



Nanyang Girls' High School
SECONDARY 3
Integrated Mathematics

Name: _____ ()

Class: _____ Date: _____

Accuracy _____

Presentation _____

Punctuality _____

Correction _____

Tier A

- Given $3x^3 + 2x^2 + x + 2 = (x^2 + 2x - 1)(3x - 4) + Ax + B$ for all values of x , find the value of A and of B .
- Given that $4x^3 - 6x^2 + ax + 3$ leaves a remainder of 7 when divided by $2x - 1$, evaluate the value of a .
- Given that $f(x) = x^3 - 2x^2 - 5x + 6$, find the remainder when $f(x)$ is
 - divided by $3x - 2$, $f(x) = (3x - 2)Q(x) + R$
 - divided by $2x + 5$. $x = -\frac{5}{2}$ $f(-\frac{5}{2}) = R$
 - Given that the expression $f(x) = ax^3 - x^2 + bx + 18$ is exactly divisible by $x^2 + x - 6$, find the value of a and of b .

Tier B

- Given that $4x^3 - 6x^2 + 1 = (x - 2)(x + 1)Q(x) + ax + b$ for all real values of x where $Q(x)$ is a polynomial, find the value of a and of b .
- Given that $(p + 6)x^2 + (p - 2)x - 2p^2$ is exactly divisible by $x + 1$ but not by $x - 1$, find the value of p .
- Given that the remainder when $x^3 - x^2 + ax$ is divided by $x + a$, where $a > 0$, is twice the remainder when it is divided by $x - 2a$, find the value of a .
- The expression $f(x) = ax^3 - 9x^2 + 7x + b$ leaves a remainder of $6(x + 1)$ when divided by $x^2 + 3x - 4$. Find the value of a and of b .
- The expression $x^3 - 7x + 6$ and $x^3 - x^2 - 4x + 24$ have the same remainder when divided by $x + p$.
 - Find the possible values of p .
 - Determine whether, for either or both of these values, $x + p$ is a factor of the expressions.
 - Given that $f(x) = x^4 - x^3 + 5x^2 + 4x - 36$, find
 - the remainder when $f(x)$ is divided by $x + 1$
 - the value of a ($a > 0$) for which $x + a$ and $x - a$ are both factors of $f(x)$.

Tier C

9. Simplify $\sqrt[3]{8x^3 - 12x^2 + 6x - 1} - \sqrt[3]{x^3 - 9x^2 + 27x - 27}$.

10.

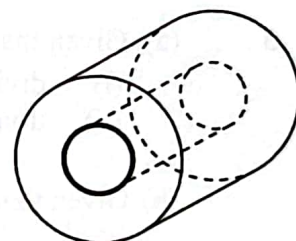
When a polynomial $f(x)$ is divided by $(x - 1)$ and $(x + 5)$, the remainders are -6 and 6 respectively. Let $r(x)$ be the remainder when $f(x)$ is divided by $x^2 + 4x - 5$. Find the value of $r(-2)$.

11.

Australian Math Competition Question 2014/25

Thanom has a roll of paper consisting of a very long sheet of thin paper tightly rolled around a cylindrical tube, forming the shape indicated in the diagram.

Initially, the diameter of the roll is 12 cm and the diameter of the tube is 4 cm. After Thanom uses half of the paper, the diameter of the remaining roll is closest to



- (A) 6 cm (B) 8 cm (C) 8.5 cm (D) 9 cm (E) 9.5 cm

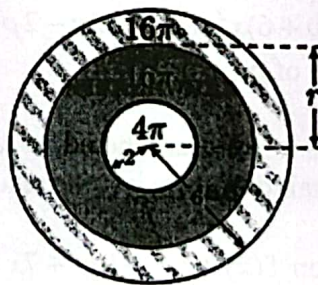
Ans to WS RnF 1

1. $A = 12, B = -2$
2. $a = 10$
- 3ai. $\frac{56}{27}$ or $2\frac{2}{27}$
- 3aii. $-\frac{77}{8}$ or $-9\frac{5}{8}$
- 3b. $a = 2, b = -15$
4. $a = 6, b = -3$
5. $p = -2$
6. $a = \frac{2}{17}$
7. $a = -\frac{28}{13}$ or $-2\frac{2}{13}$, $b = \frac{210}{13}$ or $16\frac{2}{13}$
- 8ai. $p = 3, -6$
- 8bi. -33
- 8bii. $a = 2$

Tier C

9. $x + 2$
10. $r(-2) = 0$

AMC 2014/25



Working in centimeters, let the half-roll's radius be r . From end on, the full roll had area $(6^2 - 2^2) = 32\pi$, so half the roll has area 16π . Including the tube, the end of the half-roll has area $20\pi = \pi r^2$.

Then $r^2 = 20$, but $4.5^2 = 20.25$ and

$4.4^2 = 19.36$ so $4.4 < r < 4.5$, and the diameter is twice that.

Hence (D) 9 cm.



Nanyang Girls' High School
SECONDARY 3
Integrated Mathematics

Name: _____ ()
Class: _____ Date: _____

Accuracy _____

Presentation _____

Punctuality _____

Correction _____

Tier A

1. (a) Think! A Math TB A Ex 4c Q4a

(b) Think! A Math TB A Ex 4c Q4c

2. Think! A Math TB A Ex 4c Q6b

3. Express each of the following in partial fractions.

(a) $\frac{7x+11}{x^2+x-6}$

(b) $\frac{39x^2-35x+11}{(2x+3)(3x-1)^2}$

(c) $\frac{-8x^2-7x+28}{(3x-2)(2x^2+9)}$

Tier B

4. When $ax^3 + bx^2 + cx + 6$ is divided by $x - 1$, the remainder is -8 . It is exactly divisible by $x - 2$ and $x + 3$.

(i) Find the value of a , of b and of c .

(ii) Find the remaining factor.

(iii) Hence solve the equation $ax^3 + bx^2 + cx + 6 = 0$.

5. Solve $2x^3 + 9x^2 + 7x - 6 = 0$.

Hence, find the value of u where $2(u+2)^3 + 9(u+2)^2 + 7(u+2) - 6 = 0$.

6. Express each of the following in partial fractions.

(a) $\frac{2x^2-11x+12}{x^2-5x+6}$

(b) $\frac{2x^3-5x^2-14x-17}{x^2-3x-4}$

7. Given that $f(x) = x^{2n} - (p+1)x^2 + p$ where n and p are positive integers, show that $x - 1$ is a factor of $f(x)$ for all values of p .

When $p = 4$, find the value of n for which $x - 2$ is a factor of $f(x)$ and for this case, factorise $f(x)$ completely.

Tier C

8. Solve the equation $8x^3 - 2x^2 - 25x - 6 = 0$.
Hence, by using a suitable substitution, solve the equation $3x^3 - 25x^2 + 4x + 32 = 0$.

9. Australian Math Competition Question 2013/23

How many positive integers n are there such that $2n + 1$ is a divisor of $8n + 46$?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Ans to WS RnF 2

1.(a) $(5x - 4y)(25x^2 + 20xy + 16y^2)$ (b) $(7a + 6x^2)(49a^2 - 42ax^2 + 36x^4)$

2. $x = \frac{1}{2}$ or $x = 3.19$ or $x = -2 \cdot 19$

3. (a) $\frac{2}{x+3} + \frac{5}{x-2}$

(b) $\frac{5}{2x+3} - \frac{1}{3x-1} + \frac{1}{(3x-1)^2}$

(c) $\frac{2}{3x-2} - \frac{4x+5}{2x^2+9}$

4i. $a = 3, b = 2, c = -19$

4ii. $3x - 1$

4iii. $x = 2, -3$ or $\frac{1}{3}$

5. $x = -2, -3$ or $\frac{1}{2}$
 $u = -4, -5$ or $-\frac{3}{2}$

6 (a) $2 + \frac{2}{x-2} - \frac{3}{x-3}$

(b) $2x + 1 + \frac{2}{x+1} - \frac{5}{x-4}$

7. $n = 2$

$(x+1)(x-1)(x+2)(x-2)$

8. $x = 2, -\frac{1}{4}, -1\frac{1}{2}$

$x = -1, 8, 1\frac{1}{3}$

9. If $2n + 1$ is a divisor of $8n + 46$, it is also a divisor $8n + 46 - 4(2n + 1) = 42$. Odd factors of 42 are 1, 3, 7 and 21, which have $n = 0, 1, 3, 10$
But $n > 0$ so there are three solutions. Hence (D) 3.