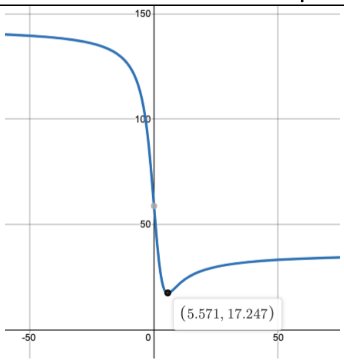
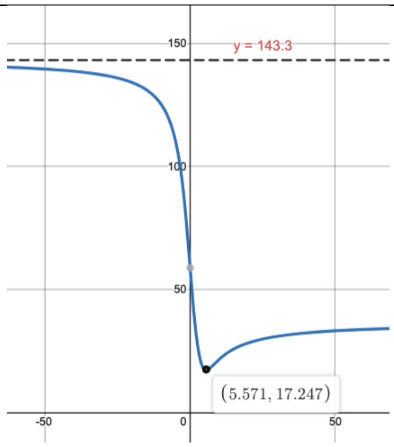
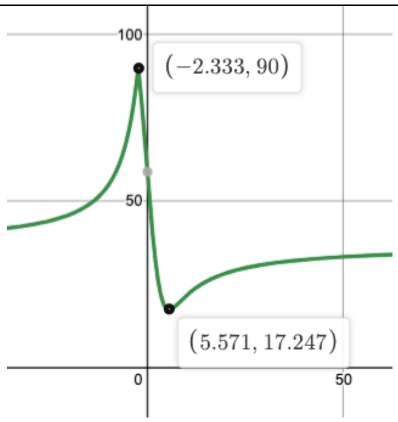
1a	Using scalar product, $$\begin{pmatrix}1\\-2\\3\end{pmatrix}\cdot\begin{pmatrix}3\\-2\\m\end{pmatrix}=\left\|\begin{pmatrix}1\\-2\\3\end{pmatrix}\right\|\left\|\begin{pmatrix}3\\-2\\m\end{pmatrix}\right\|\cos p$$ $$1(3)+(-2)(-2)+3(m)=\sqrt{1^2+(-2)^2+3^2}\sqrt{3^2+(-2)^2+m^2}\cos p$$ $$7+3m=\sqrt{14}\sqrt{13+m^2}\cos p$$ $$\cos p=\frac{7+3m}{\sqrt{14}\sqrt{13+m^2}}$$ $$p=\arccos\left(\frac{7+3m}{\sqrt{14}\sqrt{13+m^2}}\right)$$
1bi	Negative
1bii	$$\cos 150^{\circ}=\frac{7+3m}{\sqrt{14}\sqrt{13+m^2}}$$ $$-\frac{\sqrt{3}}{2}=\frac{7+3m}{\sqrt{14}\sqrt{13+m^2}}$$ $$\frac{3}{4}=\frac{(7+3m)^2}{14(13+m^2)}$$ $$\vdots$$ $$3m^2-84m+175=0$$
1biii	From (ii), m=2.27, 25.7. But when m is positive, $\frac{7+3m}{\sqrt{14}\sqrt{13+m^2}}$ is positive, implying that p is acute. Hence, there will be no m such that p is obtuse.
1c	 Smallest angle = 17.2° m = 5.57
1di	$$\lim_{m\to\infty}\frac{7+3m}{\sqrt{14}\sqrt{13+m^2}}=\lim_{m\to\infty}\frac{7/m+3}{\sqrt{14}\sqrt{13/m^2+1}}$$ $$=\frac{0+3}{\sqrt{14}\sqrt{0+1}}=\frac{3}{\sqrt{14}}$$

1dii	$\lim_{m\to\infty} p = \lim_{m\to\infty}\left[\arccos\left(g(m)\right)\right]$ $= \arccos\left[\lim_{m\to\infty} g(m)\right]$ $= \arccos\left[\frac{3}{\sqrt{14}}\right]$ $= 36.7°$
1diii	$\lim_{m\to-\infty} p = 180° - 36.7° = 143.3°$ or $\arccos\left(-\frac{3}{\sqrt{14}}\right) = 143.3°$
1e	 $17.2 \le p < 143.3$
1f	$\left\lvert \begin{pmatrix}1\\-2\\3\end{pmatrix} \times \begin{pmatrix}3\\-2\\m\end{pmatrix} \right\rvert = \left\lvert \begin{pmatrix}1\\-2\\3\end{pmatrix} \right\rvert \left\lvert \begin{pmatrix}3\\-2\\m\end{pmatrix} \right\rvert \sin p$ $\left\lvert \begin{pmatrix}-2m+6\\-(m-9)\\4\end{pmatrix} \right\rvert = \sqrt{14}\sqrt{13+m^2}\sin p$ $\sqrt{(-2m+6)^2+(9-m)^2+4^2} = \sqrt{14}\sqrt{13+m^2}\sin p$ $\vdots$ $\sin p = \sqrt{\frac{5m^2-42m+133}{14(13+m^2)}}$ $p = \arcsin\left(\sqrt{\frac{5m^2-42m+133}{14(13+m^2)}}\right)$

1g	 $17.2 \le p \le 90$
1h	Part (e) = $17.2° \le p < 143.3°$ Part (g) = $17.2° \le p \le 90°$ <u>Functions reason</u> Range of arccos(x) is $[0°, 180°]$ while range of arcsin(x) is $[-90°, 90°]$. Hence, the upper bound for part (e) is different from part (g). OR <u>Vectors reason</u> Defn of angle between 2 vectors allows for angles to lie in quadrants 1,2. Hence, scalar product finds the <u>**true**</u> angle between 2 vectors. While cross product defn is only for acute angle between 2 vectors.

2a	$F_0 = 3, F_1 = 5, F_2 = 17, F_3 = 257$
2b	LHS = F_0 = 3 RHS = F_1 – 2 = 5 – 2 = 3 So, LHS = RHS
2c	n = 2 means $F_0 F_1 = F_2 - 2$. So, LHS = 3(5) = 15 = 17 – 2 = RHS n = 3 means $F_0 F_1 F_2 = F_3 - 2$ So, LHS = 3(5)(17) = 255 = 257 – 2 = RHS

2d	Let P(n) be the statement $F_0F_1F_2 \ldots F_{n-1} = F_n - 2$ where $F_n = 2^{\left(2^n\right)} + 1, n \in \mathbb{Z}^+$ When n = 1, LHS = F_0 = 3, RHS = F_1 – 2 = 5 – 2 = 3 So, LHS = RHS, and P(1) is true. Assume P(k) is true for some $k \geq 1, k \in \mathbb{Z}$. Ie $F_0F_1F_2 \ldots F_{k-1} = F_k - 2$ Consider n = k+1, LHS $= F_0F_1F_2 \ldots F_{k-1}F_k$ $= \left(F_k - 2\right)F_k$ $= \left[2^{\left(2^k\right)} + 1 - 2\right]\left[2^{\left(2^k\right)} + 1\right]$ $= \left[2^{\left(2^k\right)}\right]\left[2^{\left(2^k\right)}\right] - 1$ $= 2^{\left(2^k\right)+\left(2^k\right)} - 1$ $= 2^{\left(2^{k+1}\right)} - 1$ $= F_{k+1} - 2$ = RHS So, P(k+1) is true. Since P(1) is ture, and P(k) true imples P(k+1) true, by MI, P(n) is true for all positive integers n.
2ei	Since $F_l = ap$, then $F_0F_1 \ldots F_l \ldots F_{m-1} = F_0F_1 \ldots \left(ap\right) \ldots F_{m-1}$ Hence, p is a factor of $F_0F_1 \ldots F_l \ldots F_{m-1}$.
2eii	Since $F_0F_1 \ldots F_l \ldots F_{m-1} = F_m - 2$, p is also a factor of F_m – 2.
2f	p is a factor of both F_m and F_m – 2.
2gi	Part (f) is impossible as F_m and F_m – 2 are consecutive odd numbers. So no numbers other than 1 can be a common factor for both F_m and F_m – 2.
2gii	By proof by contradiction, p cannot be a common factor for two Fermat numbers. OR no two Fermat numbers have a common factor greater than 1.
2h	Factors for F_1 = 5 are 1, 5 Factors for F_2 = 17 are 1, 17.
2i	1
2j	Each distinct prime factor can only appear in one and only one Fermat number.
2k	Since there are infinitely many Fermat numbers, there must be infinitely many prime numbers.