**RAFFLES INSTITUTION**
**H2 Mathematics (9758)**
**2025 Year 5**

## C6A: Differentiation Techniques (Additional Practice)

**1** Differentiate each of the following with respect to $x$:

**(a)** $(x-1)^2 e^{2x}$

**(b)** $\dfrac{\cos\left(x+\frac{\pi}{4}\right)}{\sqrt{x}}$

**(c)** $\dfrac{1}{1+\sin x}$

**(d)** $\left(\ln\left(\dfrac{2}{x}\right)\right)\tan^{-1}(\sqrt{x})$

**(e)** $\ln\sqrt{\dfrac{1+x^2}{1-x^2}}$

**(f)** $e^{x\ln x}$

**(g)** $e^{\tan^{-1}\sqrt{3}x}$

**(h)** $x^2\sec^3 2x$

**(i)** $\cot x \operatorname{cosec} 2x$

**(j)** $\sin^{-1}(\tan x)$

**(k)** $x^2\cos^{-1}(e^x)$

**(l)** $x^{\sin x}$

**2** Find $\dfrac{d}{dx}\left(x\sqrt{9-x^2}+9\sin^{-1}\left(\dfrac{x}{3}\right)\right)$, leaving your answer in the form $a\sqrt{b-x^2}$, where the values of $a$ and $b$ are to be found.

**3** For each of the following, find $\dfrac{dy}{dx}$ in terms of $x$ and $y$.

**(a)** $x^3+2y^3+3xy=0$

**(b)** $x+\tan(xy)=0$

**(c)** $\cos(xy)-\ln(x+y)=0$

**4** It is given that $x^2y-\tan^{-1}y=\frac{3}{4}\pi$ . Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$. [4]

Hence, find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $y = 1$, given that $x > 0$. [3]

**5** **(a)** Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if

**(i)** $y=\sqrt{\frac{3}{2x+1}}$

**(ii)** $y=\ln\left(\frac{x^2+x+1}{1+\cos x}\right)$

**(b)** Given that $ye^y=(x-5y)^3$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$.

**6** A curve $C$ has equation $\frac{x^2-4y^2}{x^2+xy^2+100}=\frac{1}{2}$ , $x\in\mathbb{R}$, $x\neq-8$.

Show that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x-y^2}{2xy+16y}$. [2]

**7** If $\sin y=2\sin x$, show that $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=1+3\sec^2 y$.

**8** **(a)** Given that $y=\tan^{-1}\sqrt{x}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

**(b)** Given that $\sqrt[x]{y}=\sqrt[y]{x}$, where $x>0$, $y>0$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [4]

**9** Given the curve with equation $x^2-4xy+3y^2+1=0$, find the gradient of the curve at the point where the $y$ coordinate is 1.

**10** **(a)** Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$ for $x=t^2-\frac{8}{t^2}$, $y=\frac{t^2}{2}+\frac{8}{t^2}$, where $t\neq0$.

**(b)** A curve $C$ has parametric equations

$$x=\sin 2t,\quad y=\cos^2 t+1,\quad 0\le t\le\tfrac{\pi}{2}.$$

Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in the form $a\tan bt$, where $a$ and $b$ are constants to be determined. [2]

**(c)** The parametric equations of a curve $C$ are $x=\sin^{-1}(1-t)$, $y=e^{\sqrt{2t-t^2}}$.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$. [4]