiation to show that the exact maximum volume of the container is given that $V = \left(\dfrac{1}{3}k + 1\right)\dfrac{2500\pi}{27}$ cm^3, proving that it is a maximum. **[6]**

[Volume of Cone $= \dfrac{1}{3}\pi r^2 h$, Curved Surface Area of Cone $= \pi r l$]

(i)	$\pi r^2 + 2\pi r h + \pi r(10) = 100\pi$ $h = \dfrac{100 - 10r - r^2}{2r}$ (Shown).
(ii)	$V = \dfrac{1}{3}\pi r^2 kh + \pi r^2 h = \pi\left(\dfrac{1}{3}k + 1\right) r^2 \left(\dfrac{100 - r^2 - 10r}{2r}\right)$ $V = \dfrac{\pi}{2}\left(\dfrac{1}{3}k + 1\right)\left(100r - r^3 - 10r^2\right)$ $\dfrac{dV}{dr} = \dfrac{\pi}{2}\left(\dfrac{1}{3}k + 1\right)\left(100 - 3r^2 - 20r\right)$ $\dfrac{dV}{dr} = 0$ $r = \dfrac{20 \pm \sqrt{400 + 4(3)(100)}}{2(-3)} = \dfrac{20 \pm 40}{-6} = -10$(NA as $r > 0$) or $\dfrac{10}{3}$

To verify $r=\dfrac{10}{3}$ gives maximum V:

<u>Method 1: 2nd derivative test</u>

$$\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=\pi\left(\frac{1}{3}k+1\right)(-6r-20)<0 \text{ since } r>0$$

V is max when $r=\dfrac{10}{3}$

<u>Method 2: 1st derivative test</u>

$$\frac{\mathrm{d}V}{\mathrm{d}r}=\frac{\pi}{2}\left(\frac{1}{3}k+1\right)\left(100-3r^2-20r\right)=\frac{3\pi}{2}\left(\frac{1}{3}k+1\right)\left(\frac{10}{3}-r\right)(10+r)$$

Since k>0 and r>0, $\left(\frac{1}{3}k+1\right)>0$ and $(10+r)>0$

r	$\left(\frac{10}{3}\right)^-$	$\frac{10}{3}$	$\left(\frac{10}{3}\right)^+$
$\left(\frac{10}{3}-r\right)$	>0	0	<0
$\frac{\mathrm{d}V}{\mathrm{d}r}$	>0	0	<0

V is max when $r=\dfrac{10}{3}$

Max $V=\dfrac{\pi}{2}\left(\dfrac{1}{3}k+1\right)\left(\dfrac{1000}{3}-\dfrac{1000}{27}-\dfrac{1000}{9}\right)=\dfrac{2500}{27}\pi\left(\dfrac{1}{3}k+1\right)$ (Shown).

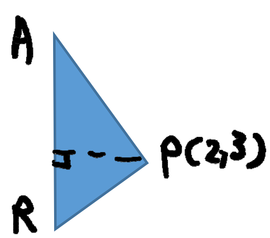
3 ASRJC Promo 9758/2021/Q4

A curve C has equation $y^3 - 2xy^2 + 3x^2 - 3 = 0$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **[2]**

(ii) Find the equation of the normal to the curve at the point $P(2, 3)$. **[2]**

(iii) Given that C meets the y-axis at the point R and the normal in **(ii)** meets the y-axis at the point A, find the area of triangle APR in the form $a - \sqrt[3]{b}$, where a and b are integers to be determined. **[3]**

(i)	$y^3 - 2xy^2 + 3x^2 - 3 = 0$ Differentiating with respect to x, $3y^2 \frac{\mathrm{d}y}{\mathrm{d}x} - (2y^2 + 4xy\frac{\mathrm{d}y}{\mathrm{d}x}) + 6x = 0$ $(3y^2 - 4xy)\frac{\mathrm{d}y}{\mathrm{d}x} = 2y^2 - 6x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y^2 - 6x}{3y^2 - 4xy}$
(ii)	Gradient of tangent to curve at $P(2, 3) = \frac{2(3)^2 - 6(2)}{3(3)^2 - 4(2)(3)} = 2$ Gradient of normal to curve at $P = -\frac{1}{2}$ Equation of normal at P: $y - 3 = -\frac{1}{2}(x - 2)$ $\Rightarrow y = -\frac{1}{2}x + 4$
(iii)	When $x = 0$, normal cuts y-axis at $A(0, 4)$. C: $y^3 - 2xy^2 + 3x^2 - 3 = 0$ When $x = 0$, $y^3 = 3 \Rightarrow y = \sqrt[3]{3}$ C meets y-axis at $R(0, \sqrt[3]{3})$.  Area of triangle APR $= \frac{1}{2} \times 2 \times (4 - \sqrt[3]{3})$ $= 4 - \sqrt[3]{3}$, where $a = 4$, $b = 3$.