Additional Notes

Topic 2: Kinematics

Content:

- Rectilinear motion
- Non-linear motion

Learning Outcomes

Candidates should be able to:

(a) show an understanding of and use the terms distance, displacement, speed, velocity and acceleration.
(b) use graphical methods to represent distance, displacement, speed, velocity and acceleration.
(c) identify and use the physical quantities from the gradients of displacement-time graphs and areas under and gradients of velocity-time graphs, including cases of non-uniform acceleration.
(d) derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line.
(e) solve problems using equations which represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance.
(f) describe qualitatively the motion of bodies falling in a uniform gravitational field with air resistance.
(g) describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular motion.

Demonstrating Science Inquiry Skills
The study of kinematics begins with the introduction of precise terminology and language for describing motion, to reduce ambiguity in expression and confusion in thought. Using multiple representations such as motion diagrams, graphs and equations, we can express motion of objects.

Introduction

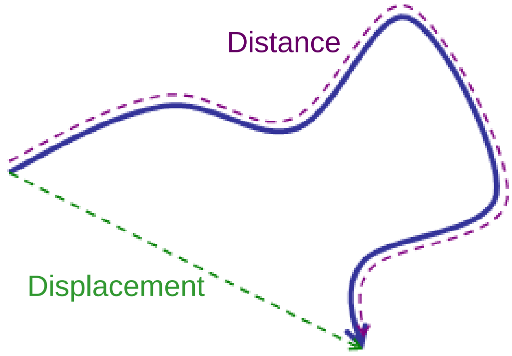
Kinematics is the branch of mechanics which deals with the description of the motion of bodies. In this chapter, two types of motion will be covered, namely,

- along a straight line (rectilinear motion), and
- on a plane (non-rectilinear motion).

A Terms of Reference for Kinematics

A.1 Distance and Displacement

- Distance is a scalar quantity while displacement is a vector quantity.
- Distance refers to the length of the path travelled by a body regardless of the direction of motion.
- Displacement is the distance of a body or a point, in a specified direction, from reference point.



Check Your Understanding 1
When displacement of a body is zero, the distance travelled by the body must be zero. (True / False)

A.2 Speed and Velocity

Speed is a scalar quantity while velocity is a vector quantity.

memorise

Definition: Speed is distance travelled divided by the time taken.

memorise

Definition: Velocity is the rate of change of displacement.

Direction of the velocity is the same as the direction of the change in displacement.

Additional Notes

A.3 Instantaneous Speed and Velocity versus Average Speed and Velocity

Both average speed and average velocity are measured over a time interval Δt. However, the phase "how fast" more commonly refer to how fast a body is moving at a particular instant. Since a moving body often changes its speed during its motion, it is common to distinguish between the average quantity and instantaneous quantity.

- **Instantaneous speed** is the speed at a particular point or a particular instant of time.

$$\text{Instantaneous speed} = \frac{\text{small distance travelled by the object}}{\text{small time taken}}$$

$$= \text{gradient of the distance-time graph at that instant}$$

- **Instantaneous velocity** v is the average velocity for a diminishing time interval Δt.

$$v = \text{rate of change of displacement } \frac{ds}{dt}$$

$$= \text{gradient of displacement-time graph at that instant}$$

Hence **instantaneous velocity** (or simply **velocity**) can be defined as the rate of change of displacement.

Note:
The instantaneous speed is always equals to the magnitude of the instantaneous velocity because distance and displacement become the same when they become infinitesimally small.

Instantaneous speed is akin to the speedometer reading at any given instance

Average speed is akin to the average of all the speedometer readings during the course of the trip.

- **Average speed** refers to the total distance travelled divided by the total time taken.

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

- **Average velocity** refers to the total change in displacement of the body divided by the total time interval.

$$\text{average velocity} = \frac{\text{total change in displacement}}{\text{total time taken}}$$

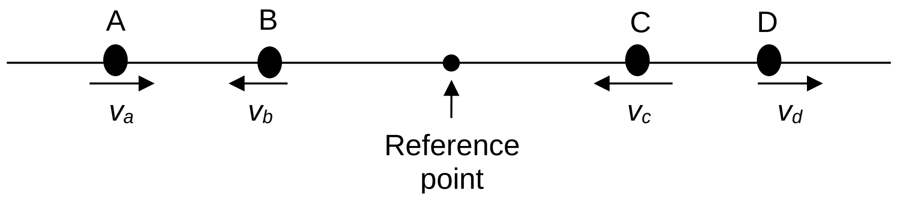
Some examples of speeds

	Speed / m s^{-1}
Light	3.0×10^8
Electron around nucleus	2.2×10^6
Earth around Sun	3.0×10^4
Commercial jet airplane	2.5×10^2 (900 km h^{-1})
Typical car speed	22 (80 km h^{-1})
Walking speed	1.5
Snails	1.0×10^{-3}

Additional Notes

Example 1

Based on the diagram below, describe the direction of velocities and displacements of the bodies A, B, C and D. (Take right as positive)



Always try to assign the same convention for all vectors along the same line. Eg. the positive direction is the same for displacement and velocity.

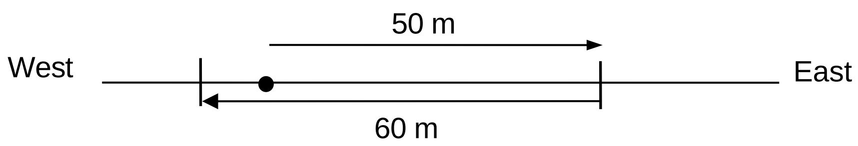
Taking the direction pointing to the right as positive. (→+ve)

A has ______ displacement (on the left of reference point), ______ velocity (moving in the ______ direction or to the ______)
B has ______ displacement, ______ velocity (moving in the ______ direction or to the ______)
C has ______ displacement, ______ velocity (moving in the ______ direction or to the ______)
D has ______ displacement, ______ velocity (moving in the ______ direction or to the ______)

The sign for displacement depends on the position of body with respect to reference point. The +/- sign of velocity indicates the direction of velocity which is also the direction of motion.

Example 2

A person walks 50 m east, then 60 m west. Suppose this walk takes 40 s to complete.



Determine the
(a) total distance travelled,
(b) total displacement,
(c) average speed,
(d) average velocity.

Additional Notes

A.4 Acceleration

Definition: Acceleration is the rate of change of velocity.

memorise

Acceleration is a vector quantity. S.I. unit is written as m s^{-2}.

- **Average acceleration** of any body is calculated using the equation:

$$<a> = \frac{\Delta v}{\Delta t} = \frac{v-u}{\Delta t}$$

where u = initial velocity v = final velocity after the time interval Δt

An astronaut in the space shuttle experience an approximately linear acceleration that may be as great as *20 m s⁻²* in the initial uplifting phase.

In practice both velocity and acceleration can be taken to mean instantaneous velocity and instantaneous acceleration.

- **Instantaneous acceleration** = $\frac{dv}{dt}$

= gradient of velocity-time graph at that instant

- A body is accelerating if and only if it is changing its velocity. This can be shown in several ways:
  - by change in magnitude only, i.e. the speed changes while the direction remains the same;
  - by a change in direction only, e.g., a body moving with constant speed in a circular path; or
  - by a change in magnitude and direction simultaneously.

Description of what it means for a body to possess acceleration.

- If a body is changing its velocity - whether by a constant amount or a varying amount - then it is accelerating.
  - Conversely, a body with a constant velocity is not accelerating.
  - Constant acceleration means that the velocity is changing by a constant amount each second and there is no change in direction of acceleration.

How do we know if the body has no acceleration?

- **Deceleration** means that body is **slowing down**, regardless of direction of motion.
  Note: While acceleration can have a positive or negative value, deceleration can only have a positive value.

Note that in the A-level syllabus, acceleration can be used to refer to both speeding up and slowing down.

Example 3

A car was travelling with an initial velocity u m s^{-1}. After a time interval of Δt, it is moving at v m s^{-1}. In the table below, calculate the acceleration for each time interval and state the corresponding type of motion.

u / m s^{-1}	v / m s^{-1}	Δt / s	a / m s^{-2}	Motion of the car
5.0	20	3.0	5.0	Speeding up
-5.0	-20	5.0		
-30	-14	2.0		
25	2.0	4.0		
10	-5.0	3.0		

- Direction of acceleration same as direction of velocity → body speeds up
- Direction of acceleration opposite to direction of velocity → body slows down

Check Your Understanding 2

Negative acceleration means the body is slowing down. (True / False)

B Motion Graphs

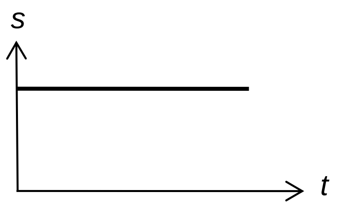
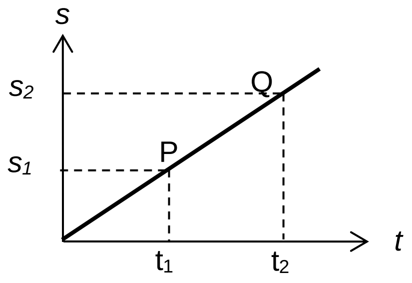
- Graphical representations offer an overall description of the motion of the bodies.
- The two main areas of focus in graphical appreciation are: <u>Area</u> and <u>Gradient.</u>
- <u>Graphically, when plotting a dependent quantity *y* against time (horizontal axis–time), the gradient represents the rate of change of *y*.</u>

- The <u>3 main types of graphs</u> and their relationships can be summarized as below.

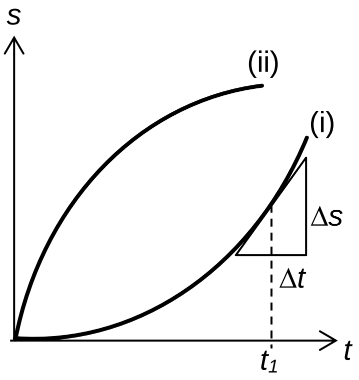
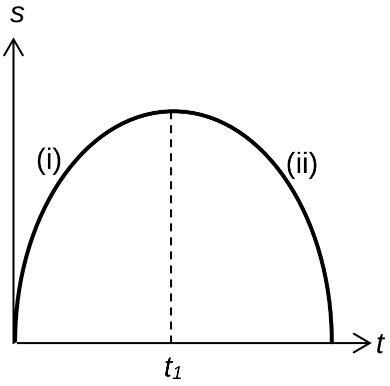
Type of graph	**Gradient represents**	**Area represents**
Displacement–time	Velocity	Not Applicable
Velocity–time	Acceleration	<u>Change in</u> Displacement
Acceleration–time	Not Applicable	<u>Change in</u> Velocity

B.1 Displacement–Time Graph (*s*–*t* graph)

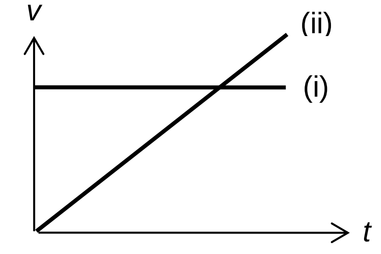
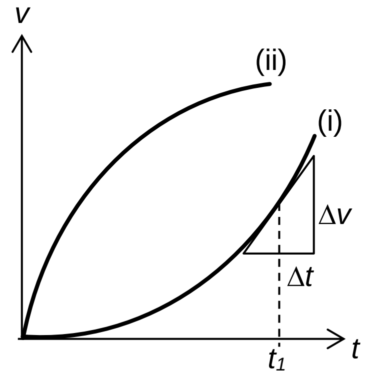
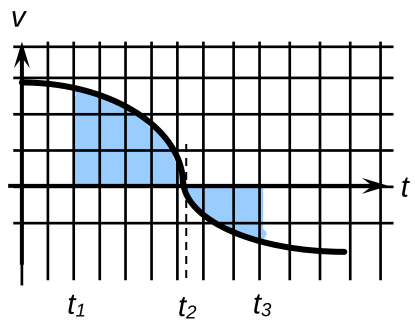
For *s*-*t* graphs, the positive direction is taken to be the direction of increasing displacement.

	***s*–*t* graph**	**Interpretation**
(a)		• *s* is constant, not changing with *t*, gradient of *s*–*t* graph = 0 ⇒ body is at rest.
(b)		• *s* increases linearly with *t*, gradient of *s*–*t* graph = +ve and constant ⇒ Body is moving in the positive direction with constant or uniform velocity • average velocity = gradient of PQ = $\dfrac{s_2 - s_1}{t_2 - t_1}$ • instantaneous velocity at any point = average velocity

Additional Notes

	s–t graph	Interpretation
(c)		• *s* increases non-linearly with *t*, gradient of *s–t* graphs = +ve and non-constant ⇒ Body is moving with non-uniform velocity. • instantaneous velocity at any point = gradient of tangent at any point $= \dfrac{\Delta s}{\Delta t}$ • Graph (i) indicates increasing gradient. ⇒ Body is moving in the positive direction with increasing velocity. ⇒ accelerating • Graph (ii) indicates decreasing gradient. ⇒ Body is moving with positive direction with decreasing velocity. ⇒ decelerating
(d)		• *s* increases and then decreases non-linearly with *t*, gradient of *s–t* graph = non-constant • At t_1, body is instantaneously at rest. • Graph (i) indicates +ve, decreasing gradient. ⇒ The body is moving in the positive direction with decreasing velocity. ⇒ decelerating • Graph (ii) indicates -ve, increasing gradient. ⇒ The body is moving in the negative direction with increasing velocity. ⇒ accelerating

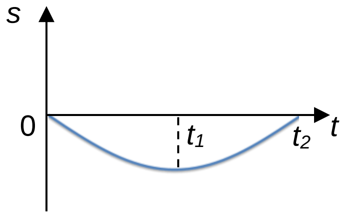
B.2 Velocity–Time Graph (*v*–*t* graph)

	***v*–*t* graph**	**Interpretation**
(a)		• Graph (i): *v* is not changing with *t*, gradient of *v*–*t* graph = 0 $\Rightarrow$ body is not accelerating, it is moving with uniform velocity. • Graph (ii): *v* increases linearly with *t*, gradient of *v*–*t* graph = +ve and constant $\Rightarrow$ Body is moving with uniform or constant acceleration $\Rightarrow$ direction of acceleration is same as the velocity
(b)		• *v* increases non-linearly with *t*, gradient of *v*–*t* graph = +ve and non-constant $\Rightarrow$ Body is moving with non-uniform acceleration. • instantaneous acceleration at any point = gradient of tangent at any point = $\dfrac{\Delta v}{\Delta t}$ • Graph (i): *v* increases non-linearly with *t*, gradient of *v*–*t* graph = +ve and increasing $\Rightarrow$ increasing acceleration. $\Rightarrow$ direction of acceleration is <u>same</u> as the velocity. • Graph (ii): velocity *v* increases non-linearly with time *t*, gradient of *v*–*t* graph = +ve and decreasing $\Rightarrow$ decreasing acceleration. $\Rightarrow$ direction of acceleration is <u>same</u> as the velocity.
(c)		• The area beneath the velocity–time (*v*–*t*) graph is the change in displacement. • The area can be determined by estimation through counting squares if the area is irregular. • From t_1 to t_2, area **above** the *t*-axis is positive, i.e. **change in displacement is <u>positive</u>**. • From t_2 to t_3, area **below** the *t*-axis is negative, i.e. **change in displacement is <u>negative</u>**. • Hence, from t_1 to t_3, **change in displacement is <u>positive</u>**.

Additional Notes

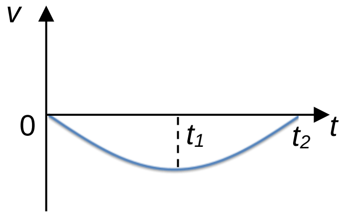
Check Your Understanding 3

Describe the motion of the object from the graph given. Circle the correct answer.



Question 1

From t = 0 to t_1, magnitude of displacement of object increases / decreases / remains constant with time.
Object moves in positive /negative direction, with magnitude of velocity (speed) increasing / decreasing / constant, until it comes to a momentarily stop and changes direction at t_1.
From $t = t_1$ to t_2, magnitude of displacement of object increases / decreases / remains constant with time.
Object moves in positive / negative direction, with magnitude of velocity (speed) increasing/ decreasing/ constant, until it returns to the origin at t_2.

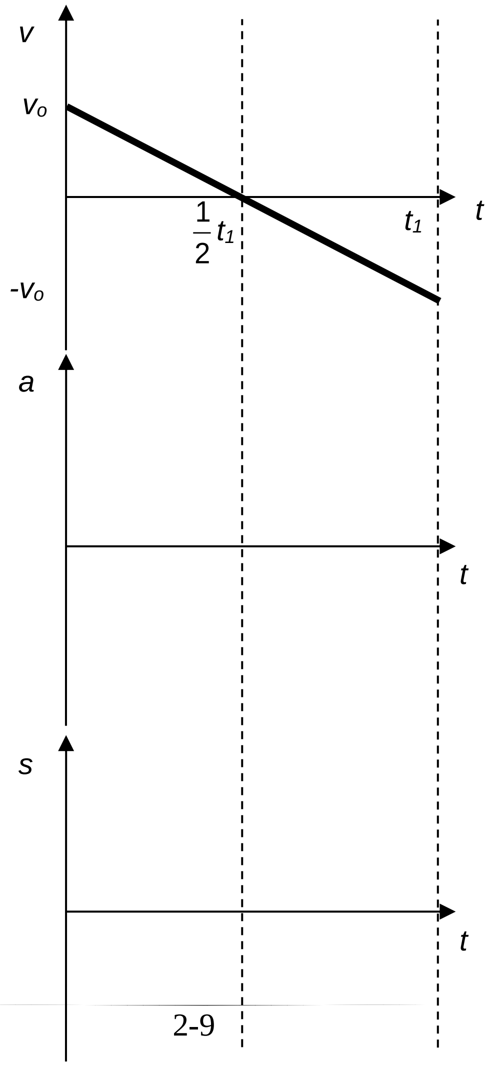


Question 2

From t = 0 to t_1, object moves in positive / negative direction, with magnitude of velocity (speed) increasing / decreasing / constant. Acceleration is positive /negative and increasing / decreasing / constant.
From $t = t_1$ to t_2, object moves in positive /negative direction, with magnitude of velocity (speed) increasing / decreasing / constant. Acceleration is positive /negative and increasing / decreasing / constant until it comes to a stop at t_2.

Example 4

For the velocity–time graph shown below, sketch the corresponding acceleration–time and displacement–time graphs. The displacement at time t = 0 s is 0 m.



The a-t graph can be determined using the gradient of v-t graph

Note:
From start to end, since total area under v-t graph is zero, Δs = 0, body returns to the starting point

The gradient of s-t graph represents the velocity
The +/- gradient of s-t graph is determined by the direction of velocity i.e +/- sign of velocity

Additional Notes

C Visualising Concepts in Physics (Multiple Representation)

Multiple representations of knowledge

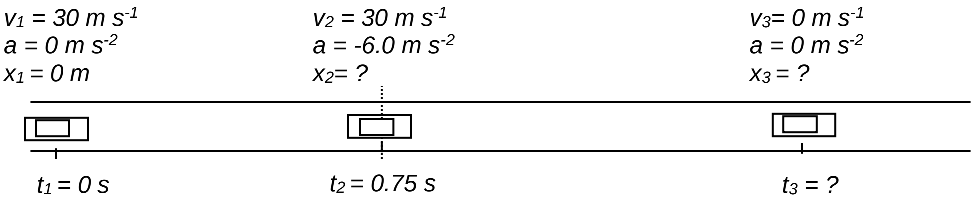
- Other than graphs (section B), we can also describe motion with verbal representation, pictorial representation, physical representation (motion diagram) and mathematical representation.

Example: Express motion of bodies using multiple representations

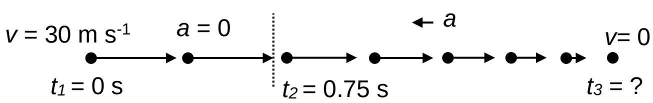
(a) Verbal representation:

A car is travelling at a speed of 30 m s^{-1}, a typical highway speed, on wet pavement. The driver sees an obstacle ahead and decides to stop. From this instant, it takes him 0.75 s to begin applying the brakes. Once the brakes are applied, the car experiences an acceleration of -6.0 m s^{-2}. How far does the car travel from the instant the driver notices the obstacle until stopping?

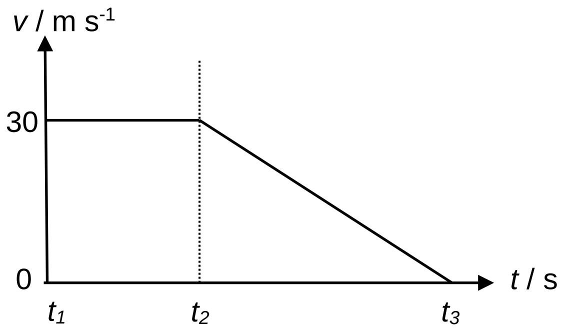
(b) Pictorial representation:



(c) Physical representation (Motion diagram)



(d) Graphical representation (refer to section B)



Describing motion from graph
Direction: positive / negative
Value: increasing / decreasing / remain constant
Rate: increasing / decreasing / remain constant
Additional information: comes to a stop / changes direction / returns to the start point
e.g. From $t = t_2$ to t_3, the car moves in positive (**D**) direction, velocity decreases (**V**) at constant rate (**R**) until it comes to a stop (**A**).

(e) Mathematical representation (refer to section D)

From t_1 to t_2 the velocity stays constant at 30 m s^{-1}, i.e. $a = 0$
distance $x_2 = x_1 + v_1(t_2 - t_1) = 0 + 30 \times 0.75 = 22.5$ m

From t_2 to t_3, a = – 6.0 m s^{-2},
$v_3^2 = v_2^2 + 2a(x_3 - x_2)$
Hence braking distance =$(x_2 - x_1) + (x_3 - x_2)$

Additional Notes

D Kinematics Equations

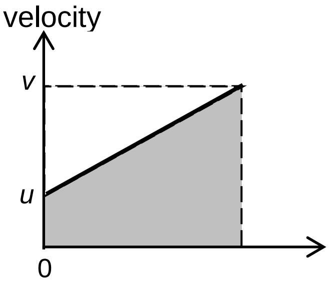
Conditions to apply kinematic equations:

1. Constant acceleration
2. Motion in straight line

Look out for keywords like 'constant acceleration' or 'constant force' to check whether kinematic equations are applicable.

Condition is **different** from assumption (e.g. negligible air resistance is an assumption that leads to the condition of constant acceleration)

D.1 Derivation of Kinematics Equations



Suppose the velocity of the body increases steadily from u to v in time t then the uniform acceleration a is given by

$$a = \text{rate of change of velocity} = \frac{dv}{dt}$$

$$= \text{gradient of velocity-time graph}$$

$$= \frac{v-u}{t}$$

Therefore $\boxed{v = u + at}$ ------------------------------(1)

Since velocity is the rate of the change of displacement, $v = \dfrac{ds}{dt}$,

the area under the v-t graph gives the change in displacement.
Assuming at t = 0, s = 0, the displacement of the body s in time t is

$$s = \frac{1}{2}(u+v)t$$

But $v = u + at$

Thus $s = \dfrac{1}{2}(u+u+at)t$

or $\boxed{s = ut + \dfrac{1}{2}at^2}$ --------------------------------(2)

From eqn (1) $t = \dfrac{v - u}{a}$ and substituting into eqn (2), we get

$\boxed{v^2 = u^2 + 2as}$ ------------------------------(3)

The equations of motion:

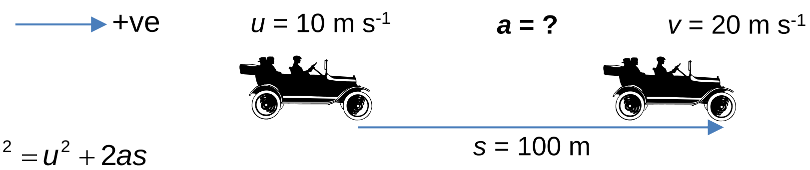
$$\boxed{\begin{gathered} v = u + at \\ s = ut + \frac{1}{2}at^2 \\ v^2 = u^2 + 2as \end{gathered}}$$

Steps to Follow in Application of Kinematics Equations

1. If diagram is not given, sketch one.
2. Assign positive direction.
3. Identify quantities given and label in diagram. Use appropriate symbols and subscripts.
4. Identify the unknown to be found.
5. Identify start and end point.
6. Apply appropriate kinematic equation(s).
7. Check that answer is sensible. (i.e. not too big/small, positive/negative).

Worked Example

A car is travelling with uniform acceleration along a straight road. The road has marker posts every 100 m. When the car passes one post, it has a speed of 10 m s^{-1}, and when it passes the next one, its speed is 20 m s^{-1}. Calculate the car's acceleration.



$v^2 = u^2 + 2as$

$20^2 = 10^2 + 2a(100)$

$a = 1.5 \text{ m s}^{-2}$

Example 5

A motorcar, starting from rest, has an acceleration of 2.6 m s^{-2}. After it has travelled a distance of 120 m, it slows down at a rate of 1.5 m s^{-2} until its velocity is 12 m s^{-1}. Calculate the final displacement of the motorcar from the starting point.

E Free Falling Bodies in a Uniform Gravitational Field

- A body is said to be in **free fall** if the only force acting on it is the gravitational pull of the Earth.
- If there is no air resistance, all bodies irrespective of mass, shape or size fall towards the Earth with approximately the same acceleration known as the **acceleration of free fall** or the **acceleration due to gravity, *g***.
- **Acceleration of free fall** is the acceleration of a body towards the surface of Earth when the only force acting on it is its weight.
- The value of g varies from place to place on the Earth's surface and is approximately 9.81 m s^{-2}. As the variation is small, it is often taken to be a constant.
- g = 9.81 m s^{-2} directed downwards (towards the centre of the Earth).

E.1 Equations of motion in free fall

Since g is considered constant, the equations of motion for uniform acceleration can be used.

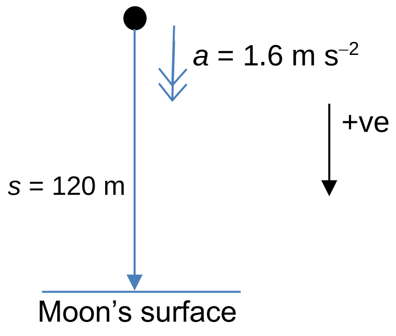
$$\boxed{\begin{aligned} v &= u + at \\ s &= ut + \frac{1}{2}at^2 \\ v^2 &= u^2 + 2as \end{aligned}}$$

In the equations of motion g replaces a.

Note: In problems dealing with free-falling bodies, it is extremely important to take note of the sign convention. Choose a direction as positive and to follow this convention consistently in the substitution of known values in the equations.

Worked Example

A lunar landing module is descending to the moon's surface at a steady velocity of 10.0 m s^{-1}. At a height of 120 m, a small body falls from its landing gear. Taking the moon's gravitational acceleration to be 1.6 m s^{-2}, calculate the speed of the body when it strikes the moon.



Using $v^2 = u^2 + 2as$

$v^2 = (10.0)^2 + 2\,(1.6)(120)$

$v^2 = 484$

$\therefore \quad v = 22.0$ m s^{-1}

The speed the body strikes the moon is 22.0 m s^{-1}

Note: When the object was on the landing gear, it had the same velocity as the landing gear, i.e. 10.0 m s^{-1} downwards. Immediately after losing contact with the landing gear, it would momentarily continue having this velocity because the net force acting on it would not have had sufficient time to alter its velocity.

Example 6

A stone is projected vertically upwards with a velocity of 10.0 m s^{-1} near the edge of a cliff. Neglect air resistance. (Take g = 9.81 m s^{-2}). Calculate

(a) the time taken to reach maximum height, (1.02 s)

(b) the maximum height reached (measured from cliff top), (5.10 m)

(c) the time taken for the stone to reach the ground, which is 20.0 m below the edge of the cliff, (3.28 s)

(d) the speed before hitting the ground. (22.2 m s^{-1})

Check Your Understanding 4

Just before an object hits the ground, its velocity is zero. (True / False)

Additional Notes

E.2 Motion in the Presence of Air Resistance

- In the presence of air resistance, a falling body will not accelerate uniformly. The air resistance opposes the motion of the body.
- Air resistance or viscous force is a function of the speed of the body. It is proportional to speed when speed is low, and is proportional to (speed)^2 when speed is high.
- Consider a body falling in air,

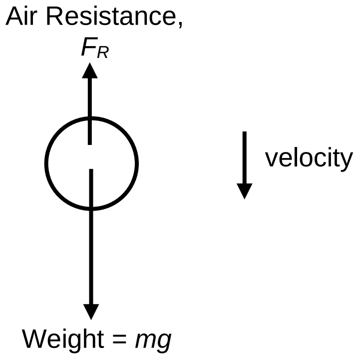
Taking downwards as positive

<u>Downward Motion</u>

$F_{net} = W - F_R$

$ma = mg - F_R$

$$a = g - \frac{F_R}{m}$$



Based on the equation, as $a = g - \frac{F_R}{m}$ as long as $\frac{F_R}{m}$ is larger zero, a will be less than g.

Also, when $\frac{F_R}{m}$ decreases, a becomes closer to g.

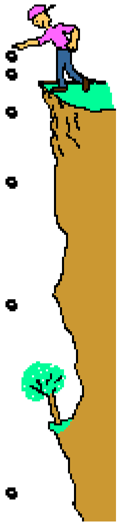
- When a body falls in air, the air resistance opposing its motion increases as its speed rises, thereby reducing its acceleration
- Eventually, air resistance acting upwards equals the weight of the body which acts downwards. The resultant force on the body is then zero (since the two opposing force balance each other) and hence the net acceleration is then zero. The body then falls with a constant velocity, called **terminal velocity**.
- The value of terminal velocity depends on the size, shape and weight of the body.
- A small dense body such as steel ball-bearing has high terminal velocity. A light body such as a raindrop or one with a large surface area such as a parachute has low terminal velocity.

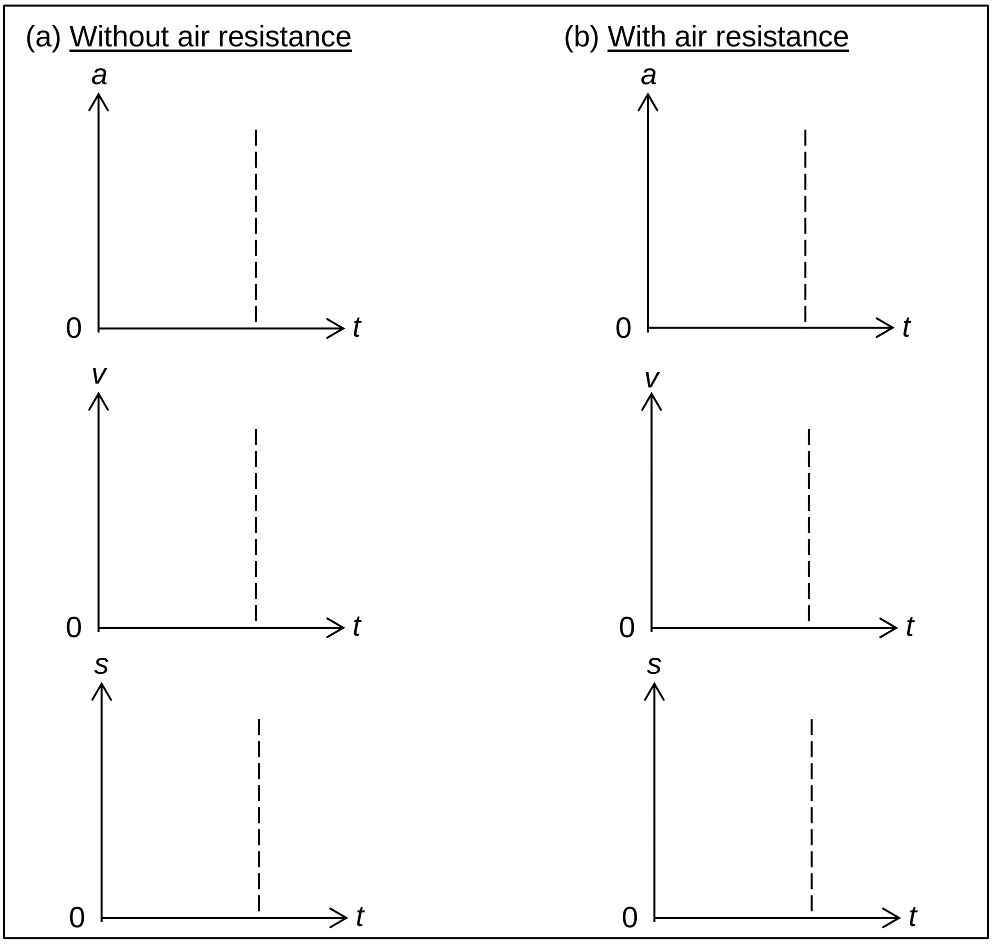
Example 7

Consider a body being dropped from rest with initial velocity u = 0. Taking the point of projection as reference and downwards as the positive direction, sketch the *a–t, v–t and s–t* graphs for motions

(a) without air resistance and

(b) with air resistance

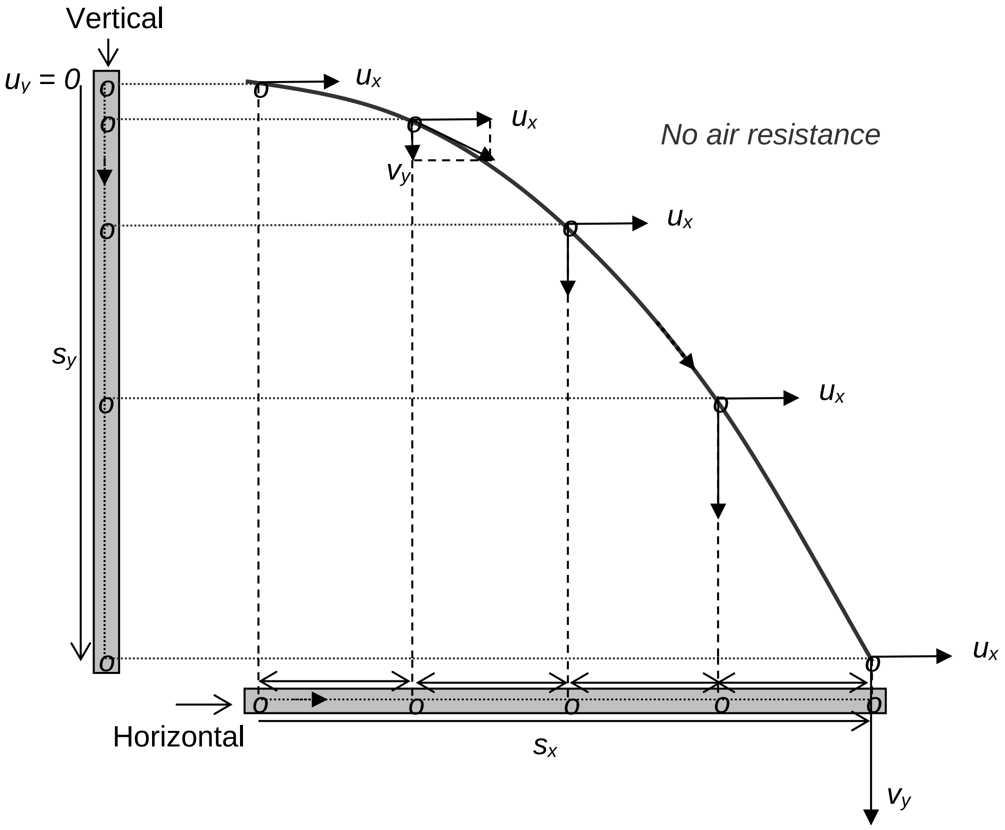




Additional Notes

F Projectile Motion (Non-Rectilinear Motion)

- Projectile motion, which generally involves curves (parabolic) motion in a plane can be analysed in terms of two straight line motions.
- One of these (horizontal motion) is moving with constant horizontal velocity while the other (vertical motion) with constant acceleration.



Note the length of the vertical and horizontal component of the velocities. They represent the magnitude of the velocities throughout the parabolic motion of the body.

- ➢ The horizontal component of the velocity does not interfere with vertical component of velocity because they are perpendicular to each other.
- ➢ The horizontal and vertical components combine to give the instantaneous velocity. The velocity at any instant is always tangential to the path.
- ➢ The *horizontal motion is not influenced by the acceleration due to gravity g* as *g* has no component in the horizontal direction.
  $a_x = 0$
  $v_x = u_x = u$
  $s_x = u_x t$
- ➢ The *vertical motion is a linear motion with constant downward acceleration due to gravity* g, thus it is free fall motion and equations of motion apply.
  $v_y = u_y + a_y t$
  $$s_y = u_y t + \frac{1}{2} a_y t^2$$
  $v_y^2 = u_y^2 + 2a_y s_y$

Range of projectile motion
$$R = \frac{u^2 \sin 2\theta}{g}$$
(no need to memorise)
Max range is achieved when $\theta = 45°$

- ➢ It takes the *same time* to travel s_x as to drop s_y in height.

Time *t* is the variable that links horizontal motion to vertical motion.

Problem solving steps for projectile motion:

Step 1: Draw a diagram

Step 2: Assign positive directions

Step 3: Identify quantities given and label in diagram. Use appropriate symbols and subscripts.
Resolve the initial velocity into horizontal and vertical components

Step 4: Identify the unknown to be found.

Step 5: Apply appropriate kinematic equation(s). (By selecting the direction and looking at the data given for quantities in that direction)

Step 6: Substitute values with appropriate signs into the equations (Take into account the **characteristics of the horizontal and vertical motion** and the **positive direction** assigned)

Example 8

A stone is thrown with a horizontal velocity of 12 m s^{-1} from a building that is 20 m high. Calculate

(a) the time when it hits the ground
(b) the speed just before it hits the ground
(c) the angle that it makes with the horizontal before hitting the ground
(d) the horizontal distance from the building when it hits the ground.

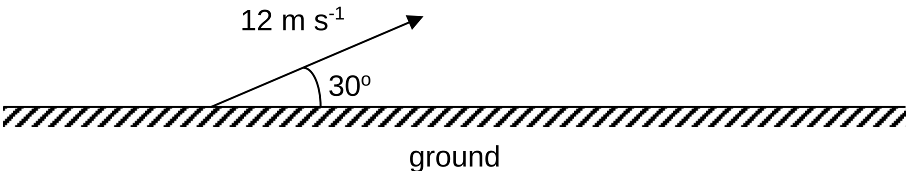
When body is projected horizontally, the initial vertical velocity is zero. i.e. $u_y = 0$.

Take note that final velocity does not refers to velocity after it hits the ground, it refers to the velocity just before it hits the ground.

Additional Notes

Example 9

A stone is thrown from horizontal ground with a velocity of 12 m s^{-1} at an angle of 30^o to the horizontal, as shown in the figure.



(a) Assuming the air resistance can be neglected, calculate

- **(i)** the time before it hits the ground
- **(ii)** the range
- **(iii)** the velocity just before impact
- **(iv)** the time it reaches maximum height
- **(v)** the maximum height that it reaches during its flight.

(b) Sketch the path of the ball,

- **(i)** assuming air resistance is negligible, and label this path N;
- **(ii)** assuming air resistance cannot be neglected, labeling this path A.

(c) Suggest an explanation for any differences between the two paths N and A.

Additional Notes

Video on vertical throw

No air resistance : https://www.youtube.com/watch?v=hMlLNWQjoU0

With air resistance : https://www.youtube.com/watch?v=Sx7kyhaJuuU

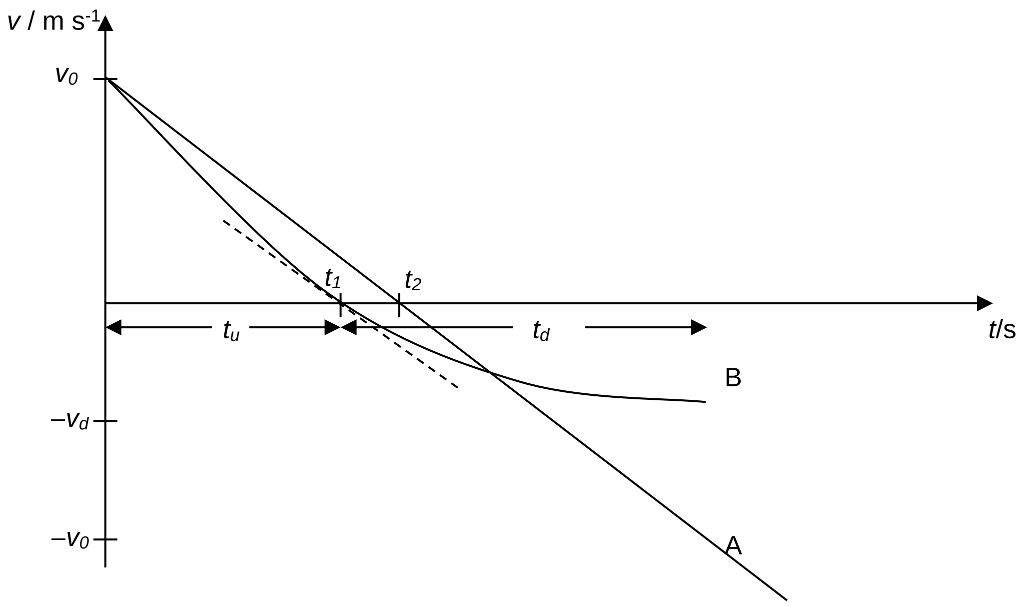
Appendix A: Detailed Comparison of Body Motions with and without air resistance

A ball is thrown vertically upwards with an initial velocity v_0.

(1) Graphical Analysis

The graphs below depicts its velocity of the ball from the point of throwing, until the ball reaches the thrower's hand in the case when

- air resistance is neglected (Graph A)
- air resistance is considered (Graph B)



(2) Detailed Explanation

Taking upward as positive

Without Air Resistance (Graph A)

- Constant negative slope because the only force acting on the ball is gravity (neglect air resistance).
- Net force is the force of gravity, magnitude of acceleration = 9.81m s^{-2}.

With Air Resistance (Graph B)

- There are two forces acting on body.
  - A downward weight.
  - Retarding force, which is downward when body is going up, and upward when body is coming down.
- Resultant acceleration is due to these two forces.

Additional Notes

There are six features to note about the graphs.

1. Slope for graph B during the upward journey is steeper than graph A

- In graph B, force of gravity and drag force act on ball in the same direction.
- Larger net force (since both forces act downwards) acting on the ball in graph B as compared to graph A.
- Graph A has only gravity as its net force.
- Larger acceleration, steeper slope on the velocity-time graph.

Most questions do not test on all features of the graph, rather part of it.

It is important to study all features of the graph, but distil the portions relevant to the question.

2. Slope for graph B during the downward journey is gentler than graph A

- Net force (mg - R) in the downwards direction.
- Acceleration (gradient of graph B) of the ball in graph B would be gentler as compared to graph A.

3. Slope for graph B equals that of graph A at the maximum height

- Drag force is proportional to the speed of the body.
- Ball at instantaneous rest
- No drag force and net force is force of gravity.
- Magnitude of acceleration is 9.81m s^{-2}.

4. Area under graph B for upward journey is less than that of graph A

- Some energy is lost due to friction when the ball travels with air resistance.
- Only part of the kinetic energy is being converted to gravitational potential energy, results in lower height
- Area under *v*-*t* graph represents change in displacement.
- Smaller area in graph B to represent lower height.
- For graph A, neglecting air resistance, all the kinetic energy converted to gravitational potential energy and the ball reaches a greater height.

5. Time to reach maximum height is less than time taken to come down for graph B

- As body is going up, retarding force and weight are in same direction.
- As body is coming down, retarding force and weight are in opposite direction.
- Magnitude of resultant acceleration as body is going up is larger than the magnitude of resultant acceleration as body is coming down.
- Area on the upward journey is equal to the area of the downwards journey, thus causing $t_u < t_d$.

6. For graph B, Magnitude of v_d is less than that for v_0

- Velocity with which the ball leaves the thrower's hand is v_0.
- Velocity of the ball just before it reaches the thrower's hand is $-v_d$.
- Magnitude of v_d is less than that for v_0.
- Throughout the journey, energy is lost as heat due to the air resistance.
- Kinetic energy when the ball returns is less than that when it leaves the hand. Since KE = ½ mv^2, smaller KE means smaller speed.

Summary

Term	Definition	Formula or important concepts
Speed	Speed is the distance travelled divided by the time taken.	average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$
Velocity, v	Velocity of a body is the rate of change of displacement.	area under an a-t graph = change in velocity, Δv gradient of displacement-time graph $= \dfrac{ds}{dt}$ = instantaneous velocity, v average velocity $= \dfrac{\text{total change in displacement}}{\text{total time taken}}$
Acceleration, a	Acceleration of a body is the rate of change of velocity.	a = gradient of v-t graph $a = \dfrac{dv}{dt}$
Equations of motion (for uniform acceleration)		$s = ut + ½ at^2$ $v = u + at$ $v^2 = u^2 + 2as$
Projectile motion	Vertical Direction	$s_y = u_y t + ½ at^2$ $v_y = u_y + at$ $v_y^2 = u_y^2 + 2as_y$
	Horizontal Direction	$s_x = u_x t$