Tutorial 19 Quantum Physics Suggested Solutions

D1 **(a)** Since $P = IA = \dfrac{dN_p}{dt} \times \dfrac{hc}{\lambda}$, the rate of incident photons

$$\frac{dN_p}{dt} = \frac{IA\lambda}{hc} = \frac{(210)(12\times10^{-6})(254\times10^{-9})}{(6.63\times10^{-34})(3.00\times10^{8})} = 3.22\times10^{15}\ \text{s}^{-1}$$

(b) Since $i_0 = \dfrac{dN_e}{dt} \times e$, the rate of emission of photoelectrons

$$\frac{dN_e}{dt} = \frac{i_0}{e} = \frac{4.8\times10^{-10}}{1.60\times10^{-19}} = 3.00\times10^{9}\ \text{s}^{-1}$$

(c) **(i)** Quantum yield

$$Q = \frac{dN_e/dt}{dN_p/dt} = \frac{3.00\times10^{9}}{3.22\times10^{15}} = 9.3\times10^{-7}$$

(ii) Any two of the following:

- Photons are **mostly reflected** off the metal surface and so are not involved in photoemission.
- Besides energy, a photon imparts momentum to the electron. These electrons are generally "**back-scattered**" (away from surface). To escape from the metal, these electrons must acquire sufficient energy by colliding with other electrons. This process has a **very low probability**.
- After acquiring energy from a photon, an electron near the surface of the metal will **lose energy** through **multiple collisions** with other electrons or ions as it migrates towards the surface of the metal.

(d) The wavelength of 313 nm is **greater** than the **threshold wavelength**. Hence, photons of such radiation have insufficient energy to overcome the work function of the metal.

D2 **(a)** Energy of photon

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{2.55\times10^{-7}} = 7.8\times10^{-19}\ \text{J}$$

(b) **(i)** The photoelectric current depends on the **rate of emission of photoelectrons**, which in turn depends on the **intensity** of the incident radiation. Increasing V accelerates the electrons but **does not affect** the rate of emission of photoelectrons.

Hence, current reaches a maximum value no matter how large V is made.

(ii) Electrons are emitted with a **range of kinetic energy** (up to $K.E._{max}$).

Some energetic electrons still have **sufficient K.E.** to overcome the retarding p.d. to reach the gauze.

(c) **(i)** The rate of incidence photon **increases** with increasing intensity of light. Increasing rate of incident photon **increases** the rate of emission of photoelectrons (assuming constant quantum yield) and hence maximum photoelectric current increases.

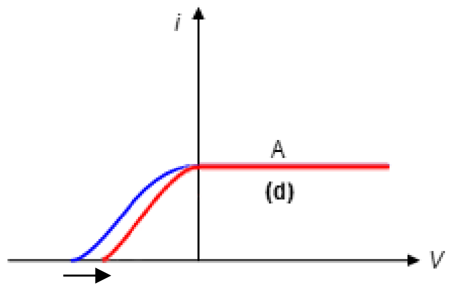
(ii) The stopping potential V_s is the minimum value of the potential difference between the gauze (collector) and plate (emitter) needed to prevent the most energetic photoelectrons from reaching the gauze.

$$eV_s = \text{K.E.}_{\text{max}}$$

K.E. of most energetic photoelectrons is **dependent only on the frequency of the light** and the **work function** of the surface but is **independent of the intensity** of light.

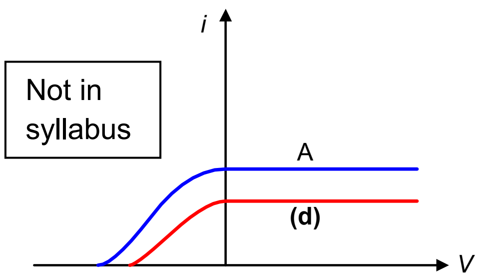
So the stopping potential remains unchanged.

(d) Since nickel has a greater work function, **K.E. of most energetic photoelectrons is lower**. Hence a **lower stopping potential** is needed to prevent the most energetic photoelectrons from reaching the gauze.



Since the intensity and frequency of the light did not change, the saturated photocurrent remains constant.

(Not in syllabus: Typically, **quantum yield decreases with increasing work function** as a smaller fraction of the incident photons is capable of ejecting electrons from the nickel surface. As a result, there is a **lower rate of emission of photoelectron** and hence a lower maximum photoelectric current.)



D3 **(a)** $V = -1.0 \text{ V}$

(b) Maximum K.E.

$$\text{K.E.}_{\text{max}} = eV_S = 1.0 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

(c) Work function

$$\Phi = \frac{hc}{\lambda} - \text{K.E.}_{\text{max}}$$

$$= \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{365 \times 10^{-9}} \times 1.60 \times 10^{-19} = 3.85 \times 10^{-19} \text{ J}$$

(d) Rate of emission of photoelectrons

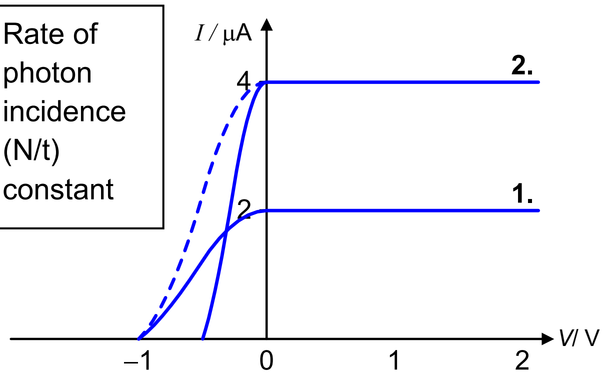
$$\frac{dN_e}{dt} = \frac{i_0}{e} = \frac{4.0 \times 10^{-6}}{1.60 \times 10^{-19}} = 2.5 \times 10^{13} \text{ s}^{-1}$$

Hence rate of incidence of photons

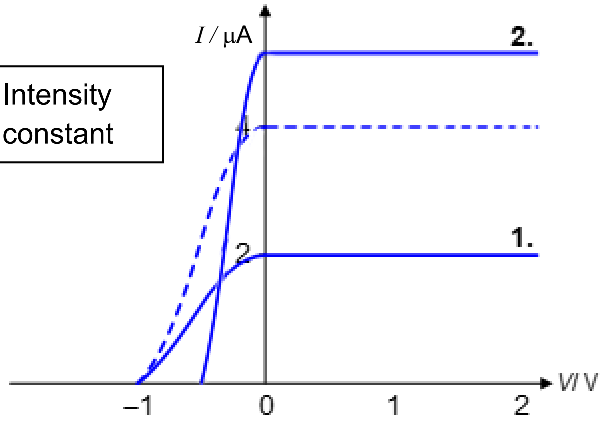
$$\frac{dN_p}{dt} = 3 \times \frac{dN_e}{dt} = 7.5 \times 10^{13} \text{ s}^{-1}$$

(e)

Rate of photon incidence (N/t) constant



Intensity constant



Recall: Intensity $= \dfrac{Nhf}{tA}$

If frequency is decreased, and rate of photon incidence (N/t) stays constant, then intensity will decrease. But because N/t is constant, the rate of electron emission is also constant, hence the photocurrent stays constant.

If frequency is decreased, and intensity stays constant, the rate of photon incidence (N/t) must increase. This leads to an increase in rate of electron emission, and hence higher photocurrent.

This question here (a very old A-level question) is vague with what quantity is being kept constant. **Future questions should be more specific.**

Note: if you face such vague questions in exams, write down your assumption(s) and proceed accordingly.

D4 **(a)** **(i)** Threshold frequency

$$f_0 = 6.4 \times 10^{14} \text{ Hz}$$

(ii) From the graph,

$$\text{K.E.}_{\text{max}} = 4.8 \text{ eV} = 7.7 \times 10^{-19} \text{ J}$$

(b) Einstein's photoelectric equation

$$\text{K.E.}_{\text{max}} = hf - hf_0$$

Rearranging,

$$h = \frac{\text{K.E.}_{\text{max}}}{f - f_0} = \frac{7.68 \times 10^{-19}}{(18.0 - 6.4) \times 10^{14}} = 6.62 \times 10^{-34} \text{ J s}$$

D5 **(a)** **(i)** Energy of photon

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{4.5 \times 10^{-7}} = 4.42 \times 10^{-19} \text{ J}$$

(ii) Since $P = IA = \dfrac{dN_p}{dt} \times E$, the rate of incident photons

$$\frac{dN_p}{dt} = \frac{IA}{E} = \frac{(700)(1 \times 10^{-4})}{4.42 \times 10^{-19}} = 1.58 \times 10^{17} \text{ s}^{-1}$$

(b) **(i)** de Broglie relation

$$\lambda = \frac{h}{p}$$

(ii) Momentum of photon

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{4.5 \times 10^{-7}} = 1.47 \times 10^{-27}\ \text{kg m s}^{-1}$$

(c) **(i)** Change in momentum of each photon on reflection is $-2p$.

Rate of change in momentum

$$\frac{dp}{dt} = \frac{dN_p}{dt} \times (-2p) = -(1.58 \times 10^{17})(2)(1.47 \times 10^{-27})$$

$$= -4.66 \times 10^{-10}\ \text{N}$$

Therefore the change in momentum in 1 second

$$\Delta p = -4.66 \times 10^{-10}\ \text{N s}$$

(ii) Photons experienced a force when they are reflected off the surface. By Newton's 3rd law, the photons exert an **equal but opposite** force on the surface. The **force per unit area** exerted on the surface is the radiation pressure.

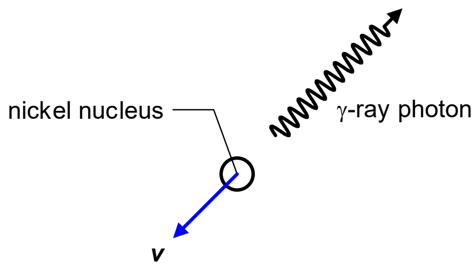
D6 **(a)** **(i)** Wavelength of γ-ray photon is

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{(1.17 \times 10^{6})(1.60 \times 10^{-19})} = 1.06 \times 10^{-12}\ \text{m}$$

(ii) Using de Broglie equation, the momentum of the photon is

$$p_\gamma = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{1.06 \times 10^{-12}} = 6.24 \times 10^{-22}\ \text{kg m s}^{-1}$$

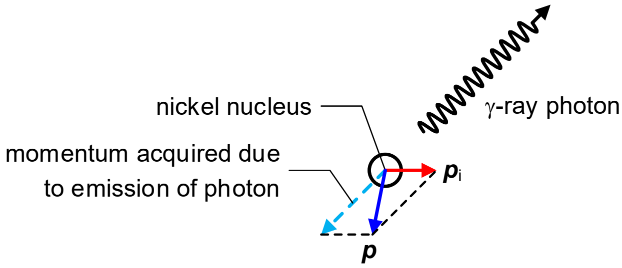
(b) **(i)**



(ii) By the principle of conservation of momentum,

$$mv = p_\gamma \text{ or } v = \frac{p_\gamma}{m} = \frac{6.24 \times 10^{-22}}{9.95 \times 10^{-26}} = 6.27 \times 10^{3}\ \text{m s}^{-1}$$

(c) The angle between the final direction of motion of the nucleus and that of the emitted photon will only be either zero or $180°$ if the second nickel nucleus emits photon parallel to its momentum.



If photon is emitted in other directions, then the initial momentum of the nucleus must be added to the final momentum of the nucleus determined in (b). Hence the angle between **zero** and $180°$.

D7 **(a)** **(i)** The lowest and most stable energy state. $n = 1$

(ii) The energy needed to cause an electron in an atom in a lower energy state to transit to a higher energy state.

(iii) The minimum energy needed to completely remove the valence electron of an atom in the ground state. $n = 1$ to $n = \infty$

(b) **(i)** No transition

(ii) $n = 1$ to $n = 2$

(iii) No transition

(iv) $n = 1$ to $n = 3$

(c) **(i)** No transition

(ii) $n = 1$ to $n = 2$

(iii) $n = 1$ to $n = 2$

(iv) $n = 1$ to $n = 2$ or $n = 1$ to $n = 3$

D8 **(a)** **(i)** Level 5 to Level 1

(ii) 10

(b) 4 spectral absorption lines.

(c) **(i)** No transition

(ii) Level 2 to Level 1

(iii) Level 3 to Level 1, Level 3 to Level 2, Level 2 to Level 1

(d) **(i)** Ionisation energy of sodium

(ii) To produce spectral line emission but no free electrons, the bombarding electrons must possess energy from 3.38×10^{-19} J to 8.21×10^{-19} J .
The corresponding accelerating p.d. is $2.11\text{ V} \leq \text{p.d.} < 5.13\text{ V}$.

(e) **(i)** Let $\lambda_{D1} = 589.0$ nm and $\lambda_{D2} = 589.6$ nm. The corresponding energy of the photons are

$$E_{D1} = \frac{hc}{\lambda_{D1}} = 3.3769 \times 10^{-19}\ \text{J};\ E_{D2} = \frac{hc}{\lambda_{D2}} = 3.3735 \times 10^{-19}\ \text{J}$$

So energy difference between the two levels is

$$\Delta E = 0.0034 \times 10^{-19} = 3.4 \times 10^{-22}\ \text{J}$$

(ii) The ground state.

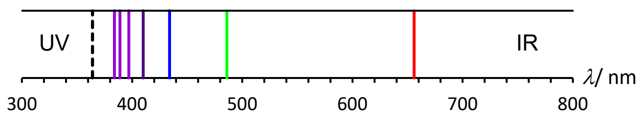
D9 **(a)** When $m = 4$,

$$\lambda_m = \frac{1}{R}\left(\frac{1}{2^2} - \frac{1}{4^2}\right)^{-1} = 486 \times 10^{-9}\ \text{m}$$

(b) Minimum wavelength occurs when $m = \infty$.

$$\lambda_m = \frac{1}{R}\left(\frac{1}{4} - \frac{1}{\infty^2}\right)^{-1} = 365 \times 10^{-9}\ \text{m}$$

(c)



(d) The **probability** that atoms exist in states corresponding to **large *m*** is **very low**. As such, transitions that produce lines with short wavelengths near 364 nm are **far fewer** than those involving lower *m*. So although there are in theory an infinite number of lines towards 364 nm (these would have formed a **wide band** of wavelengths), these **lines are too faint** to be seen and only those of **longer wavelengths are visible**.

D10 **(a)** Momentum of electron

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2.4 \times 10^{-10}} = 2.76 \times 10^{-24}\ \text{N s}$$

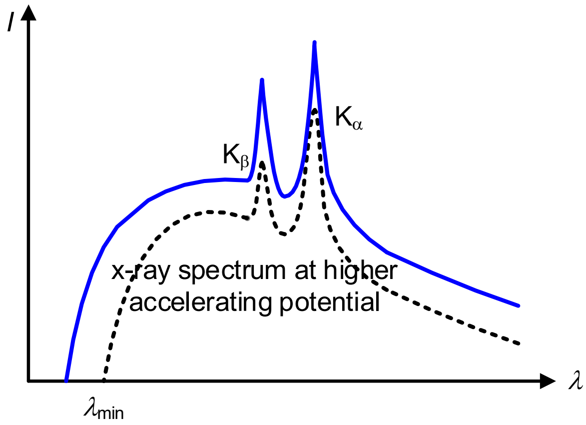
(b) The K.E. of the electron accelerated by a p.d. *V* is

$$\frac{p^2}{2m} = eV$$

$$\Rightarrow \quad V = \frac{p^2}{2me} = \frac{\left(2.76 \times 10^{-24}\right)^2}{(2)(9.11 \times 10^{-31})(1.60 \times 10^{-19})} = 26.1\ \text{V (exact value 26.2 V)}$$

D11 **(a)** When electrons are decelerated in their interactions with nuclei in a metal target, the electrons lose kinetic energy. Due to conservation of energy, the loss of kinetic energy of the electron is equal to the energy of the X-ray. By assuming that an electron that is stopped in a single collision emits an X-ray that has energy that is equal to the maximum kinetic energy of the electrons, and that an X-ray behaves as a photon, the maximum kinetic energy of the electron works out to be the same value as the energy of a photon emitted with the minimum wavelength i.e. $KE_{max} = \dfrac{hc}{\lambda_{min}}$. Hence, this supports the particulate nature of light.

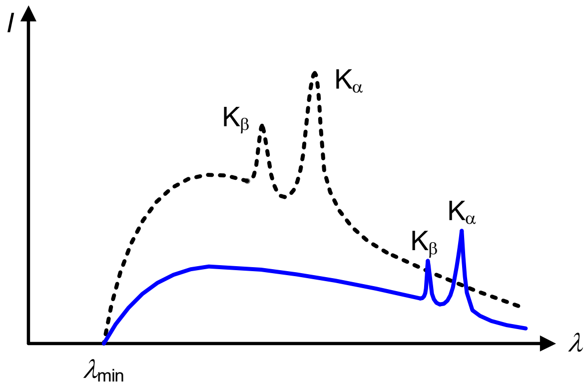
(b) **(i)**



When the accelerating potential is increased to 250 kV,

- the **minimum wavelength** $\lambda_{min} = hc/eV$ decreases with higher V.
- more energetic incident electrons increase the rate of emission of X-ray photons so intensity increases for all wavelengths
- **No change** in **wavelengths** of **characteristic X-rays** but peaks are higher.

(ii)



When a copper target is used,

- different **wavelengths** of the **characteristic X-rays**
- **no change** in **minimum wavelength** λ_{min}
- **lower intensity** for all wavelengths due to lower rate of x-rays photons emission from a copper target. (for your info only)

D12 The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

So for a given energy E, the uncertainty in position is

$$\Delta x \approx \lambda \propto \frac{1}{\sqrt{m}}$$

Since $\Delta x \Delta p \approx h = \text{constant}$,

$$\Delta p \propto \sqrt{m}$$

Therefore

$$\frac{\Delta p_p}{\Delta p_e} \approx \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67 \times 10^{-27}}{9.11 \times 10^{-31}}} = 42.8$$

D13 Using de Broglie relation,

$$p = \frac{h}{\lambda}$$

we can deduced that

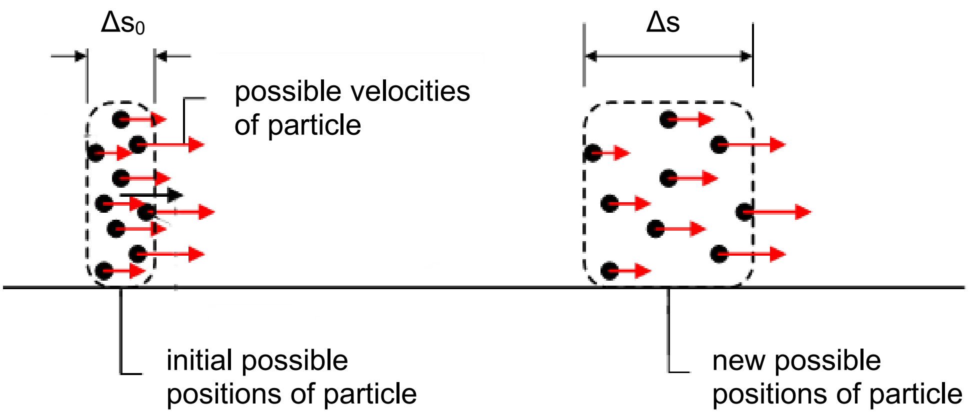
$$\frac{\Delta p}{p} = \frac{\Delta \lambda}{\lambda} \quad \text{or} \quad \Delta p = \frac{\Delta \lambda}{\lambda} \times p = \frac{h \Delta \lambda}{\lambda^2}$$

Hence the uncertainty in the location

$$\Delta x \Delta p \gtrsim h$$

$$\Delta x \gtrsim \frac{h}{\Delta p} = \frac{\lambda^2}{\Delta \lambda} = \frac{\left(300 \times 10^{-9}\right)^2}{\left(300 \times 10^{-9}\right)/10^6} = 0.300 \text{ m}$$

D14



$$\Delta s_0 \Delta p \gtrsim h \quad \Rightarrow \quad \Delta s_0 \left(m \Delta u\right) \gtrsim h \quad \Rightarrow \quad \Delta u \gtrsim \frac{h}{m \Delta s_0}$$

since $s = ut$

$$\Delta s = \Delta u t \approx \frac{h}{m \Delta s_0} \cdot t = \frac{6.63 \times 10^{-34}}{(1.67 \times 10^{-27})(1.5 \times 10^{-11})} \times 1.00 = 2.65 \times 10^{4} \text{ m}$$

C1 **(a)** By having the photon source move toward the metal, the incident photons are Doppler shifted to higher frequencies, and hence, higher energy.

(b) For $v = 0.280c$,

$$f' = f\sqrt{\frac{1+v/c}{1-v/c}} = 9.33\times10^{14}\ \text{Hz}$$

Therefore, work function

$$\Phi = hf_0 = 6.19\times10^{-19}\ \text{J} = 3.87\ \text{eV}$$

(c) For $v = 0.900c$,

$$f' = f\sqrt{\frac{1+v/c}{1-v/c}} = 3.05\times10^{15}\ \text{Hz}$$

So maximum K.E.

$$\text{K.E.}_{\text{max}} = hf - \Phi = 1.40\times10^{-18}\ \text{J} = 8.77\ \text{eV}$$

C2 **(a)** For a classical atom, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2 m_e}$$

Total energy

$$E = -\frac{e^2}{4\pi\varepsilon_0 r} + \tfrac{1}{2}mv^2 = -\frac{e^2}{8\pi\varepsilon_0 r}$$

Taking the derivative of E w.r.t. t,

$$\frac{dE}{dt} = \frac{e^2}{8\pi\varepsilon_0 r^2}\frac{dr}{dt} = -\frac{1}{6\pi\varepsilon_0}\frac{e^2a^2}{c^3} = -\frac{1}{6\pi\varepsilon_0}\frac{e^2}{c^3}\left(\frac{e^2}{4\pi\varepsilon_0 r^2 m_e}\right)^2$$

So

$$\frac{dr}{dt} = -\frac{e^4}{12\pi^2\varepsilon_0^2 r^2 m_e^2 c^3}$$

(b) Rearranging the terms, we have

$$dt = -\frac{12\pi^2\varepsilon_0^2 r^2 m_e^2 c^3}{e^4}dr$$

So time taken is

$$T = \int_{2.00\times10^{-10}}^{0} -\frac{12\pi^2\varepsilon_0^2 r^2 m_e^2 c^3}{e^4}dr = \left[-\frac{12\pi^2\varepsilon_0^2 r^3 m_e^2 c^3}{3e^4}\right]_{2.00\times10^{-10}}^{0} = 8.46\times10^{-10}\ \text{s}$$

C3 **(a)** Let the momentum atom be p, then

$$hf' + \frac{p^2}{2m} = \Delta E$$

(b) By the principle of conservation of momentum,

$$p = \frac{hf'}{c}$$

So

$$\text{K.E.}_{\text{atom}} = \frac{p^2}{2m} = \frac{(hf')^2}{2mc^2}$$

Hence the required ratio is

$$\frac{\text{K.E.}_{\text{atom}}}{hf'} = \frac{hf'}{2mc^2}$$

To get an estimate of this ratio, we can consider using f instead of hf, where f > f' and ΔE = hf (energy of a photon emitted equal to the energy level difference).

$$\frac{hf}{2mc^2} = \frac{\Delta E}{2mc^2} = \frac{(1.9)(1.6\times10^{-19})}{2(1.67\times10^{-27})(3\times10^{8})^2} = 1.0\times10^{-9}$$

Which shows that the ratio is very small. Hence, we can infer that using f', which is smaller than f, it will only make the ratio even smaller. Hence, we can conclude that the effect is not very significant. (In reality, f' is only very slightly smaller than f.)

C4 We first make the following assumptions:

$$r \sim \tfrac{1}{2}\Delta x$$

$$p \sim \Delta p \approx \frac{\hbar}{2\Delta x} \approx \frac{\hbar}{r}$$

The energy of an electron near a proton is

$$U = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r}$$

Replacing *r* with *p*,

$$U = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0\hbar}\cdot p$$

Differentiate w.r.t. *p* and equating to zero to find the minimum energy,

$$\frac{dU}{dp} = \frac{p}{m} - \frac{e^2}{4\pi\varepsilon_0\hbar} = 0$$

Minimum energy occurs when

$$p = \frac{me^2}{4\pi\varepsilon_0\hbar}$$

Therefore, the energy of the electron is

$$U = \frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2} - \frac{me^4}{16\pi^2\varepsilon_0^2\hbar^2} = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2} = -13.6\text{ eV}$$

The radius of the hydrogen atom is

$$r \approx \frac{\hbar}{p} = \frac{4\pi\varepsilon_0\hbar^2}{me^2} = 0.53\times10^{-10}\text{ m}$$

So the hydrogen atom is large because if the electron is confined to a smaller space, its momentum uncertainty would increase, causing its momentum and hence energy to increase.