Topical Revision Worksheet 7

Topic: Linear Law

Question 1 [2008 EOY, Q4] | marks

Two variables x and y are such that when $\lg y$ is drawn against $\lg x$, a straight line of gradient -3 is obtained, passing through the point $(-2,9)$.

(i) Express y in terms of x.

[4]

Solution:

$$\lg y-9=-3(\lg x+2)$$

$$\Rightarrow \lg y=-3\lg x+3 \Rightarrow \lg y=\lg x^{-3}+\lg 10^3 \Rightarrow \lg y=\lg\left(\frac{10}{x}\right)^3$$

Hence $y=\left(\frac{10}{x}\right)^3$

(ii) Find the value of x when $y=-1$.

[2]

Solution:

When $y=-1$, $\left(\frac{10}{x}\right)^3=-1 \Rightarrow \frac{10}{x}=-1 \Rightarrow x=-10$

Question 2 [2009 EOY, Q15] | marks

Answer the whole of this question on a sheet of graph paper.

The table shows experimental values of two variables, x and y, which are known to be related by the equation $y-1=px+\frac{q}{x}$.

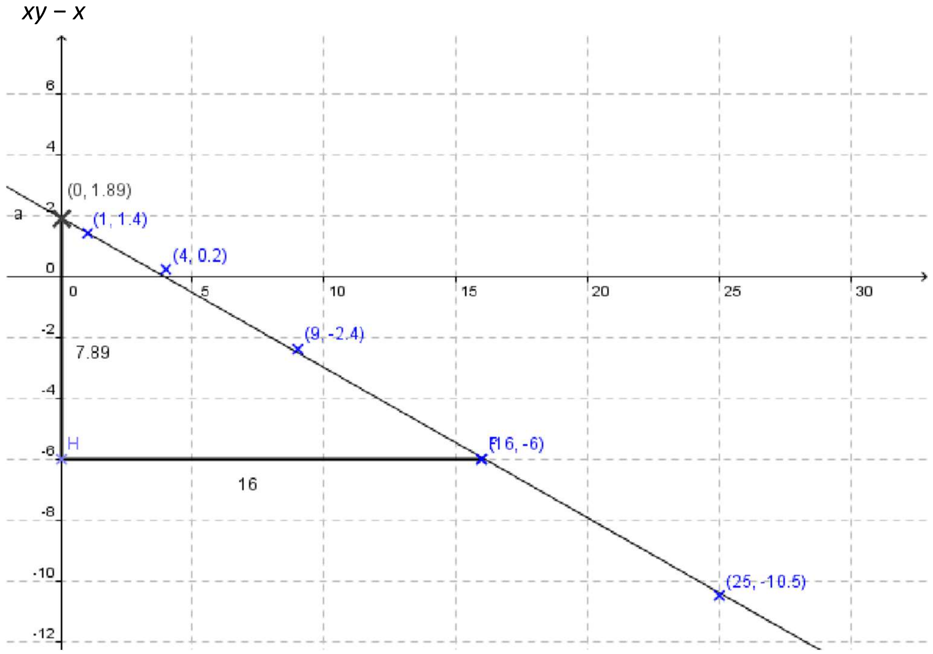
x x	1	2	3	4	5
y	2.4	1.1	0.2	– 0.5	– 1.1

(i) Choosing a suitable scale, plot $(xy-x)$ against x^2 and draw a straight line.

[5]

Solution:

$xy-x$	1.4	0.2	– 2.4	– 6	– 10.5
x^2	1	4	9	16	25



(ii) Use your graph to estimate the values of *p* and *q*.

Solution:

$$x(y-1)=px^2+q$$

Fr graph, q = 1.89, $p=-0.493$

[2]

(iii) Explain how you would use your graph to determine if the set of experimental values x = 10 and y = − 4 are related by the equation $y-1=px+\frac{q}{x}$.

Solution:

$xy-x=-50$, x^2 = 100. Extend the line. If it passes through (100, −50), then the values are related by the equation $y-1=px+\frac{q}{x}$.

[1]

Question 3 [2010 EOY, Q8] | marks

The table below shows experimental readings of y for given values of x such that $xy = ay + b\sqrt{x}$ where a and b are real constants.

x	0	1	2	3	4	5	6
y	0	2.21	1.96	−8.95	−3.88	−2.67	−2.11

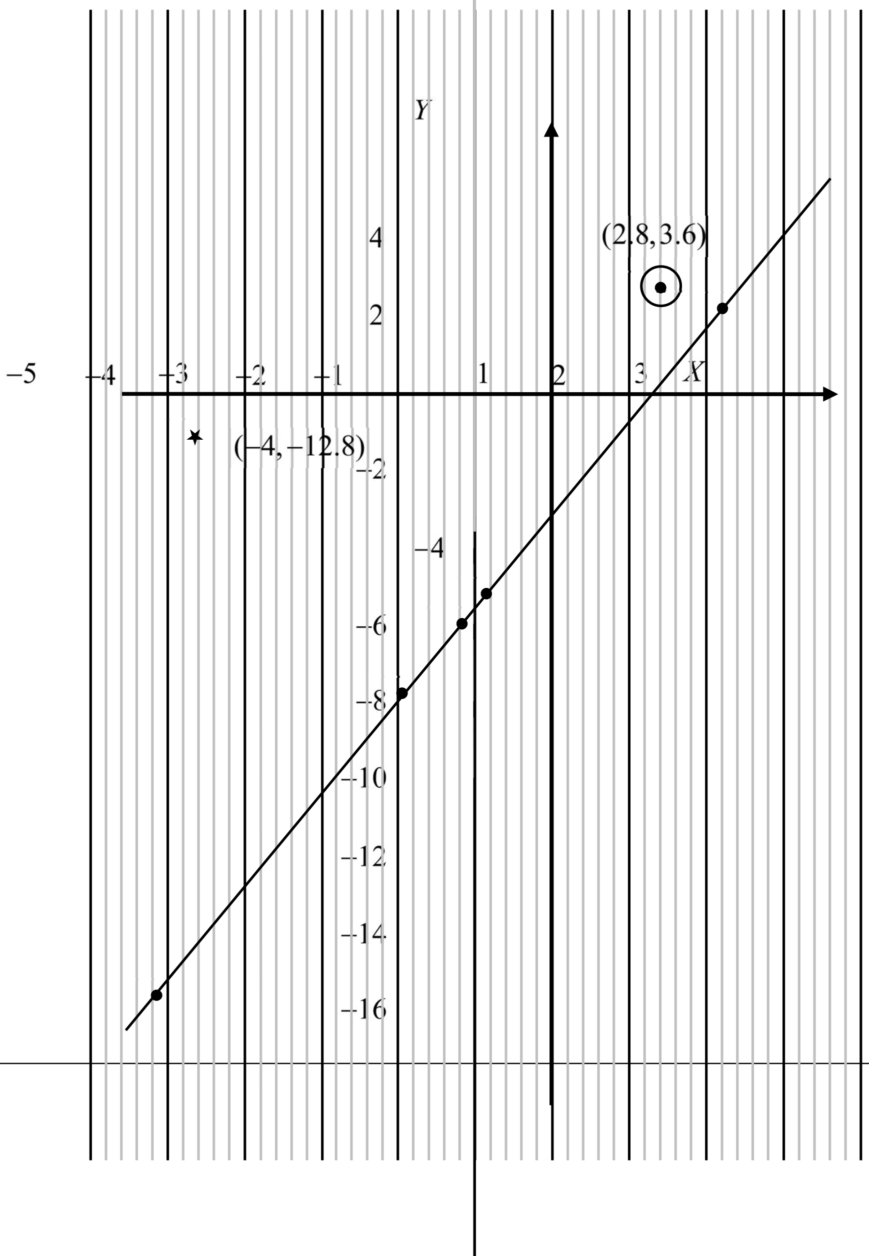
By drawing a straight line graph of $\sqrt{x}y$ against $\frac{y}{\sqrt{x}}$, identify the incorrect reading,

[4]

Solution: $xy = ay + b\sqrt{x} \Rightarrow \sqrt{x}y = a\frac{y}{\sqrt{x}} + b$ so let $X = \frac{y}{\sqrt{x}}$ and $Y = \sqrt{x}y$

X	2.21	1.39	−5.17	−1.94	−1.19	−0.86
Y	2.21	2.77	−15.5	−7.76	−5.97	−5.17

From graph, the incorrect reading is 1.96



(ii) estimate the values of a and b.

[2]

Solution:

From the graph, $a \approx \dfrac{3.6-(-12.8)}{2.8-(-4)}$

$$= 2.4 \text{ (1 d.p.)}$$

$$b \approx -3.2 \text{ (1 d.p.)}$$

Question 4 [2012 EOY, Q12]

marks

The table shows experimental values of two variables, x and y, which are known to be related by the equation $\sqrt{y}\left(x^{b}\right)=a$, where a and b are constants.

x	1.0	1.5	2.0	2.5	3.0
y	7.3	3.5	2.0	1.3	0.9

(i)

Express this equation in a form suitable for drawing a straight line graph.

[2]

Solution:

$$\sqrt{y}\left(x^{b}\right)=a \Rightarrow \frac{1}{2}\lg y + b\lg x = \lg a \Rightarrow \lg y = -2b\lg x + 2\lg a$$

(ii)

Using a suitable scale, plot a straight line graph of $\lg y$ against $\lg x$.

Use your graph to estimate the value of a and of b.

[6]

Solution:

$\lg x$	0		0.18	0.30	0.40	0.48
$\lg y$	0.86		0.54	0.30	0.11	−0.05

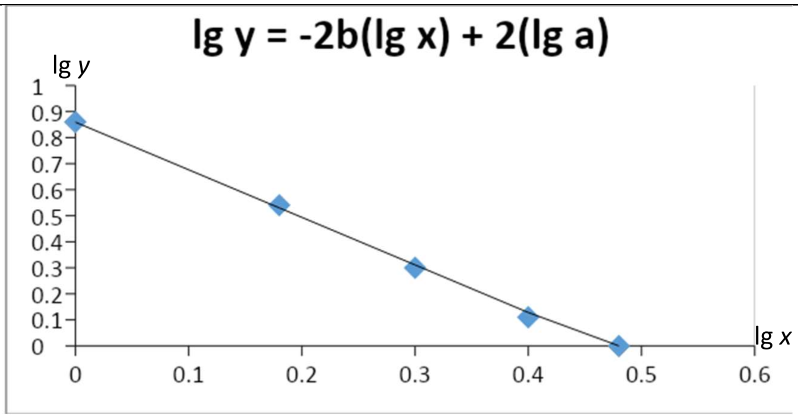


Table of values
Label axes
Plot points correctly
Best fit line
From graph,

$$2\lg a \approx 0.86$$

$$a = 2.69$$

$$-2b \approx \frac{0.86-0}{0-0.47}$$

$$b = 0.915$$

Question 5 [2013 EOY, Q15] marks

The table shows the experimental values of two variables x and y.

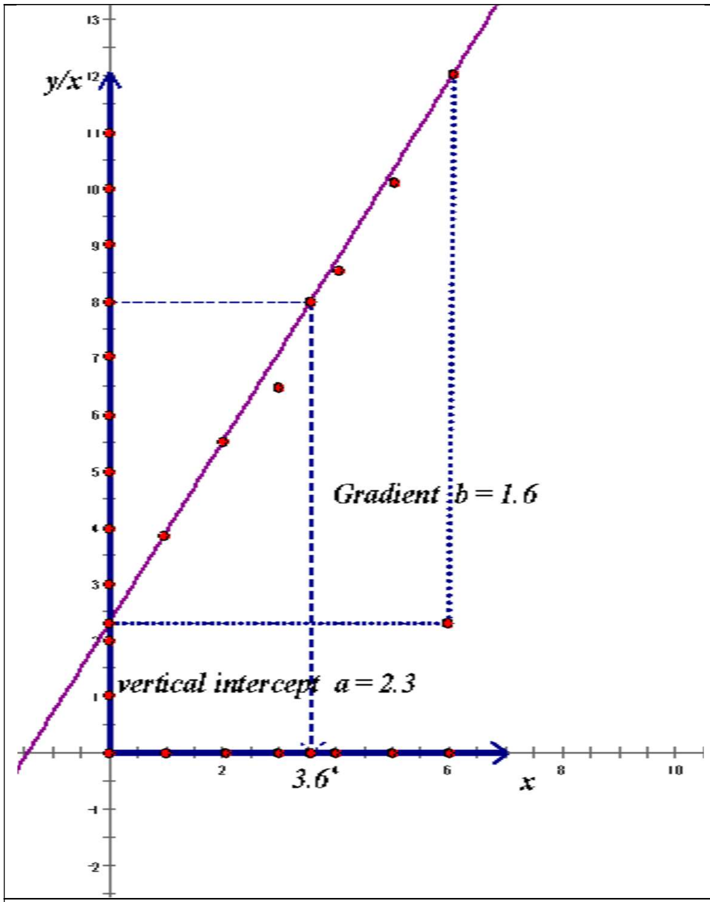
x	1	2	3	4	5
y	3.8	11.2	20.5	33.2	50.6
$\frac{y}{x}$	3.8	5.6		8.3	

It is known that x and y are related by the equation $y = ax + bx^2$

Plot $\frac{y}{x}$ against x and draw a suitable straight line.

Solution:

x	1	2	3	4	5
y	3.8	11.2	20.5	33.2	50.6
$\frac{y}{x}$	3.8	5.6	***6.8***	8.3	***10.1***



(a) [4]

Use the graph to estimate the value of a and of b,

Solution:

(a) Plot $\frac{y}{x}$ against x. From the diagram,

The gradient $b = 1.6$ (±0.2) and

the vertical intercept is $a = 2.3$. (±0.1)

(ii) the value of x when $y = 8x$.

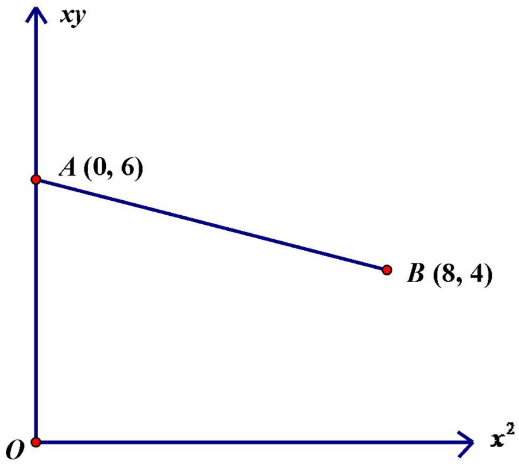
Solution: [1]

When $y = 8x \Rightarrow \frac{y}{x} = 8$.

From the diagram, the value of x is 3.6 (±0.1)

(b) The diagram shows the straight line graph of xy against x^2 passing through the points $A(0, 6)$ and $B(8, 4)$. Express y in terms of x.

[3]



Solution:

xy – intercept is 6

gradient $\frac{6-4}{0-8}=-\frac{1}{4}$

equation of straight line is of the form

" $y = mx + c$ "

i.e. $xy = mx^2+c$

$$xy=-\frac{1}{4}x^2+6$$

Hence $y=-\frac{1}{4}x+\frac{6}{x}$ or $-\frac{x^2+24}{4x}$

Question 6 [2014 EOY, Q10] marks

Answer the whole of this question on a sheet of graph paper.

The table shows experimental values of two variables x and y. It is known that x and y are related by an equation $y=ae^{-bx}$, and that one of these values of y given in the table is subject to an abnormally large error.

x	1.5	2.5	3.5	4.5	5.0	6.0
y	9.9	6.0	3.6	2.2	1.7	1.4

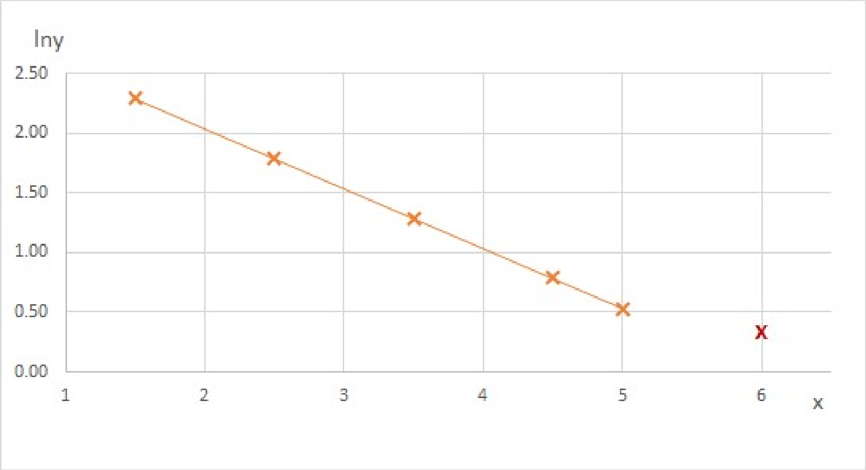
(i) Draw a straight line graph of $\ln y$ against x, using a scale of 2 cm to 1 unit on the x-axis and 1 cm to 0.2 units on the $\ln y$ -axis.

Solution:

[4]

$y = ae^{-bx} \Rightarrow \ln y = \ln a - bx$

x	1.5	2.5	3.5	4.5	5.0	6.0
y	9.9	6.0	3.6	2.2	1.7	1.4
$\ln y$	2.29	1.79	1.28	0.79	0.53	0.34



(ii)

Use your graph to estimate the value of *a* and of *b*.

[3]

Solution:

$\ln a \approx 3.04$

=> $a \approx 20.9$ (accept 20.0 to 21.8)

$b \approx 0.500$ (accept 0.47 to 0.53)

(iii)

Identify the abnormal reading of *y* and estimate its correct value.

[2]

Solution:

The abnormal reading is 1.4. [B1]

When $x = 6.0$, $\ln y$ is estimated to be 0.04.

Hence, the correct value of $y \approx e^{0.04} \approx 1.04$. (accept 1.00 to 1.04)

Question 7 [2015 EOY, Q8]	marks
Answer the whole of this question on a sheet of graph paper. Two variables x and y are related such that $y^2 = ax^2 + bx$, where a and b are unknown constants. In an experiment, values of y are found for several values of x and the values are tabulated as shown below. It is also found that one particular value of y is incorrectly recorded.	

x	1	2	3	4	5	6
y	2.07	3.77	5.45	6.81	8.81	10.48

(i)

Using a scale of 2 cm to 1 unit for the horizontal axis and 1 cm to 1 unit for the vertical axes, plot $\frac{y^2}{x}$ against x and draw a straight line graph, and use it to identify the incorrect value of y recorded. [4]

Solution:

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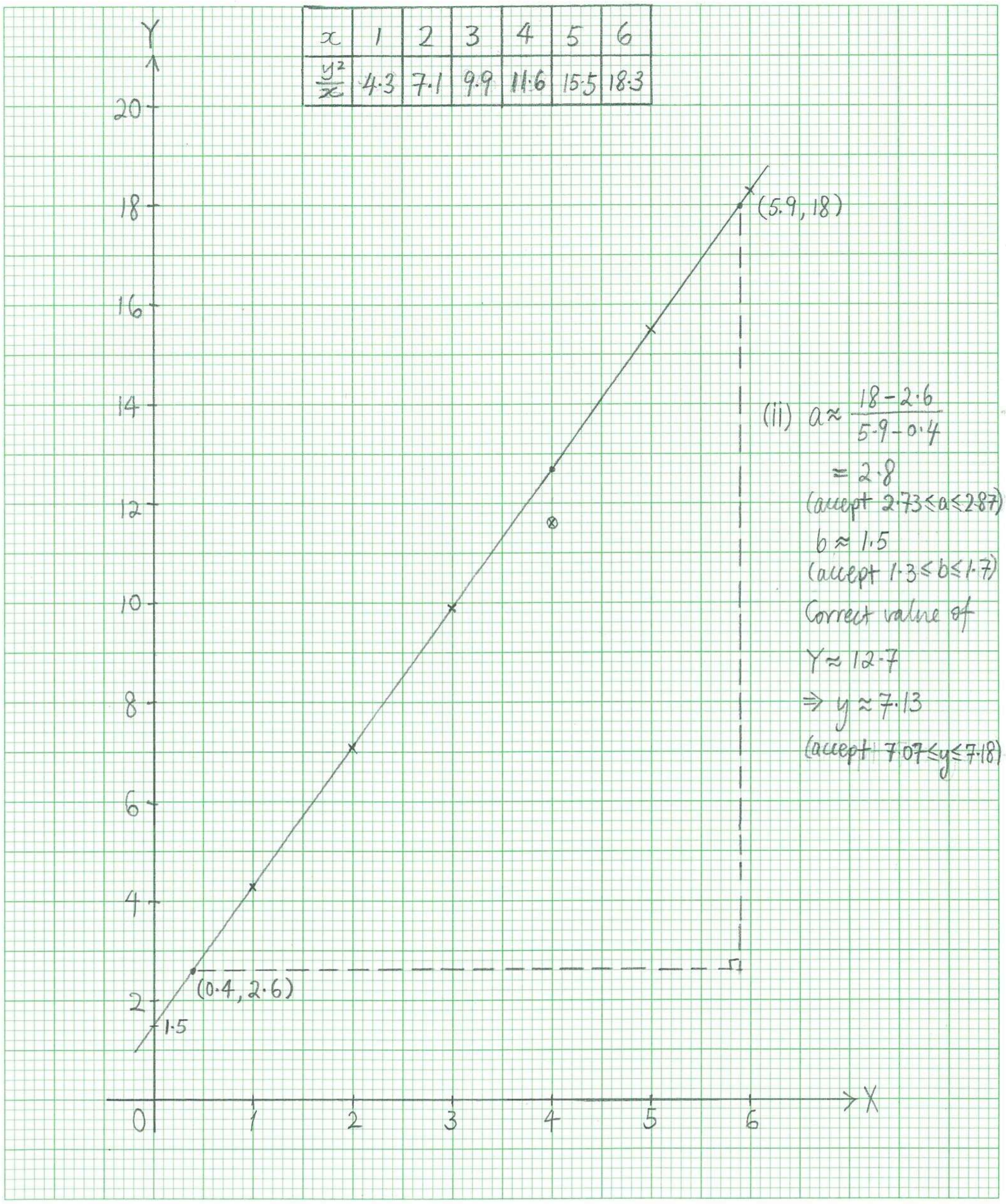
x	1	2	3	4	5	6
$\frac{y^2}{x}$	4.3	7.1	9.9	11.6	15.5	18.3



(ii) $a \approx \frac{18-2.6}{5.9-0.4}$

$= 2.8$

(accept $2.73 \leq a \leq 2.87$)

$b \approx 1.5$

(accept $1.3 \leq b \leq 1.7$)

Correct value of

$Y \approx 12.7$

$\Rightarrow y \approx 7.13$

(accept $7.07 \leq y \leq 7.18$)

Incorrect value of *y* recorded is 6.81.

(ii)

Use your graph to estimate, to one decimal place, the values of *a* and *b*, and hence estimate, to two decimal places, the correct value of *y* in **(i)**. [4]

Solution:

The gradient represents *a*, so $a \approx 2.8$ (accept $2.73 \leq a \leq 2.87$)

The vertical intercept represents *b*, so $b \approx 1.5$ (accept $1.3 \leq b \leq 1.7$)

$\frac{y^2}{x} \approx 12.7 \Rightarrow y \approx 7.13$ (accept $7.07 \leq y \leq 7.18$)